THE FRICTION IN THE OUT-OF-PLANE FAILURE MECHANISMS OF MASONRY WALLS

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SUMMARY

The paper deals with the out-of-plane response of masonry walls. The analysed schemes feature slender masonry piers of exterior walls belonging to factory buildings. The walls are characterized by the presence of a concrete beam at mid height and bear roofs that have different horizontal constraints. Several out-of-plane failure mechanisms are analysed and the horizontal load multiplier is computed through linear and nonlinear kinematic procedures. The dependence of failure mechanisms on roof constraint and the influence of the friction strength between wall and roof on the seismic capacity of walls are provided.

INTRODUCTION

Safety assessment of existing masonry buildings in seismic prone areas requires the analysis of local failure mechanisms in addition to the global seismic analysis. In fact, partial collapses generally due to loss of equilibrium in masonry portions, caused by forces orthogonal to the masonry wall plane involving out-of-plane response, often occur under earthquakes.

As it is well known, the seismic analysis of masonry buildings can be performed by using more than one method whose choice depends on the structural model. Finite element methods are usually used for the modelling of complex building geometries in detail through three-dimensional elements, or through bi-dimensional elements if structural morphology and expected stresses allow the seismic response to be reproduced with accuracy. However, several remarkable difficulties can arise from selection and calibration of the masonry constitutive model. In fact, nonlinear analyses are mainly required to achieve results accounting for the cracking. Another critical modelling issue can be the selection of the internal and external constraints that can not be easy to identify. The above-mentioned difficulties, together with the ones due to the numerical control of nonlinear solutions, imply that finite element analysis cannot be yet considered as a suitable tool in the everyday engineering practice. For those reasons, simplified methods based on some essential assumptions and requiring relatively lower computational demand are improving.

The recent Italian seismic code (OPCM 3431 2005) proposes a displacement-based method to evaluate the ultimate load bearing capacity of masonry walls subjected to orthogonal loads. Specifically, it is advised to evaluate local collapse mechanisms through the equilibrium limit analysis, and kinematic analyses are recommended for assessing the horizontal acceleration that activates the mechanism and for estimating the ultimate displacement capacity. Such an assumption is also based on theoretical and experimental research that confirmed how the

structural capacity of masonry walls subjected to out-of-plane actions should be evaluated by comparing displacement capacity and demand rather than the customary stresses and strengths (Doherty et al. 2002, Griffith et al. 2003). In fact, the damage mechanisms involving specific building parts or components frequently develop as loss of equilibrium of blocks that slide and rotate, instead of exceeding the material strength.

Safety verification with respect to local mechanisms through the kinematic approach is meaningful if the monolithic behaviour of masonry walls is ensured so that local collapse by masonry disintegration is prevented. In that case, it is possible to consider some masonry portions as rigid blocks and the structural capacity under horizontal forces can be computed through the equilibrium limit analysis. The procedure is based on some simple and extreme hypotheses, coincident with the assumptions proposed by Kooharian (Kooharian 1952) and Heyman (Heyman 1966) to analyse masonry block structures using the plastic limit analysis theorems (unlimited compressive strength of blocks, non-tensile strength of masonry and joints, sliding failures not permitted). However, the reliability of results has been proved more than once, even though they have been only recently the object of research. In (Lagomarsino et al. 2004) results obtained by kinematic analyses are compared with the ones supplied by finite element analyses with reference to a transversal section of a church, showing that the two analysis methods lead to similar indications of the seismic vulnerability. Similarly, regarding perimeter walls of factories, in (Guadagnuolo et al. 2007) it is shown that there is a substantially good agreement between the results of nonlinear kinematic analyses and nonlinear finite elements analyses as regards both the probable failure mechanism and the structural capacity. Reliability of kinematic analyses is also demonstrated in (Giovinazzi et al. 2006), where a comparison with results from dynamic analyses is carried out with reference to façades of seismic damaged churches.

