DESIGN PROVISIONS FOR POST-TENSIONED MASONRY WALLS SUBJECT TO LATERAL LOADING

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SUMMARY

This paper reviews the expressions provided by the 2002 and 2005 Masonry Standards Joint Committee (MSJC) design provisions for the calculation of tendon stress at nominal flexural strength of post-tensioned masonry walls. The accuracy of the design provisions is investigated with 127 finite element analyses. This study was motivated by questions that arose regarding accuracy when the MSJC 2005 tendon stress formulas, which were developed for moderately stocky or moderately slender ($10 < h/t < 25$) were applied to either very slender ($h/t > 25$) or very stocky ($h/t < 10$) out-of-plane walls. A new expression for predicting the tendon stress at nominal flexural strength for post-tensioned masonry walls is developed and verified. The proposed equation applies to out-of-plane walls with a broad range of variables including aspect ratio, reinforcement ratio, magnitude of effective prestress, magnitude of external axial load, material properties, tendon restraint conditions, and support conditions.

INTRODUCTION

In strength design of masonry walls, nominal flexural strength is compared to required capacity under anticipated service loads. If the calculated nominal flexural strength exceeds the required strength, the design should ensure safe sectional performance, even if overloaded [Drysdale et al 1999]. Accurate prediction of tensile stress magnitude in the tendons at nominal flexural strength is crucial for accurate estimation of member strength. Although the use of bonded tendons is rare, the tendon stress can be found in this case using analyses that ensure strain compatibility with the surrounding masonry. In the case of unbonded tendons, tendon stress at nominal flexural strength results from wall deformation between the tendon anchorages at the top and bottom of the wall. Only out-of-plane loading of walls with unbonded tendons is discussed in this paper.

The current masonry code provisions in the United States [MSJC 2005] are intended to ensure that post-tensioned masonry sections remain uncracked under service loads and avoid collapse

under ultimate loads. The 2005 edition of the MSJC design provisions provides an updated version of the 2002 tendon stress formula and addresses both unrestrained and restrained tendons [MSJC 2005]. The updated version is based on work by Bean and Schultz [2003] who demonstrated that the 1999 and 2002 MSJC provisions did not always accurately predict the wall flexural strength when unbonded tendons were used, and that the predicted flexural strength could be overly conservative. Based on these comparisons, modifications were made to the ultimate tendon stress and compression stress block depth formulas, and an additional equation estimating the effective depth of an unrestrained tendon at ultimate was proposed. These changes, accepted into the 2005 MSJC Provisions, improved the overall accuracy of flexural strength calculations for out-of-plane walls with stocky or moderately stocky profiles [Bean and Schultz 2003, Bean 2003].

The 2005 Masonry Standards Joint Committee design equation (Equation 1) for the tendon stress at ultimate strength is equivalent to the effective stress after all losses, f_{se} (i.e., creep, shrinkage, elastic shortening, relaxation) plus an additional term dependant on the tendon restraint condition, χ, depth-to-span ratio, d_{eff}/l_p, and the ratio of the ultimate force in the steel, $f_{pu}A_{ps}$, to the ultimate strength in the concrete, $f'_m bd_{eff}$.

$$f_{ps} = f_{se} + \chi\left(\frac{d_{eff}}{l_p}\right)\left[1 - 1.4\frac{f_{pu}A_{ps}}{f'_m bd_{eff}}\right]^n \le f_{py} \qquad (1)$$

This study was motivated by questions that arose regarding accuracy when the MSJC 2005 tendon stress formulas, which were developed for moderately stocky or moderately slender ($10 < h/t < 25$) were applied to either very slender ($h/t > 25$) or very stocky ($h/t < 10$) out-of-plane walls.

DATABASE FOR EQUATION VERIFICATION

In order to study the accuracy of the current MSJC 2005 design provisions, finite element studies were conducted on simply supported walls containing both restrained and unrestrained tendons. Additionally, a few finite element studies on cantilever walls were conducted to investigate the impact of a different support condition on the increase in tendon stress. The finite element modeling procedure was verified using existing experimental data (Bean Popehn 2007).

