LOAD DISTRIBUTION AND STABILITY IN MASONRY BRIDGES, VAULTS AND DOMES.

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SUMMARY

Established analyses of masonry arch bridges use a two dimensional model. The loading on the model is computed by distributiong live loads to an effective strip using simple rules. This model is clearly wrong in concept and produces unreliable results.

A new distribution model is described which allows the bridge to be considered as a whole. It also allows consideration of skew bridges and the application of complex load patterns. The new method uses Boussinesq formulae for two dimensional distribution of a point load and applies it both longitudinally through the fill and transversely through the fill and the arch.

An outline is presented of how the method might be used to explore the effects of localised faults in bridges.

PROBLEMS WITH ANALYSIS

Arch bridge analysis is fraught with considerably more difficulty than may at first appear. Masonry bridges are complex structures in which the various elements are all, in some way, continua. The arch barrel itself has, since at least the time of Hooke and Wren, been considered as a two dimensional rib whereas it is, for many purposes, better represented as a membrane in three dimensional space. The edges of the bridge support, and are stiffened by spandrel walls which are in some senses two dimensional but occupy a vertical plane. Above the arch and between the walls is often filled with soil which is a three dimensional continuum. The fill also has a complex interaction with both the walls and the arch.

These problems are compounded by the fact that the loads are typically wheels which therefore move. They can be applied at any point on the upper surface of the bridge and may not be set square to the abutments, especially in a skew bridge.

Most of these issues have long been assumed unimportant, on the grounds that the bridges exhibit adequate strength and the analyses (or pseudo analyses such as the MEXE method, (Highways Agency 2004) confirm adequate capacity. In recent years there has been evidence of bridges failing at loads much lower than those predicted by traditional analysis. A major aspect of this seems to be the distribution models which are the central issue of this paper.

Another long established tradition is to use ultimate capacity as a surrogate for serviceability limits, simply by using an enhanced factor of safety. The damaged bridges referred to above, fail progressively, beginning with breakdown of mortar joints and progressing into general articulation and wear with stones progressively descending. There is no doubt that this process will eventually lead to failure but it is not a failure which can be identified by traditional approaches to analysis. More will be said about this later.

WHAT IS DISTRIBUTION

Distribution means many things in engineering and the particular application here must be defined. In fact what has been treated as a single as a single issue till now requires at least two levels of thinking before progress can be made. In many examples of analysis of continuous structures under concentrated loads, it is normal to use an effective width model to reduce the problem to two dimensions. This is true, for example of a concentrated load on a cantilever slab where the moment is assumed to be distributed along a certain length of the cantilever root.

In arch bridges, the concentrated loads are dealt with by a complex set of rules which first define and effective strip of arch to carry a particular load and then distribute the pressure induced by the load over part of the span. Both these processes reduce the peak bending moment.

The effective strip model is even less appropriate for an arch than it is for a slab. Across the width, the concentrated patch load under a wheel will spread a little through the fill but will be seen by the arch as still concentrated. At the arch, it will be resisted by a live load thrust which must also be concentrated at the load point but which will distribute as it flows through the arch towards the abutment. An exact analogy is the design assumptions for a heavy load and pad-stone in a wall. Stress is concentrated under the pad but distributes over ever greater width as it flows down through the wall

LOAD AND THRUST DISTRIBUTION

In the established models, transverse and longitudinal distribution through the soil are treated separately. Indeed, since the whole of distribution , through the fill and through the structure, is lumped into a single model. The transverse distribution may be taken to be intended to include effects in the arch itself. Two schemes are in common use, one for Highways (Highways Agency 2004) and one for Railways (Network Rail 2006). Both use a uniform (rectangular) distribution at the arch extrados. In the longitudinal (span-wise) direction, both rules use a side slope of 1 horizontal to 2 vertical so that the patch width is equal to the surface patch plus the depth to the arch. In the transverse direction, the Highway rules use the same side slope but adds 1.5m to represent the distribution at the upper surface and the distribution in the arch itself. On the railways, the surface patch is increased at a rate of twice the depth. The two results coincide at 0.7m depth (Figure 1).

