EMPIRICAL REDISTRIBUTION PROCEDURE TO IMPROVE ACCURACY OF LINEAR ELASTIC ANALYSIS OF SHEAR WALLS IN LOAD-BEARING MASONRY

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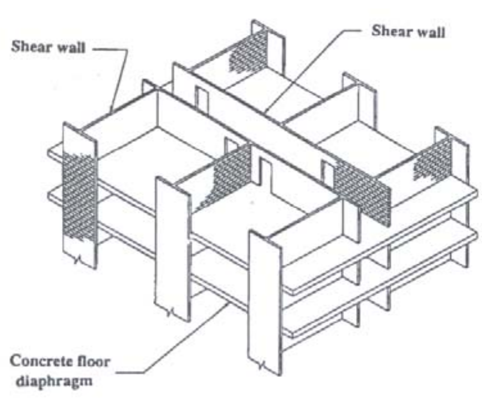
SUMMARY

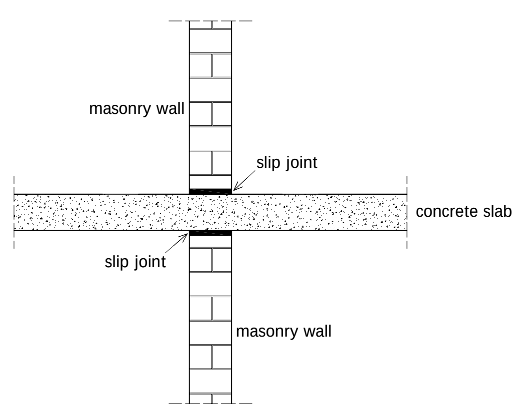
An empirical procedure for more accurate calculation of the shear wall forces has been developed. The improvements in accuracy are achieved by taking into account the level of the design compressive stress on slip joints and the potential of the joint to slip. The procedure has been validated by applying it to calculate shear forces in three different building types and then comparing results to the nonlinear analysis. It appears from this validation study that the proposed empirical procedure has the potential to double the accuracy of the standard method, with is the most popular method in practical design.

INTRODUCTION

A typical load-bearing masonry building (LMB) is composed of masonry walls and concrete floor diaphragms (Figure 1a), with the masonry walls supporting the slabs and acting as the main structural element resisting the lateral loads. In Australia, these buildings incorporate slip joints, which are placed between the walls and floor slabs (Figure 1b), to allow for long term differential movements between the walls and the slabs. This form of construction creates a challenge for earthquake design, as clear load paths need to be established for earthquake forces despite the limited shear capacity of the slip joints whose design has been governed by serviceability requirements.



a) Load-bearing masonry building



b) Slip joint

Figure 1. Typical designs

Current Design Procedures

The major load considered in the in-plane design of an un-reinforced masonry (URM) wall is wind and earthquake induced shear load. Due to the complexities of the response history dynamic analysis, current design procedures for LMB (AS1170.4, 1993) are based on equivalent static analysis. The standard requires that lateral load should be distributed between the shear walls according to their relative stiffness. The stiffness of the shear walls remains constant in the analysis.

Masonry however is a nonlinear and anisotropic material exhibiting softening behaviour, and slip in the slip joints. Both softening of masonry and slip in the joints result in a reduction of the wall stiffness, leading to a potential redistribution of shear loads between the walls. Current design methods, which consider URM as a linear elastic material, are unable to account for this redistribution of shear loads. This may consequently lead to unrealistic shear load predictions resulting in either uneconomical or unsafe design.

The use of more accurate nonlinear analysis methods is not a preferred option in practical design. There are several reasons for this: most designers have limited access to nonlinear design tools, limited computer resources and, most importantly, they have limited experience in using complex design methods and assessing accuracy of numerical results. The linear equivalent static analysis is the method specified by the current Australian standard for shear load calculation and it is most commonly used.

Previous Research in this Area

The numerical study of shear distribution in LMB requires a model that can simulate the behaviour of masonry and slip joints. Masonry is a complex material which exhibits different strengths and strain softening properties along axes parallel and normal to the bed joints. In LMB the masonry behaviour is usually modeled through a macro-model. In the macro-model masonry is treated as a composite and one type of finite element can be used, resulting in better computing efficiency.