Finite Element Data Set

A total of 65 simply supported walls with restrained tendons and 42 simply supported walls with unrestrained tendons were modeled using DRAIN-2DX (Prakash, et al. 1993). Twenty finite element studies were also conducted for cantilever walls with restrained tendons, where the walls were loaded out-of-plane with a lateral force applied at the top of the cantilever, as shown in Figure 1. The FE formulation incorporated the effects of material nonlinearity, uncracked regions of the masonry, various load and restraint conditions, and solid and hollow cross-sections.

The single wythe walls investigated had aspect ratios (h/t) between 5 and 50 for the simply supported walls, and between 3 and 25 (h_e/t or kh/t) for the cantilever walls, where the effective

height factor, k, for a cantilever is 2. Three types of steel tendons were considered, namely 105 ksi (724 MPa), 130 ksi (896 MPa) and 234 ksi (1610 MPa) yield strengths with corresponding ultimate strengths of 125 ksi (862 MPa), 150 ksi (1035 MPa), and 270 ksi (1860 MPa), respectively. These steel strengths correspond to high strength threaded bar (125 ksi (862 MPa)), high-strength prestressing bars such as the Dywidag and Williams bars (150 ksi (1035 MPa)), and high-strength strand (270 ksi (1860 MPa)). The unbonded length of steel was assumed to be equal to the wall height in all cases. The reinforcement ratios varied between 0.002 and 0.006. Four masonry compressive strengths were used, where these values represent typical strength ranges of both concrete block and clay brick (i.e., 2000 psi (13.8 MPa), 3,000 psi (20.7 MPa), 4,000 psi (27.6 MPa), and 5,000 psi (34.5 MPa)). In order to understand the influence of masonry compressive strength on tendon stress increase due to bending, a broad range of strengths were considered even though some of these strengths may exceed typical values. Initial tendon stresses varied from $0.1f_{py}$ to $0.8f_{py}$, and the external axial load varied between 0 and 15 kips (66.7 kN)

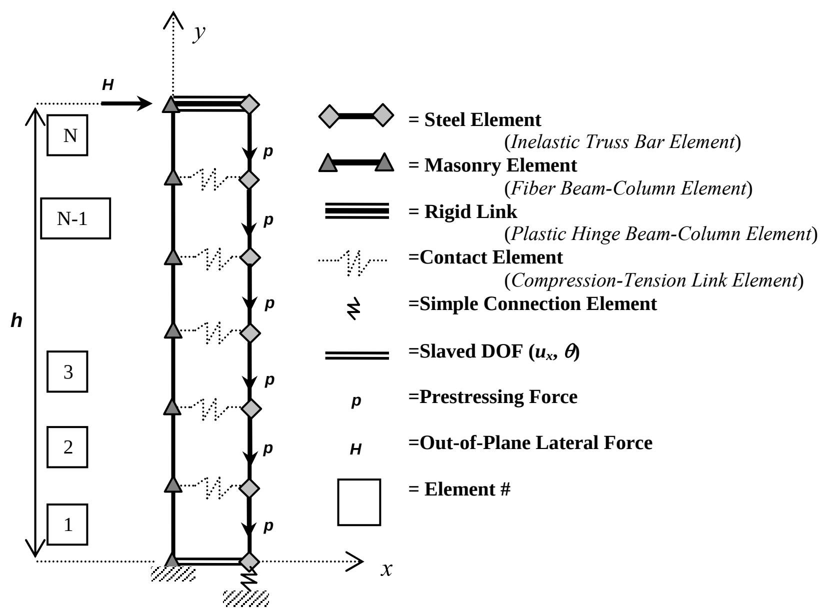


Figure 1. Schematic of DRAIN-2DX Cantilever Model

PREDICTING TENDON STRESS

Using the database of finite element analyses of simply supported and cantilever walls, the accuracy of the 2005 MSJC tendon stress equation was investigated. Additionally, the tendon stress from the finite element analyses was compared to values calculated using the MSJC 2002 formula, (Equation 2 with units of pounds per square inch), where Equation 2 is equivalent to the British expression (BS 5628-2:2000) and differs from the Australian equation (AS 3700-2001) only by the 1.4 coefficient.

$$f_{ps} = f_{se} + 100000\left(\frac{d_{eff}}{l_p}\right)\left[1 - 1.4\frac{f_{pu}A_{ps}}{f'_m\, bd_{eff}}\right] \le f_{py} \tag{2}$$