Nevertheless, a realistic simulation of the masonry wall behaviour needs to envisage the sliding between blocks taking into account the presence of friction. The inclusion of sliding mechanisms and frictional resistances poses some issues in the use of the standard kinematic analysis, which are dealt with in the paper. Therefore, the paper first provides a concise state-of-the-art aimed at clarifying modelling issues of friction and the dependence of failure mechanisms on wall constraints and friction between wall and roof. Subsequently, the out-of-plane response of masonry walls is analysed through linear and nonlinear kinematic procedures in order to determinate the seismic capacity. The schemes analysed feature slender masonry piers supporting roofs with or without horizontal constraint. Masonry walls of factory buildings having a concrete beam at mid height are considered and several possible failure mechanisms are analysed. The influence and role of frictional forces on the out-of-plane seismic response of walls are provided by comparing the seismic load multiplier obtained with and without friction.

KINEMATIC ANALYSIS

The analysis requires the selection of the local mechanisms considered significant for the building, the identification of rigid bocks defined by possible fracture planes and the evaluation of external and internal forces that act on the blocks. The mechanisms to be examined depend on external constraints, quality of connections between orthogonal walls and between walls and floors, presence of tie-rods and masonry arrangement. The forces to apply are the dead load of the blocks, the vertical loads carried by the blocks, the horizontal forces proportional to the supported vertical loads if not efficiently transmitted to other parts

of the building, the possible external forces (e.g. due to metallic tie-rods) and internal forces (e.g. actions related to interlocking of masonry units).

By assigning a virtual rotation to a generic block, it is possible to determine the block's virtual displacements on the basis of the structural geometry of the analysed masonry portion. The horizontal load multiplier λ that causes the activation of the mechanism is obtained applying the Virtual Work Principle, by equating the total work done by the external forces to that done by the internal forces (linear kinematic analysis). The load multiplier can be related to the maximum soil acceleration in order to perform the safety verification. However, the actual seismic load is a dynamic action and thus the computed acceleration that activates the collapse mechanism is generally lower than the one that really leads to the collapse. For this reason, the framework of OPCM 3431/2005 advises the development of the horizontal load multiplier to be assesses with increasing displacements until the enforceable horizontal seismic forces are equal to zero (nonlinear kinematic analysis). Therefore, a safety verification in terms of displacements is performed by defining the displacement capacity of the structure throughout the evolution of the mechanism. Then the multiplier λ is estimated also on configurations defined by finite rotations of blocks, that is by considering a sequence of virtual finite rotations, applying the Virtual Work Principle for each configuration of the kinematic mechanism and progressively updating the system geometry.

A suitable evaluation of the horizontal load multiplier λ needs to take into account the sliding between blocks considering the presence of friction, even though the procedures are strongly approximated. This is compulsory especially if the absence of sliding or the large underestimation of friction can lead to absolutely inaccurate assessment of the possible failure mechanisms, as is shown below.

FRICTION FORCES BETWEEN BLOCKS

Masonry is often characterised by large dead loads and consequently by large friction forces. Therefore, ignoring crushing, the local mechanics of masonry structures should be studied envisaging the action of friction forces when two rough blocks pass over each other.

The analysis of sliding among blocks is still quite complex if friction is reckoned with, in spite of the several items of theoretical and experimental researches, mainly concerning masonry arches. In fact, the actual behaviour of joints is usually characterized by non-associative friction, i.e. the normal displacement d_n is different from $d_t \cdot \tan\varphi$ (where d_t is the tangential relative displacement between the sliding surfaces and φ the angle of friction), and the bounding theorems of plastic limit analysis do not generally provide unique solutions for the collapse load factor if a non-associative flow rule is assumed.

Usually simple and conservative models of behaviour can be accepted, such as the Coulomb's friction sliding model. Nevertheless, it is well known that in plastic limit analysis the Coulomb's law does not satisfy the normality flow rule. This means that admissible equilibrium conditions could not assure a safe state of the structures.

Dealing with the uniqueness of the solution with reference to frictional materials, Drucker (Drucker 1954) pointed out the difficulty in computing dissipation due to friction in sliding mechanisms, where admissible normal forces often are not known, and he proposed a modified upper-bound condition.