Slip joints typically consist of one or two layers of membrane type damp-proof-course material of bitumen coated aluminium or polyethylene. Experiments on slip joints by Totoev et al (2001, 2003), Trajkovski and Totoev (2002), Totoev and Simundic (2005) have shown that their shear behaviour is almost linear elastic until a peak stress is reached followed by a residual shear capacity, which is almost constant. The load at which the slip occurs is also directly related to the level of pre-compression in the joint (Page and Griffith, 1998). This behaviour can be accurately predicted using zero-thickness interface elements and a Mohr-Coulomb yield surface.

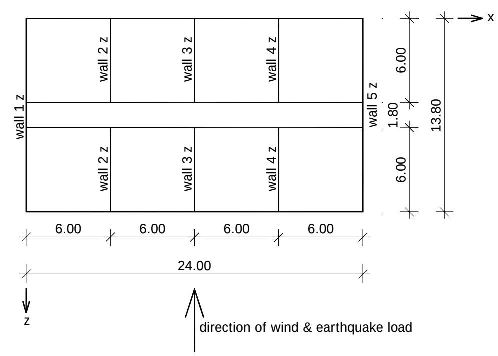
Despite there being extensive knowledge on the behaviour and modelling of masonry and slip joints, there has been little research on the lateral load distribution in LMB taking into account the nonlinear effects of the masonry and slip joints. Some previous research has been carried out by Chen (1999), Sutcliffe and Page (2001), and Sing-Sang et al (2006a and 2006b).

The study by Chen (1999) was of preliminary nature and consisted of numerical analysis of a series of idealised single storey buildings. Slip joints were represented through continuum elements and their behaviour was simulated by a Mohr-Coulomb yield surface. The study concluded that current elastic design procedures can lead to inaccurate predictions of wall load distributions at ultimate load.

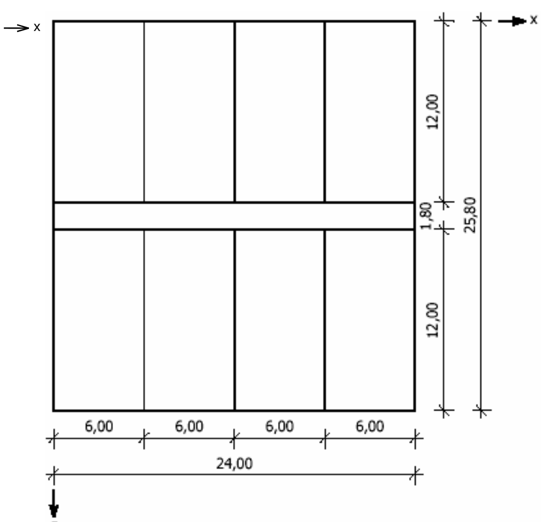
Sutcliffe and Page (2001) investigated the effects of slip joints on the failure load of LMB. Their research was based on the same assumptions as Chen's (1999), however with some improvements in layout of the walls, and the characteristic of the elements used to simulate the slip joints. Their investigation concluded that the application of the current design procedures can lead to an underestimation of wall failure load. However, this investigation was developed only for two buildings, each of one-storey, and with some limitations in the modelling.

Recent Results of Study on Wind and Earthquake Induced Shear Forces in Masonry

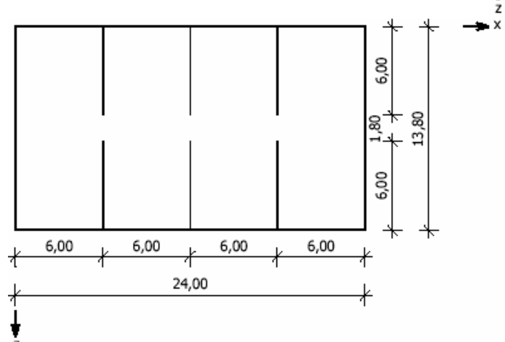
An investigation on the distribution of wind and earthquake induced shear in load-bearing masonry was carried out at the University of Newcastle. This study consisted of numerical analysis performed on twelve idealized four storey "walk up" apartment buildings, six of them symmetrical (Sing-Sang et al, 2007a) and another six asymmetrical (Sing-Sang et al, 2007b). The total number of masonry shear walls considered in this study was 432. The geometry and typology of several investigated buildings is shown in Figure 2.