Investigation into Tendon Stress Equation Accuracy

The finite element studies indicated that the majority of the walls reached a strain on the compression face of the masonry exceeding nominal crushing values (i.e., 0.0025 for concrete block and 0.0035 for clay brick [MSJC 2005]). However, the maximum mid-height lateral deflection of the wall corresponding to the point when the crushing strain was attained varied greatly. For instance, Wall B-10-0.18, a simply-supported stocky wall with an aspect ratio (height-to-thickness) equal to 10, reached a crushing strain and peak moment capacity at a mid-height lateral displacement of 1.05 in. (26.7 mm). On the other hand, Wall B-50-0.18, with an aspect ratio of 50, attained a crushing strain and peak moment capacity at a displacement of 12.3 in. (312 mm). On the basis of this observation, use of the crushing strain as the definition of nominal flexural strength was not deemed rational for all walls, particularly for slender walls.

As an alternative to using the crushing strain, the nominal flexural capacity of the walls in the finite element studies was determined based upon a deflection limit. The procedure for defining nominal flexural strength at deformations consistent with deflection limits is based on the notion that out-of-plane walls are seldom designed to undergo large inelastic deformations (i.e., consistent with the formation of plastic hinges). As such, maximum flexural capacity is defined as the resistance that the wall is likely to exhibit under expected conditions rather than on the resistance it could develop if allowed to deform in an unrestricted manner.

The MSJC provisions [2005] limit the horizontal mid-height deflection of a wall under service lateral and service axial loads to $h_e/143$ (i.e., $0.007h_e$). Since the MSJC Provisions do not address a deflection limit at nominal flexural strength, the deflection limit for service loads was amplified for loads at ultimate (Equation 3), assuming an effective load factor for wind loading (i.e., γ_e = 1.6). Using this limit, flexural strength comparisons of the 2005 MSJC (Equation 1) and 2002 MSJC formulas (Equation 2) were made in the range of wall displacements consistent with MSJC deflection limits.

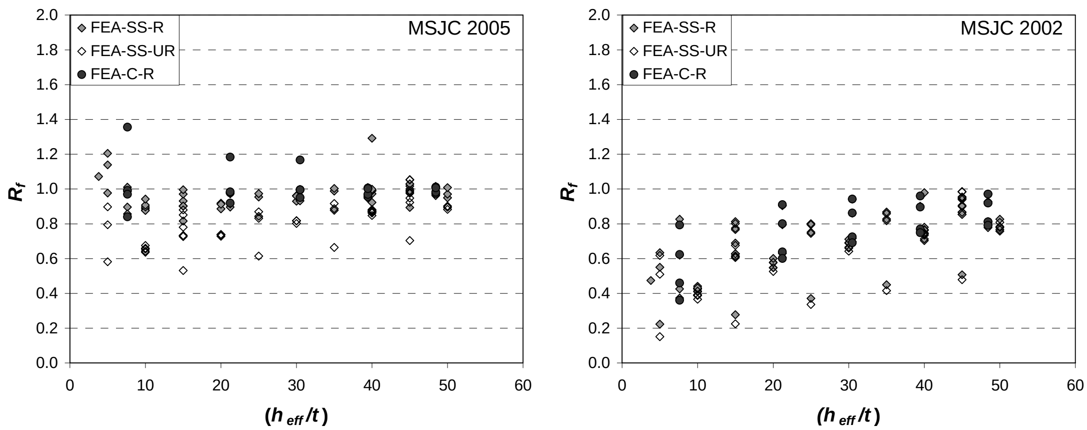
$$\delta_{u,\max} = \gamma_e \delta_{s,\max} = \gamma_e \frac{h_e}{143} = \frac{h_e}{90} \qquad (3)$$

The predicted tendon stress at nominal flexural strength was compared to the results from the finite element analyses, where a ratio of calculated-to-finite element total stress R_f is defined as the ratio of total stress calculated using the code formula ($f_{ps,calc} = f_{se} + \Delta f_{ps,calc}$) to the total stress obtained from the finite element analysis ($f_{ps,fea} = f_{se} + \Delta f_{ps,fea}$). The effective tendon stress, f_{se}, was identical for both of these cases.

The tendon stress formula in the 2005 edition of the MSJC design provisions is generally conservative regardless of the tendon restraint condition and support condition (Figure 2a). The average R_f was 1.01 (COV=0.11), 0.97 (COV=0.08), and 0.79 (COV=0.14) for the cantilever walls (FEA-C-R), simply supported walls with restrained tendons (FEA-SS-R), and simply supported walls with unrestrained tendons (FEA-SS-UR), respectively.