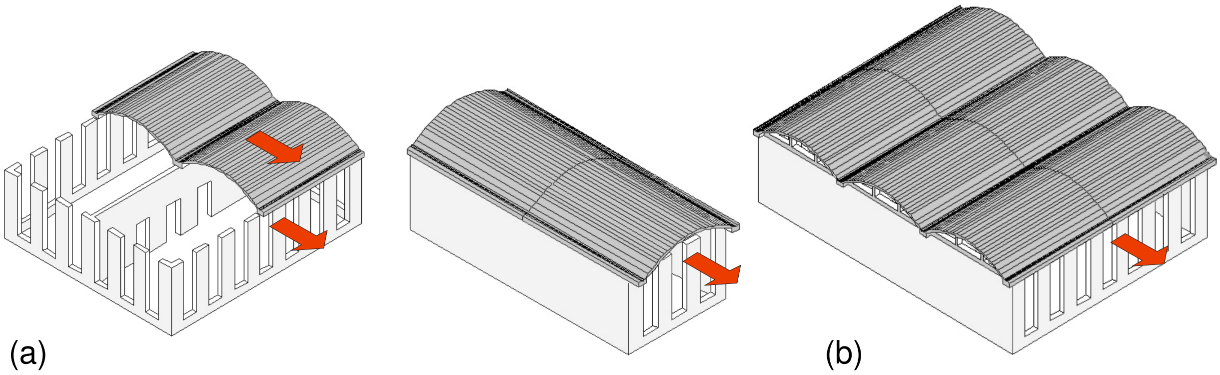

Figure 1.

In the paper, simplified hypotheses in treating Coulomb sliding friction are assumed. The upper-bound condition, having complete attachment (no sliding), the lower-bound condition that assumes zero friction coefficient and a condition assuming predetermined friction coefficient are considered. Furthermore, the dilatancy is assumed to be equal to zero. Generally, if the dilatancy of blocks is zero it is difficult to assess the load factor since the normal force transmitted between blocks can be indeterminate. Nonetheless, supposing that the normal forces are somehow evaluated, the load factor may easily be computed. In fact, an assessment of the friction coefficient through a very simplified approach can be adequate to take into account the sliding among some key contact surfaces in the kinematic analysis of out-of-plane failure mechanisms.

In presence of sliding, the friction coefficient is assumed to be constant along the interface surface so the friction resistance depends only on the resultant of the normal forces acting on the interface and not on their distribution. This assumption agrees with the hypothesis that the resultant of the frictional resistance does not depend on the size of the contact surface but only on the resultant of the normal stresses on the surface.

DEPENDENCE OF OVERTURNING FAILURE MECHANISMS ON WALL CONSTRAINTS AND FRICTION

In the out-of-plane masonry wall response, masses associated with floor gravitational loads can participate or not to the failure mechanisms of piers, since their involvement depends on the building structure geometry and constraints.

With reference to the simple case of Figure 1a, if the roof can translate, the mass due to the roof load is involved in the overturning mechanism of piers and this is usually considered including a horizontal force λV in the model (V being the total dead and live load due to the roof). In such a case (condition C1) the upper points of piers have the same displacement of the roof untill the sliding force at the interface wall-roof is smaller than the friction strength. Therefore, sliding between wall and roof occurs only if the sliding force is larger than the frictional resistance. Frequently it is assumed that the friction strength between roof and masonry wall is very large such as to exclude relative displacements until collapse.

If the roof is kept from horizontal translation by rear constraints, as an example illustrated in Figure 1b, the wall overturning mechanism can only be activated if the friction strength at the interface between the upper side of wall and the roof is exceeded (condition C2). Therefore, in

this case it is necessary to consider the friction issue to analyse possible failure mechanisms, even if by an approximate approach. In fact, disregarding possible mechanisms could lead to overestimating the wall's seismic capacity, whereas neglecting the friction resistance could imply unjustified underestimations. As a matter of fact, the roof could not be subjected to horizontal displacement if it is well connected to the masonry walls that are orthogonal to the one examined and the walls have a large stiffness, or, generally, if the wall under investigation is part of a large building and the roof above such a wall is somehow prevented from translating.

The two aforesaid simple constraint conditions are representative of numerous circumstances so that widespread results can be drawn from the analysis of simple but representative masonry walls, as is described in the following section.