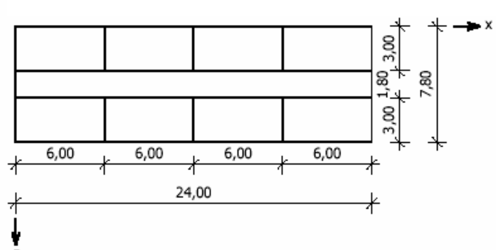
a) Typical floor plan of building S6



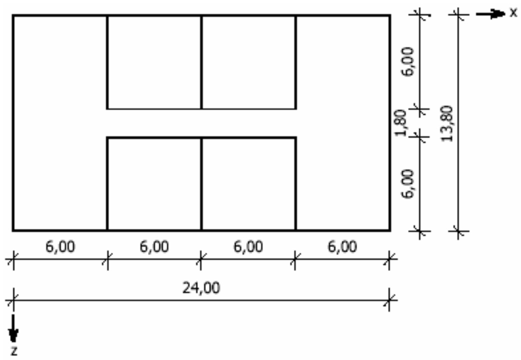
b) Typical floor plan of building S12



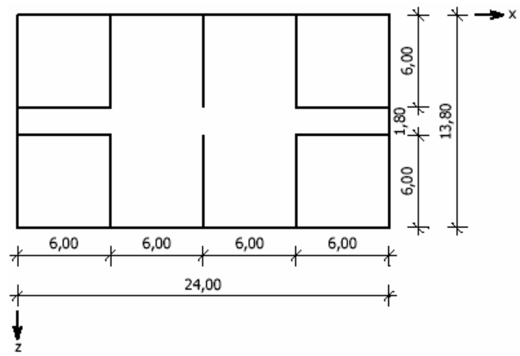
c) Typical floor plan of building SI



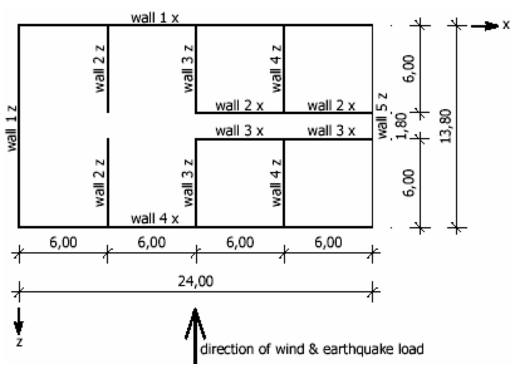
d) Typical floor plan of building S3



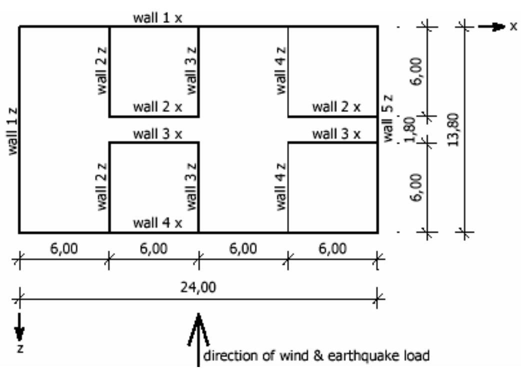
e) Typical floor plan of building SH



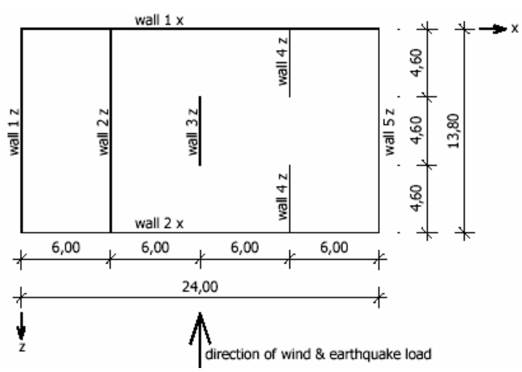
f) Typical floor plan of building ST



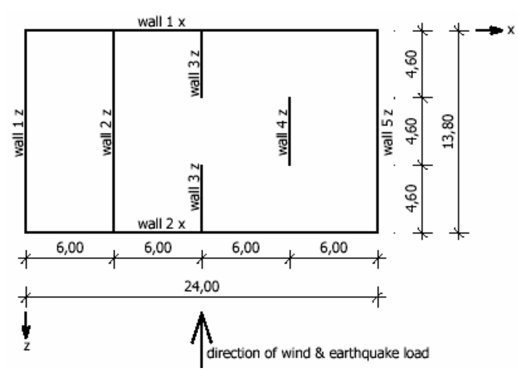
g) Typical floor plan of building C1



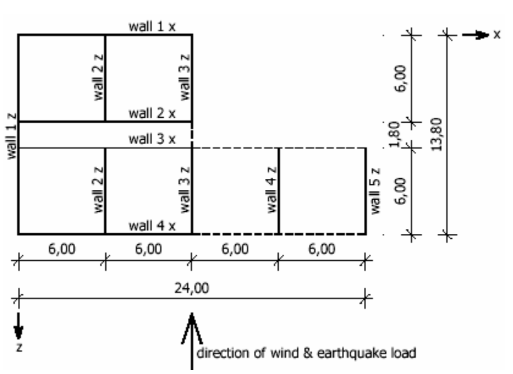
h) Typical floor plan of building H



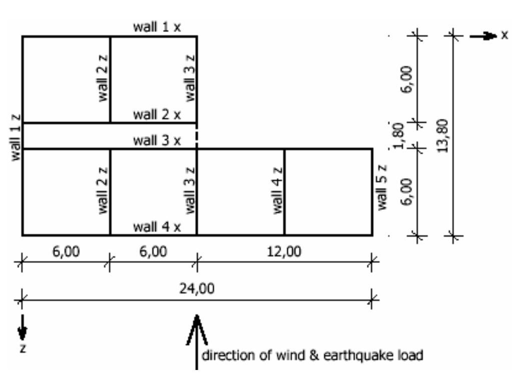
i) Typical floor plan of building C3



j) Typical floor plan of building C4



k) Typical floor plan of building L1



l) Typical floor plan of building L2

Figure 2. Building case studies

All buildings were assumed to be built from solid clay brick walls and reinforced concrete slabs. The slip joints had one layer of embossed polythene. The thicknesses of walls and slabs were 140mm and 200mm, respectively. The buildings were designed in accordance with the current Australian standard procedures. Wind and earthquake actions were defined for building built in Newcastle (Australia) on a soil profile containing 10m of loose sands. Building was classified as "ordinary" for consequences of failure and assumed to contain a large number of people. These assumptions led to an earthquake action equivalent to a base acceleration of 0.11g, and to wind pressures of 1.16 kN/m^2 and 0.97 kN/m^2 on windward and leeward walls respectively. Permanent loads were defined using the volumetric weights of solid clay brick masonry and reinforced concrete of 19kN/m3 and 25kN/m3, respectively. The live load was assumed as a uniform pressure of 2kN/m2 on the floors, as typically required for an apartment building.

Three different methods specified in AS1170.4 (1993) were used to perform numerical analysis of buildings and determine the distribution of shear forces in shear walls:

- Linear equivalent static analysis (LEA);
- Nonlinear static analysis (NSA);