The 2002 MSJC design provisions also provide conservative estimates of the tendon stress at nominal flexural strength (Figure 2b), but it is noted the variance is much higher. The average R_f

was 0.69 (COV=0.27), 0.63 (COV=0.31), and 0.76 (COV=0.22) for FEA-SS-R, FEA-SS-UR, and FEA-C-R, respectively. It is noted here that the MSJC 2005 formula provides a marked improvement over the 2002 MSJC expression through smaller variations and closer predictions.



a) MSJC 2005 b) MSJC 2002

Figure 2. Comparison of Prediction and FE Model Tendon Stress at Nominal Flexural Strength

PROPOSED TENDON STRESS EQUATION

A new expression was developed for calculating the tendon stress at nominal flexural strength for walls subject to out-of-plane loading. This proposed formula is compared to finite element studies and is shown to be more conservative and have less variation over the range of variables in the finite element database than the MSJC 2005 formula.

Development of Proposed Tendon Stress Equation

The change in stress Δf_{ps} for an unbonded tendon is a function of the total change in length, δ_p, of the masonry adjacent to the tendon (Figure 3a). This change in length produces an average change in strain $\Delta\varepsilon_{ps}$ that is constant over the length l_p of the tendon. If the steel is assumed to have a linear, elastic stress-strain (constitutive) relation, then the change in tendon stress can be expressed with Equation 4.

$$\Delta f_{ps} = E_{ps}\Delta\varepsilon_{ps} = \frac{E_{ps}\delta_p}{l_p} \tag{4}$$

The tensile deformation (i.e., extension) δ_p of the masonry adjacent to the tendon, accumulated over the length l_p of the tendon, can be computed from the cumulative compression deformation (i.e., shortening) δ_o of the masonry along the compression face. From proportionality of the deformation distribution in Figure 3a, the change in tendon stress can be expressed (Equation 5).

$$\delta_p = \delta_o\left(\frac{d_{eff} - c}{c}\right) \tag{5}$$

The British formula and the 2005 MSJC expression include an approximation by Phipps [1992] concerning compression shortening of the masonry after an observation made by Pannell [1969] of unbonded tendons in concrete beams. He assumed the compression strain along the length of the member was concentrated over a portion of the length equal to the neutral axis depth *c*, and that the compression strain in this region was equal to the crushing strain ε_{cu} for the concrete. When applied to masonry flexural members, the crushing strain ε_{mu} for masonry is used, and shortening of the compression fiber is given by $\delta_o = \varepsilon_{mu} c$. By defining the compression face shortening in terms of a shortening factor, Ψ, and the neutral axis depth, c, Equation 5 can be expanded to give Equation 6.