ANALYSED MASONRY WALLS AND MECHANISMS

The out-of-plain response of exterior masonry walls of factory buildings is examined, illustrated in Figure 2. The walls are 24.70 m wide and are characterized by a succession of eight piers 1.60 m wide and vertical window openings 1.70 m wide and almost the total wall height. To cover a representative range of several situations, walls having total height H of 7.50 m and 10.00 m are considered. The wall thickness t is varied between 0.40 m and 0.70 m for the walls 7.50 m high and between 0.55 and 1.00 m for the walls 10.00 m high. In all the walls, the pier vertical continuity is broken by one r/c beam, over the whole wall length and located at a height of 4.00 m in the wall 7.5 m high and of 5.50 m in the wall 10 m high. The intermediate r/c beam has rectangular cross section of 0.25 m high; the width is equal to the wall thickness t. The walls are well bonded, so failures due to masonry separation are prevented. The wall ends are not toothed at their ends and so the possible failure mechanisms do not involve portions of orthogonal masonry walls. As is shown in Figure 2, the walls are subjected to loads from an r/c vault 0.20 m thick, bounded at the bearing by an r/c ring beam and by transversal steel tie-rods, so the thrust is equal to zero.

Four local mechanisms involving simple overturning and vertical bending of piers are envisaged in the paper, as is shown in Figure 3. Obviously, these mechanisms do not cover the totality of the possible ones, since mechanisms that imply horizontal bending and overturning can theoretically be activated also. The first mechanism (MEC-1) relates to the

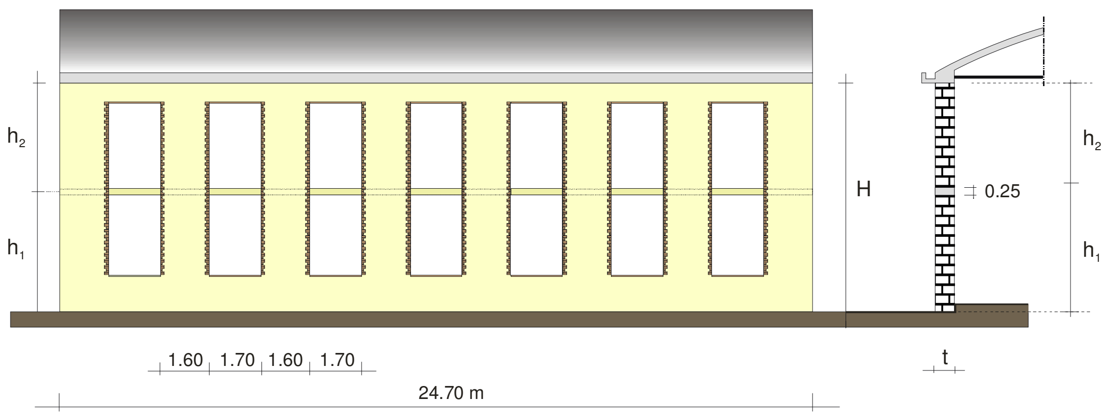

Figure 2.