- Nonlinear response history dynamic analysis (NDA).

In the equivalent static analysis both masonry and concrete were assumed as isotropic materials with a linear elastic behaviour. Walls were assumed positively connected to slabs; slip on the joints was impossible.

The nonlinear static analysis considered the anisotropic and strain softening properties of masonry, and the limited shear capacity of the slip joints. Reinforced concrete was assumed as an isotropic material with a linear elastic behaviour. The softening of masonry was simulated by the yield composite criteria proposed by Lourenço (1997), in which masonry compressive and tensile behaviours were modelled by the yield surfaces of Hill and Rankine. The slip in the joints was simulated by the yield surface of Mohr-Coulomb.

The nonlinear dynamic analysis was performed with the same material properties as the nonlinear static analysis. The earthquake action was simulated through a time variation of ground motion acceleration at the level of the building foundation. This procedure is included in the Australian Standard but without specifying any ground motion acceleration. Dynamic loading used for simulation was consistent with the "El Centro 1940 Ground Motion" acceleration (Chopra, 2001).

The numerical simulations were performed with commercial software package DIANA 9.1 (TNO DIANA, 2005), which is based on the finite element method. The Newmark method, with $\beta = 0.25$ and $\gamma = 0.50$, was used to integrate the differential equation of motion. Consistent mass and damping matrices were applied in the analysis. A Rayleigh damping, defined for a building damping ratio of 3%, was considered. This assumption led to damping coefficients of a_o=0.934 and a_1=0.001. A constant time step of 0.02s was employed in the analysis. Within time steps, the Newton-Raphson procedure was applied to consider the nonlinear behaviour of masonry and slip joints. The building mesh was drawn using elements with an average length of 700mm.

The following conclusions (relevant for this type of LMBs) which are important for development of the empirical redistribution procedure have been made from these case studies.