$$\delta_p = \varepsilon_m \Psi \left(d_{eff} - c \right) \qquad (6)$$

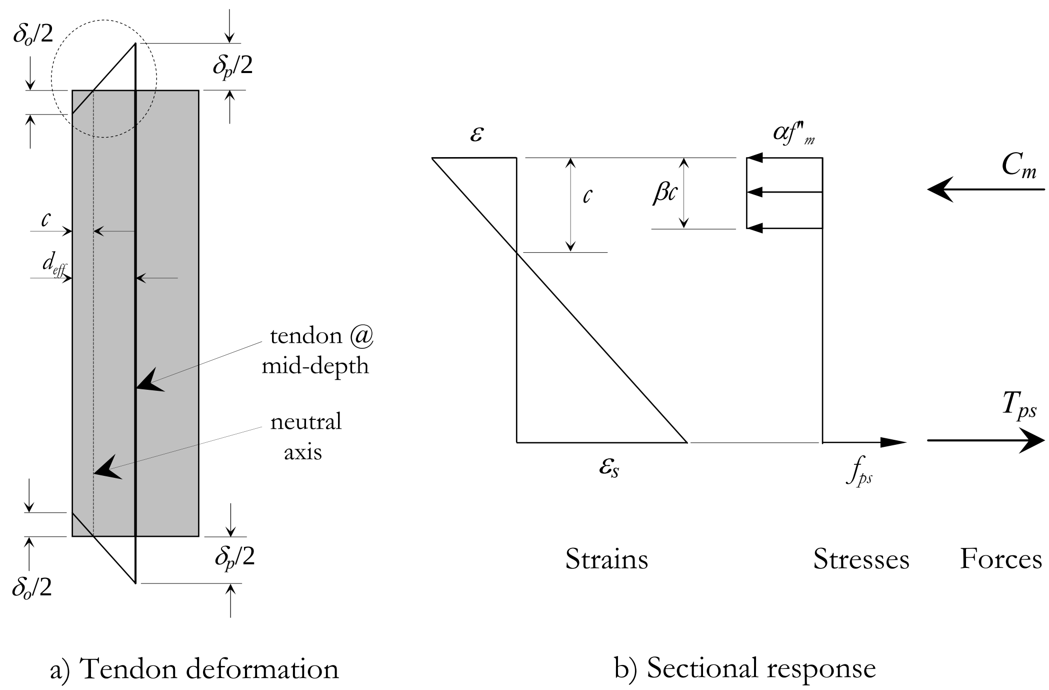


Figure 3. Schematic of Assumed Tendon Deformation and Corresponding Sectional Response

The neutral axis depth can be obtained from equilibrium of internal masonry compression (C_m) and prestressing steel tension (T_{ps}) forces as depicted in Figure 3b. In the British formula and 2005 MJSC equation, the implicit compression stress block parameters for the masonry were α =0.5 and β =1. It is proposed here that the MSJC Code stress block parameters stated in the 2005 Provisions, (i.e., $\alpha = \beta$ = 0.8) be used to define the neutral axis depth.

If Equations 4, 5, and 6 are combined and simplified, the change in tendon stress reduces to Equation 7. Note, external axial loads (including self-weight), P_v, were included in the definition of the neutral axis depth in the last term of Equation 7. External axial loads were not included in British or MSJC 2005 expression because they are not a common occurrence in lightly loaded walls that can benefit from post-tensioning.

$$\Delta f_{ps} = E_{ps}\varepsilon_m \Psi\left(\frac{d_{eff}}{l_p}\right)\left(1 - 1.56\frac{A_{ps}f_{ps} + P_v}{f'_m b d_{eff}}\right) \tag{7}$$

As noted earlier, maximum moment capacity was determined when the maximum deflection reached a specified limit, namely $h_e/90$. Using the finite element database, a trendline was fit to the maximum compression strain at the extreme masonry fiber corresponding to this deflection limit. Using the compression strain and the results from the finite element studies, a trendline was determined for the inferred shortening factor, ψ, where values of the inferred shortening factor varied between 5 and 140 depending upon the wall height, width, masonry compressive strength, and the tensile force in tendons. Combining the results of the masonry compression strain and the shortening factor, the result was a constant value equal to 0.03 [Bean Popehn 2007].

The change in stress Δf_{ps} is the difference between the effective stress after losses f_{se} and the tendon stress at ultimate moment capacity of the member f_{ps}, and this change is invariably an increase (Equation 8). The term f_{ps} appears on both sides of Equation 8. For simplification purposes, the British and the 2005 MSJC equations replaced f_{ps} in the bracketed term with the largest value allowed for tendon stress, namely $0.7f_{pu}$ [Phipps 1992]. In the proposed formula, the numerator of the stress block is left in terms of f_{ps}.

$$f_{ps} = f_{se} + 0.03\left(\frac{E_{ps}d_{eff}}{l_p}\right)\left(1 - 1.56\frac{A_{ps}f_{ps} + P_v}{f'_m b d_{eff}}\right) \tag{8}$$