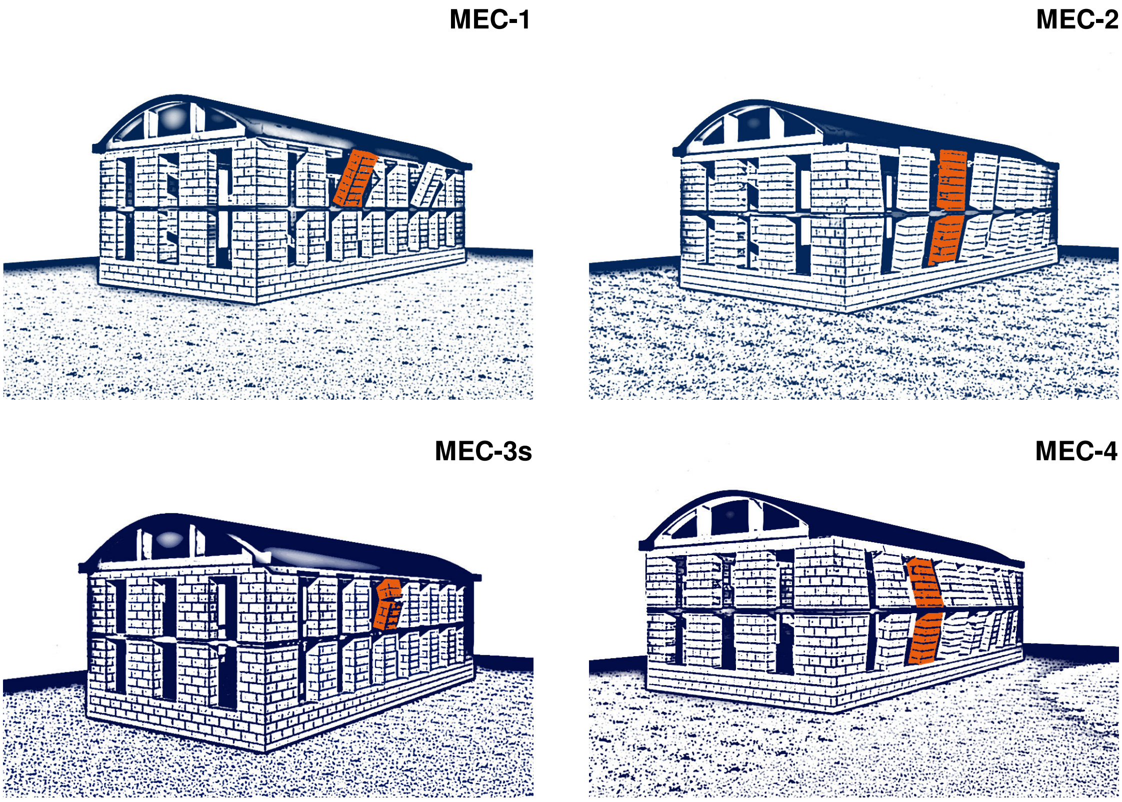

Figure 3.

overturning of the upper portion of masonry piers, assumed to be rigid, with the creation of a cylindrical hinge at intermediate r/c beam height. The MEC-2 mechanism concerns the overturning of the whole pier with respect to the foundation, restrained by the intermediate r/c beam. The other three mechanisms relate to vertical bending with the formation of intermediate horizontal hinges. The hinge location assumed is unknown in the mechanisms MEC-3i (concerning the pier portion below the intermediate r/c beam) and MEC-3s (regarding the above portion of pier), whereas it is known in MEC-4 (at the r/c beam height); consequently, these mechanisms are characterized by the creation of two rigid blocks and then by a composed kinematic chain.

The following forces are considered to act (Figure 4): the block self-weight P_i applied at the centre of each block, the dead and the live load V_2 due to the roof, the horizontal resistant force F_c provided by the intermediate r/c beam, the horizontal forces λP_i and λV_2. Besides, the friction strength at the interface between wall and roof in the roof constraint condition C2 is considered. The forces due to the tie-rods are not considered since they are attached to the upper r/c ring beam; obviously, the thrust force is assumed equal to zero.

The force V_2 is applied at the point C located at 2/3 of the wall thickness t, except in the models relevant to the mechanisms MEC-1 and MEC-2 as the roof is well-restrained (constraint condition C2). In such cases, the force is applied at the internal edge of the wall thickness. The analyses are carried out assuming the force V_2 equal to a percentage of the entire pier self-weight $P=\Sigma P_i$, and specifically to 10%, 20%, 30%, 40% and 50% of P.

An elastic, perfectly-plastic, force-displacement relationship is assumed for the force F_c. The yield and the ultimate forces and displacements characterizing the relationship are computed with reference to a simple one-bay simply-supported beam scheme. The beam has a length

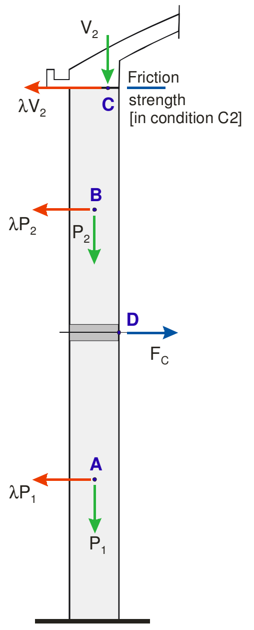

Figure 4.