1. The shear load distribution determined by the NSA is conservative overall.
2. The shear load distribution determined by the most realistic NDA correlate well with considerably more simple NSA results. The average differences for the peak shear in the wall were less than 5% and the maximum differences of 9.7% (external walls of building S6 at 2.6s) and -12.8% (wall 3z of building C3 at 1.76s).
3. The shear load distribution determined by the LEA was found unsatisfactory with the average differences to the more realistic NSA of approximately 37% and the maximum differences of 75.1% (wall 5z of building C4) and -62.4% (wall 3z of building C3).
4. Wind load did not cause any softening of masonry and only a minor slip in some joints of one building. Earthquake action also did not cause any softening of masonry but it caused significant slip in the joints followed by considerable redistribution of forces in shear walls.
5. Typically, the sequence of slip was directly related to the level of vertical prestress in the joints. Slip first occurred in less compressed joints, for example, in external walls and walls on the upper floors. In asymmetrical buildings, this behaviour was additionally influenced by the building twist.
6. Apparently, the distribution of earthquake induced load in shear walls was influenced not only by the elastic stiffness and arrangement of the walls, but also by the frictional capacity of slip joints, with depends on their vertical prestress.

Problem Statement

Based on the analysis of current design practices and on the conclusions from the recent study on the distribution of wind and earthquake induced shear in LMB (see previous section), the authors believe that there is a need to develop a simple empirical procedure for use in conjunction with the linear equivalent static analysis to improve its accuracy. It should be based on calculation of the redistribution factors for shear forces determined by the LEA. These redistribution factors should account for the level of precompression in joints and the potential slip in joints stressed beyond their frictional capacity.

REDISTRIBUTION OF SHEAR FORCES

Proposed Design Procedure

The proposed design procedure is based on the most commonly used the equivalent LEA. As in many other empirical methods, load redistribution resulting from nonlinear material behaviour or other nonlinear effects such as sliding of the joints is taken into account by the application of appropriate redistribution factors to the elastic loads. This approach could be a simple alternative to use a nonlinear modelling. In this particular case, loads in shear walls could first be determined using the equivalent LEA, and then made more accurate through the application of redistribution factors that account for the level of vertical compression and the limited frictional capacity of the joints. Thus, the problem is reduced to establishing the appropriate redistribution factor.

The difference Δ between the design shear force V_d in walls calculated by the NSA and the LEA is determined as follows

$$\Delta = \frac{V_d^{LEA} - V_d^{NSA}}{V_d^{NSA}} 100\%. \tag{1}$$

Equation (1) may be rewritten for calculation of the shear load V_d^{NSA}

$$V_d^{NSA} = \frac{1}{1 + \frac{\Delta}{100\%}} V_d^{LEA}, \tag{2}$$

where expression 1/(1+Δ/100%) is the exact value for the redistribution factor C_{RF}

$$V_d^{NSA} = C_{RF} V_d^{LEA}. \tag{3}$$

The exact value of the redistribution factors can be determined only when the NSA is performed. However, it is possible to estimate an approximate value for the redistribution factors $\widetilde{C}_{RF}$ without performing the NSA. This approximate value is based on the average difference $\overline{\Delta}$, the level of the design compressive stress on the joint f_d, and the average compressive stress $\bar{f}_d$

$$\widetilde{C}_{RF} = \frac{1}{1 + \frac{\overline{\Delta}}{100\%}} \times \left(1 + \frac{f_d - \bar{f}_d}{\bar{f}_d} \right). \tag{4}$$

From analysis of 432 shear walls in the previous study it was found that the differences between the shear loads V_d^{NSA} and V_d^{LEA} range from 17.9% to 75.1% and from -0.8% to -62.4%. The positive differences or overestimation of the shear load are registered in walls which are likely to slip on the joint due to the lower level of vertical precompression. On the other hand, the negative differences are associated with underestimation of the shear load in walls which are unlikely to slip due to the higher level of vertical precompression. The average overestimation of the wall shear load by the LEA was 50.3% and the average underestimation was -23.3%. Substitution of these averages will simplify Equation (4) to the form:

$$\widetilde{C}_{RF} = 0.67 \left(1 + \frac{f_d - \bar{f}_{d+}}{\bar{f}_{d+}} \right) \quad \text{for } f_d - \bar{f}_d \le 0, \tag{5}$$

$$\widetilde{C}_{RF} = 1.30 \left(1 + \frac{f_d - \bar{f}_{d-}}{\bar{f}_{d-}} \right) \quad \text{for } f_d - \bar{f}_d > 0; \tag{6}$$

where $\bar{f}_{d+}$ and $\bar{f}_{d-}$ are the average compressive stress in walls with overestimated and underestimated shear load, respectively.