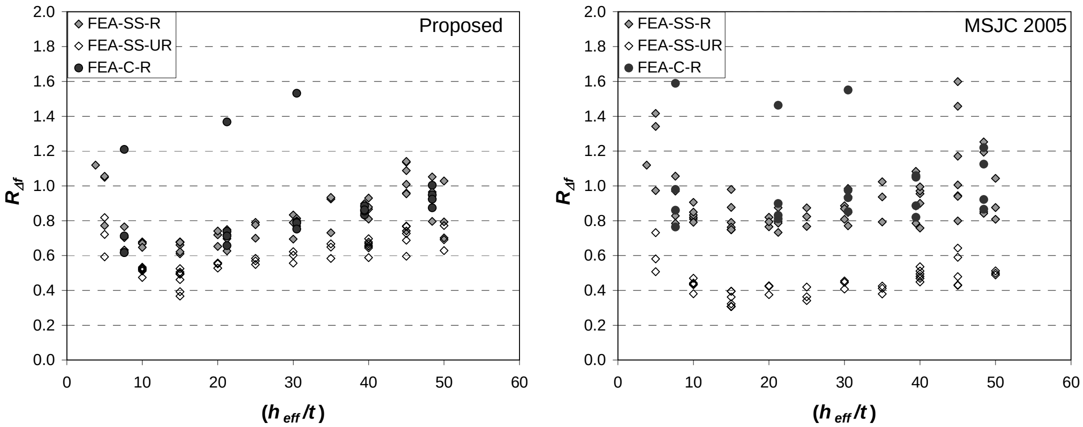
The value of the modulus of elasticity of the steel (i.e., E_{ps} = 29,000 ksi = 200 MPa) may be substituted into Equation 8 for simplification, as was done for the British and 2005 MSJC formulas. However, in the proposed equation, this remains a variable to be supplied by the designer.

Comparisons to the Finite Element Database

The proposed expression (Equation 8) was used to calculate the increase in tendon stress, Δf_{ps}, for the walls in the finite element database, and it was compared to the results from the finite element studies (Figure 4a). For the simply supported walls with restrained tendons (FEA-SS-R), the proposed equation generally provides a conservative approximation to the increase in tendon stress documented in the finite element results. A smaller variation is seen for the proposed formula: the average calculated-to-finite element ratio $R_{\Delta f}$ was 0.88 with a standard deviation on the mean of 0.65 and a coefficient of variation of 0.74 (Figure 4a). These values are an improvement on the MSJC 2005 (i.e., average = 1.02, standard deviation = 0.76, COV = 0.75) shown in Figure 4b.

The MSJC 2005 also provides a conservative estimate of the increase in tendon stress for walls with unrestrained tendons, FEA-SS-UR, in Figure 4b. The average $R_{\Delta f}$ was 0.45 (standard deviation = 0.09 and COV = 0.20). However, using the proposed formula (Figure 4a), a better correlation to the finite element results is noted: the prediction for walls with unrestrained

tendons is still conservative with an average calculated-to-finite element ratio of 0.60 (COV = 0.17).



a) Proposed Equation b) MSJC 2005

Figure 4. Comparison of Prediction and FE Model Increase in Tendon Stress using a) the Proposed and b) MSJC 2005 Expression

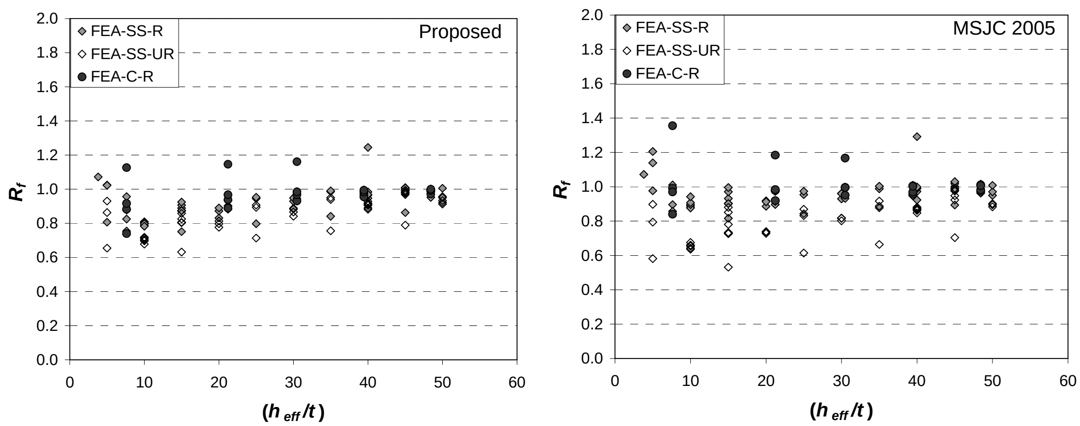
The tendon stress increase predicted for the cantilever walls with restrained tendons using the 2005 MSJC was acceptable (average $R_{\Delta f}$ = 1.02, standard deviation = 0.25, COV = 0.24), as shown in Figure 4b. As seen in this figure, tendon stress increase in three walls was over-predicted. The proposed equation estimated a more conservative increase in tendon stress compared to the 2005 MSJC, and had an average $R_{\Delta f}$ equal to 0.88 with a coefficient of variation of 0.27.