Wall thickness [m]	d_{cy} [m]	F_{cy} [kN]	d_{cu} [m]
0.40	0.0340	4.83	0.0876
0.55	0.0180	6.20	0.1500
0.70	0.0126	8.95	0.1526
1.00	0.0065	13.09	0.2160

Table 1.

equal to the wall width L and is subjected to a force in correspondence of one central pier. Therefore, the non-validity of the principle of effects superposition in the nonlinear range of behaviour is neglected in the evaluation of the force-displacement relationship. On the basis of these assumptions, the yield displacements d_{cy} and forces F_{cy} given in Table 1 have been computed. The failure displacement d_{cu} (at which the r/c beam restoring force is zero) has been evaluated assuming a failure deformation of 2% for the reinforcement bars; Table 1 contains the computed failure displacements.

LOAD MULTIPLIER AND INFLUENCE OF FRICTION

Figures 5 and 6 show the values of the load multiplier λ versus the ratio of the load V_2 to the pier self-weight P obtained for the four mechanisms examined. Both figures relate to piers belonging to walls having length equal to 24.70 m, height H equal to 10 m, thickness t equal to 0.70 m with the intermediate r/c beam. Specifically, Figure 5 concerns walls in constraint condition C1 while Figure 6 is related to walls below a restrained roof (condition C2). Obviously, values of λ for mechanisms MEC-3 and MEC-4 do not change from condition C1 to condition C2 since the roof constraint is the same. Values of λ for MEC-1 and MEC-2 in condition C2 are computed assuming the friction coefficient μ equal to 0.30, value widely adopted in literature.

From Figure 5 it can be seen that the mechanisms MEC-1 and MEC-2 are characterized by similar and small values of λ, equal to about 0.10; therefore, in the condition C1 such mechanisms are the most likely among the ones examined. Furthermore, an increment in the

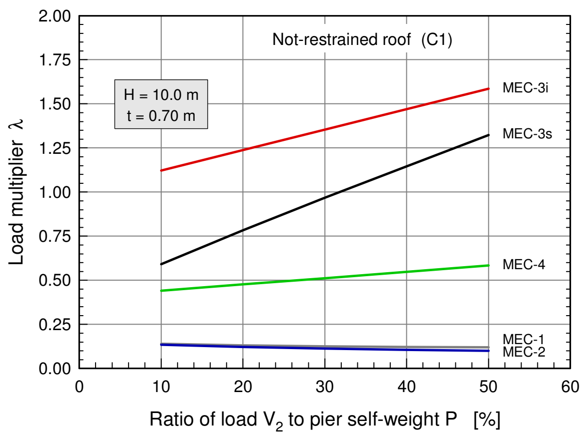

Figure 5.

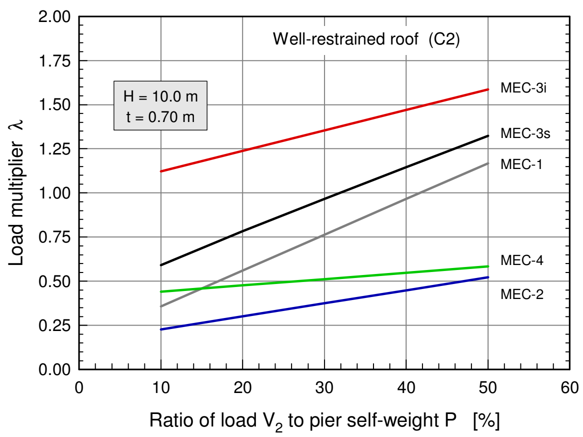

Figure 6.

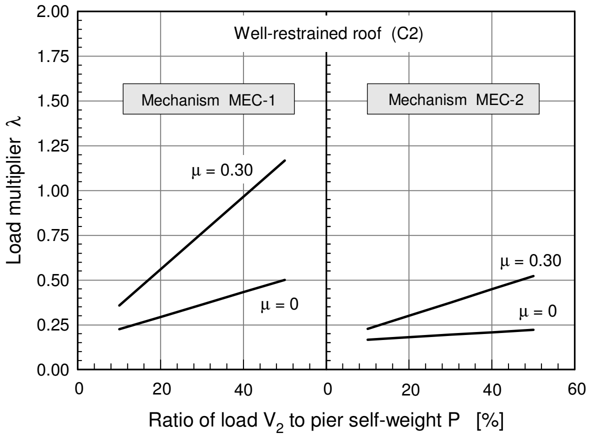

Figure 7

vertical load V_2 does not involve notable benefits in these two failure mechanisms since the corresponding horizontal force λV_2 contemporaneously increases.