Validation of the Proposed Procedure - Numerical Examples

To validate the proposed empirical procedure for redistribution of shear wall forces determined by the LEA it was applied to three buildings previously researched in the case studies. These were:

- a building with symmetrical arrangement of shear walls - S6;
- a building with relatively uniform distribution of mass but unsymmetrical arrangement of shear walls - C4;
- a building with non-uniform distribution of mass and unsymmetrical arrangement of shear walls - L2.

Results of the validation and comparison to the NSA results are presented in Tables 1 – 3.

Table 1. Application of the redistribution factors to improve accuracy of shear wall forces calculated by the LEA in the building S6

Building S6	Shear walls				
	1z	2z	3z	4z	5z
V_d^{LEA} (kN)	539.0	459.3	463.3	459.3	539.0
f_d (MPa)	0.21	0.31	0.32	0.31	0.21
$\bar{f}_{d+/-}$ (MPa)	0.21	0.315	0.315	0.315	0.21
$\widetilde{C}_{RF}$ (-)	0.67	1.29	1.31	1.29	0.67
$V_d^{corrected}$ (kN)	361.1	592.5	606.9	592.5	361.1
V_d^{NSA} (kN)	367.5	565.7	574.0	565.7	367.5
Δ (%)	46.7	-18.8	-19.3	-18.8	46.7
$\Delta^{corrected}$ (%)	1.7	-4.7	-5.7	-4.7	1.7

Table 2. Application of the redistribution factors to improve accuracy of shear wall forces calculated by the LEA in the building C4

Building S6	Shear walls				
	1z	2z	3z	4z	5z
V_d^{LEA} (kN)	634.0	737.3	337.7	131.9	790.0
f_d (MPa)	0.22	0.47	0.54	0.73	0.26
$\bar{f}_{d+/-}$ (MPa)	0.24	0.58	0.58	0.58	0.24
$\widetilde{C}_{RF}$ (-)	0.61	1.05	1.21	1.64	0.73
$V_d^{corrected}$ (kN)	386.7	774.2	408.6	216.3	576.7
V_d^{NSA} (kN)	413.0	748.5	648.2	350.8	451.2
Δ (%)	53.5	-1.5	-47.9	-62.4	-75.1
$\Delta^{corrected}$ (%)	-6.3	3.4	-36.9	-38.3	27.8

Table 3. Application of the redistribution factors to improve accuracy of shear wall forces calculated by the LEA in the building L2

Building S6	Shear walls				
	1z	2z	3z	4z	5z
V_d^{LEA} (kN)	493.0	448.7	447.7	230.6	197.1
f_d (MPa)	0.21	0.31	0.31	0.32	0.21
$\bar{f}_{d+/-}$ (MPa)	0.21	0.313	0.313	0.313	0.21
$\widetilde{C}_{RF}$ (-)	0.67	1.29	1.29	1.32	0.67
$V_d^{corrected}$ (kN)	330.3	578.8	577.5	304.4	132.1
V_d^{NSA} (kN)	365.2	562.3	451.3	267.8	157.2
Δ (%)	35.0	-20.2	-0.8	-13.9	25.4
$\Delta^{corrected}$ (%)	-9.6	2.9	28.0	13.7	-16

In the three numerical examples considered proposed empirical procedure improved the accuracy of the linear elastic analysis for all shear walls with significant difference between NSA and LEA. The average absolute difference between NSA and LEA results for theses three buildings is 32.4% with standard deviation of 22.1%. The average absolute difference between NSA and proposed empirical method is 13.4% with standard deviation of 13.0%. This means that the shear wall force calculated by the proposed empirical procedure is about two times more accurate than if calculated by the equivalent linear elastic method alone. It is, however, important to note that the proposed procedure developed to improve the shear wall design accuracy and is not suitable for any other purpose because it does not satisfy overall equilibrium.

CONCLUSION

An empirical procedure for more accurate calculation of the design shear wall forces has been developed at the University of Newcastle. It should be used in conjunction with the equivalent elastic analysis specified by Australian design standards. It should not be used for any other purpose rather than design of shear walls. The improvements in accuracy are achieved by taking into account the level of the design compressive stress on slip joints and the potential of the joint to slip. The procedure does not require any additional information about a LBM building apart from the average overestimation and underestimation of shear forces by the traditional method, which are 50.3% and -23.3%, respectively.

The procedure has been validated by applying it to calculate shear forces in three different building types and then comparing results to the proper NSA. It appears from this limited validation study that the proposed empirical procedure has the potential to double the accuracy of the standard design method, with is also the most popular option in practical design.

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