A comparison of the tendon stress at nominal flexural strength was also made using both the proposed formula (Figure 5a) and the MSJC 2005 expression (Figure 5b). Both approaches provide reasonable approximations with only minor differences apparent between the two formulas. This effect occurs because the change in stress, Δf_{ps}, is often a small component of the total tendon stress (f_{ps}), with the effective stress after losses (f_{se}) dominating total stress. The coefficients of variation of 0.08, 0.14, and 0.11 for the MSJC 2005 compare well with the values of 0.09, 0.12, and 0.10 for the proposed expression for FEA-SS-R, FEA-SS-UR, and FEA-C-R, respectively. However, the proposed equation more accurately predicts the tendons stress for stocky walls (h/t < 10).

Some tendon stress estimates, for both the MSJC 2005 and the proposed equation, are unconservative with R_f values above unity. However, using the proposed formula, there are only four data points significantly above an R_f value of unity (i.e., greater than 1.1) out of 127 walls (about 3%). Three of these data points are stocky to moderately stocky (h/t < 30) cantilever walls with a low compressive strength of 2,000 psi (13.8 MPa). The fourth point is a slender wall with a restrained tendon and an external axial load of 5 kip (22.2 kN).

The most notable difference between the MSJC 2005 and the proposed equation is in the comparison of walls with unrestrained tendons. The MSJC 2005 has a lower average R_f ratio,

0.79 compared to 0.84 using the proposed equation, and a higher coefficient of variation, 0.14 compared to 0.12 using the proposed expression.



a) Proposed Equation b) MSJC 2005

Figure 5. Comparison of Prediction and FE Model Tendon Stress at Nominal Flexural Strength

RECOMMENDATIONS

This paper presents an equation for predicting the tendon stress at nominal flexural strength for post-tensioned masonry walls. The equation was used to predict the tendon stress for simply supported and cantilever walls subject to out-of-plane loading. This equation was shown to be generally conservative for a broad range of variables including the aspect ratio, reinforcement ratio, magnitude of effective prestress, external axial loads, material properties, tendon restraint conditions, and support conditions.

The proposed formula offers one expression to predict the tendon stress for restrained and unrestrained tendons, as well as simply-supported and cantilever walls. Assuming that prediction accuracy can be ensured, a single formula for stress in both restrained and unrestrained tendons is preferred because it leads to a simpler, and therefore more reliable, design procedure. The proposed expression was developed based on out-of-plane finite element analyses, the latter which were verified with experimental wall tests. The results from cantilever walls with low aspect ratios may give similar results to a shear wall, where the aspect ratio is also low and the applied loading is in-plane. With further studies, it may be possible to extend the proposed formula for use with in-plane loading.

NOTATION

A_{ps} area of the prestressing steel
b width of the cross-section
c distance from extreme compression fiber to neutral axis
d_{eff} distance from the extreme compression fiber to centroid of tension reinforcement
E_{ps} modulus of elasticity of post-tensioned steel
f'_m specified compressive strength of masonry

f_{se} effective stress in tendon after all prestress losses have occurred
f_{ps} tensile stress in prestressing tendon at nominal flexural strength
f_{pu} ultimate tensile strength of prestressing steel
f_{py} tensile yield strength of prestressing steel
h_e effective wall height
l_p unbonded length of prestressing steel
n exponential term (i.e., 0.5)
P_v external vertical load producing axial compression on the masonry
α parameter used to define the magnitude of an equivalent rectangular stress block
β parameter used to define the width of the equivalent rectangular stress block
χ parameter dependent upon restraint condition (i.e., 700,000 for laterally unrestrained unbonded tendons, 1,000,000 for laterally restrained unbonded tendons)
$\Delta\varepsilon_{ps}$ average change in axial strain of the prestressing steel
$\delta_{s,\max}$ maximum allowable mid-height lateral deflection at service
$\delta_{u,\max}$ maximum allowable mid-height lateral deflection at ultimate
δ_o compressive deformation of the masonry along the compression face
δ_p tensile deformation of the masonry adjacent to the steel
ε_m masonry compression strain
γ_e effective load factor
Ψ compression shortening factor

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