If the roof is well-restrained (condition C2), values of the load multiplier λ for the mechanisms MEC-1 and MEC-2 are larger (Figure 6), especially for the mechanism MEC-1, since the overturning force λV_2 is equal to zero. Consequently, in the condition C2, the most probable failure mechanism is the MEC-2 that is characterized by values of λ ranging from 0.25 to 0.50. However, it has to be emphasized that the computed values of λ are affected by approximations, especially in the assumptions relevant to the friction, and thus the closeness of the multiplier value for the mechanisms MEC-2 and MEC-4 must indicate that both of the failure mechanisms can be the possible ones.

Figure 7 compares the load multiplier computed for the mechanisms MEC-1 and MEC-2 assuming the friction coefficient μ at the interface between wall and roof equal to 0.30 and equal to zero. As expected, the figure shows a noteworthy reduction in the failure load multiplier λ in the absence of friction strength (μ=0). If the load V_2 is small (V_2/P=10%), the reduction in λ is equal to about 30% whereas if V_2 is larger (V_2/P=50%) the λ values are more than halved. Therefore, an underestimation of the friction coefficient can lead to the mechanism MEC-1 being one of the likely ones, whereas the actual friction strength implies that it is less probable.

If the roof is well-restrained (constraint condition C2), the lesser benefit due to the increment in load V2 in mechanism MEC-2, with respect to the MEC-1 one, should be counterbalanced by the resistant action of the intermediate r/c beam. However, the analyses performed lead to the conclusion that the influence of the r/c beam stiffness on λ values is generally lesser than that due to the friction strength.

The above results have wide reliability since they are confirmed by varying both the wall height H and thickness t. This is confirmed by Figures 8 and 9 that show values of the horizontal load multiplier λ for piers more slender (H=7.50 m, t=0.40 m) and more squat (H=10.00 m, t=1.00 m) respectively than the ones of Figures 5 and 6.

Finally, the nonlinear kinematic analyses have allowed the displacement capacity of the structure to be assessed throughout the development of the mechanisms, and specifically the evolution of the load multiplier λ, even though more approximately than with respect to the

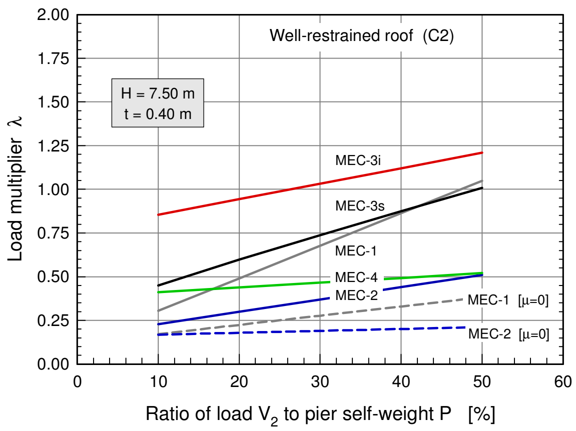

Figure 8.

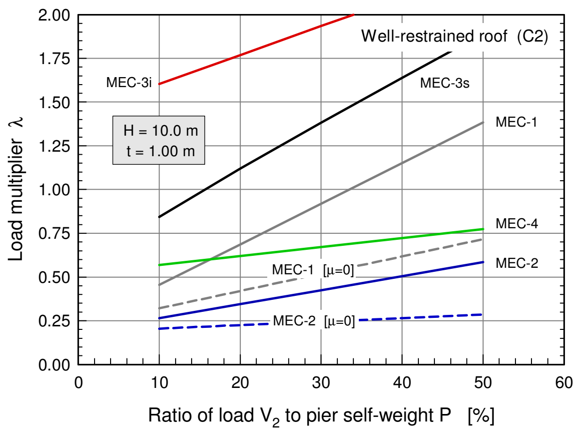

Figure 9

linear kinematic analyses. The analyses performed have shown that, under the constraint condition C1, the sliding forces at the interface wall-roof are always less than the friction strength during the evolution of both the mechanisms MEC-1 and MEC-2. If the roof is assumed well-restrained (condition C2), the horizontal top displacement of piers was always less than the wall thickness, so the roof is constantly carried by the wall.

As an example, Figure 10 shows the results of the nonlinear kinematic analysis representative of the mechanisms MEC-1 and MEC-2 for the wall 10 m high, 0.70 m thick, underneath a well-restrained roof. Specifically, the horizontal load multiplier is plotted on the y-axis and the horizontal displacement of the mass centre is plotted on the x-axis. The main remark that emerges from the figure is the need to perform suitable safety verifications in this case. In fact, the usual procedure, as advised by seismic codes, requires that the verification is performed accounting for the resistant forces that are present until the collapse only. On the contrary, Figure 10 shows the initial decrease in load multiplier λ due to the friction strength being exceeded, which is necessary to activate the mechanism. This implies a discrepancy between verifications based on linear and nonlinear kinematic analyses, which needs further investigation. In fact, the verification performed by linear procedure refers to λ values taking into account the friction strength, larger than the ones that define the curve used in the nonlinear procedure. Finally, Figure 10 shows also the influence of the intermediate r/c beam in the mechanism MEC-2 on the seismic wall capacity.

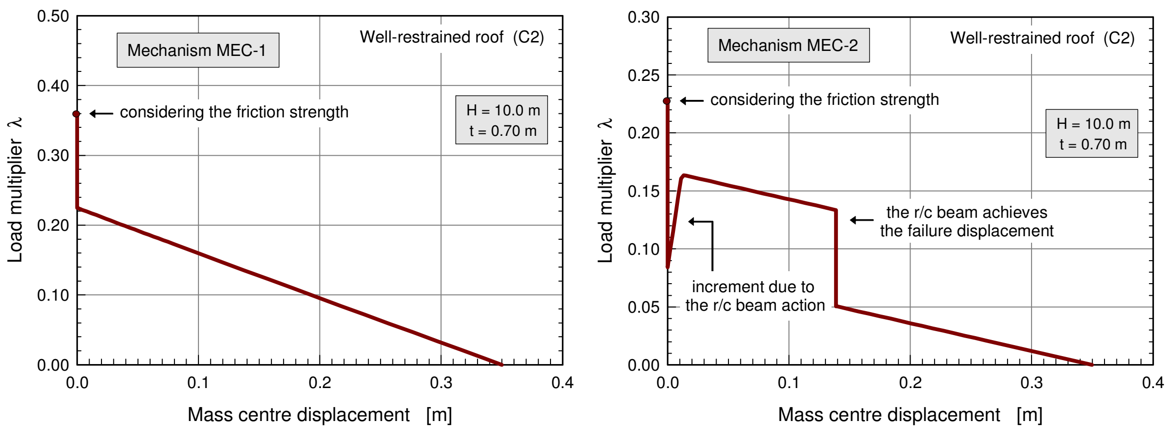

Figure 10.

CONCLUSIONS

The out-of-plane behaviour of several slender masonry piers belonging to factory buildings has been analysed in the paper. The analyses performed have enabled three key issues regarding the expected out-of-plane failure mechanism to be highlighted, specifically the wall and roof constraints, the friction at the interface between wall and roof, the load carried by the wall due to the roof. Results of linear kinematic analyses have shown that an incorrect assessment of the above input data can lead to noteworthy erroneous evaluation of the failure seismic load multiplier. The roof constraints influence the likely failure mechanism of the piers below in relation to the friction coefficient that is assumed at the interface wall-roof. As expected, if the roof is well-restrained, the failure seismic load multiplier can be more than halved as the friction coefficient vanishes. Increments in the wall load from the roof of up to fifty per cent of the pier self-weight imply that the failure seismic load multiplier can double. Finally, the discrepancy between verifications based on results from linear and nonlinear kinematic analyses is underlined if the friction strength between wall and roof is considered in the model.

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