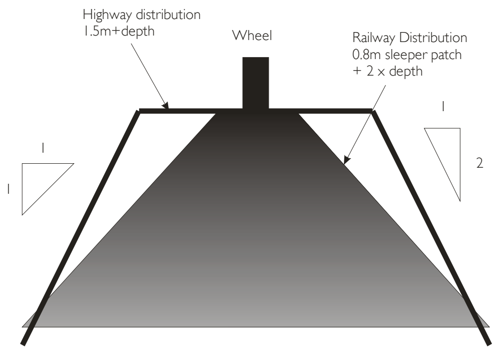

Figure 1 Two established transverse distribution models

An Alternative Model for Transverse Distribution

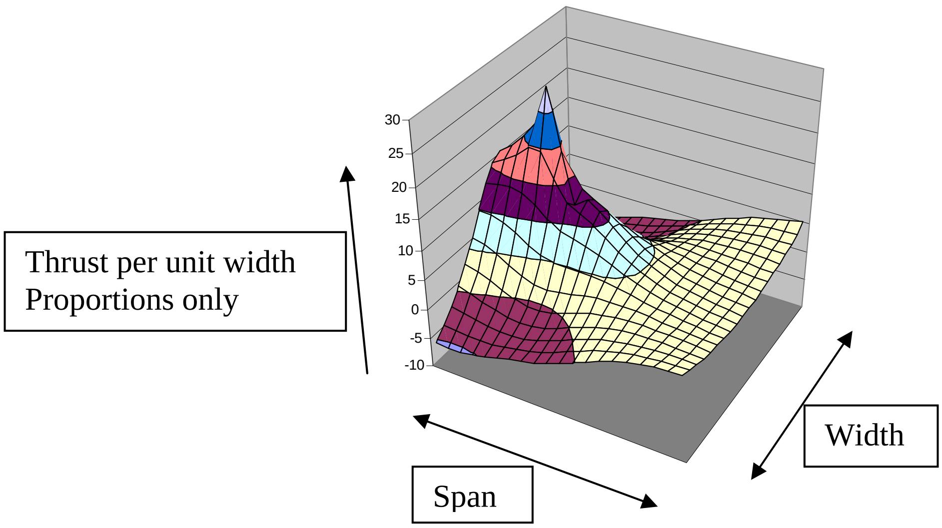

Figure 2 FE analysis for thrust due to patch load. Arch spanning from top left to bottom right.

Tests and simple analysis have indicated that a substantial amount of distribution takes place in the arch as the force flows from the load point towards the abutment. A simple fan delivering a rectangular force pattern may seem to be suitable, but such models create considerable difficulties in coding for a computer and are, in any case, far from realistic. Boussinesq (1885) developed a formulation for distribution of stress in an elastic medium (Equation 1,2 & 3), in particular an elastic half space.

$$\sigma_z = \frac{2Q.z^3}{\pi(x^2+z^2)^2} \quad \text{Equation 1}$$

$$\sigma_x = \frac{2Q.xz^2}{\pi(x^2+z^2)^2} \quad \text{Equation 2}$$

$$\tau_{xz} = \frac{2Q.x^2 z}{\pi(x^2+z^2)^2} \quad \text{Equation 3}$$

Where:

z is the distance from the load to the arch and round to the section concerned

x is the transverse offset as shown

and Q is the applied load.

In the context of the arch barrel, we may consider two half spaces back to back with a concentrated thrust at the interface which then distributes in each direction as it flows towards the abutments. This is essentially a two dimensional phenomenon.

In fact the pattern is more complex yet because there is also a measure of distribution between the surface and the arch. The patch load applied to the arch and the corresponding thrust is not rectangular but bell shaped. A reasonable correlation with experimental results is found by using the Boussinesq distribution in the fill as well. Effectively, this means using a "depth" in the Boussinesq formula measured from the surface to the arch extrados and then round the curve of the arch to the point of interest (Figure 4).

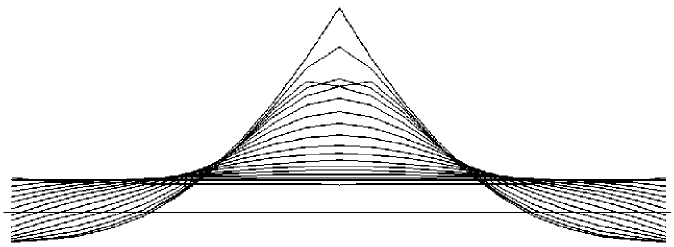

Figure 3 Distribution of thrust progressing from load to abutment
This is a series of sections from Fig 2

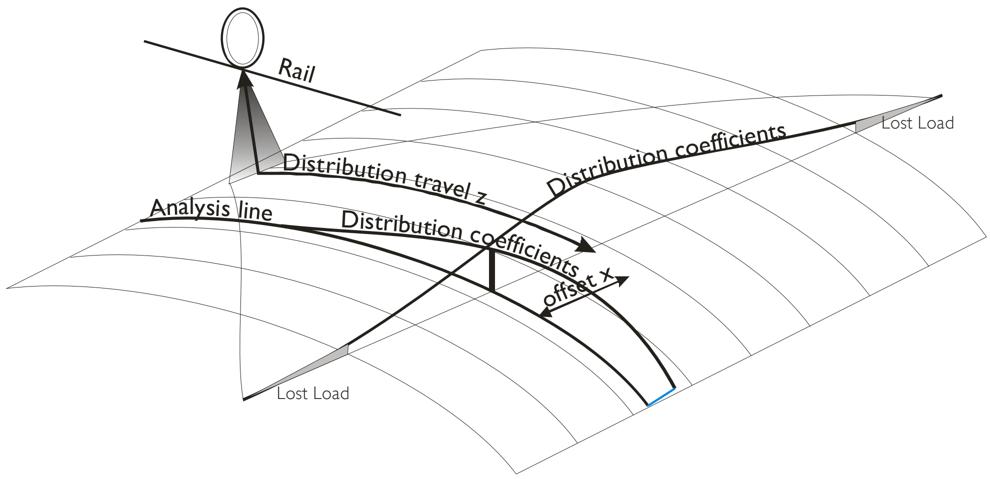

Figure 4 Transverse distribution from the surface and round the arch

Because the Boussinesq distribution actually deals with a half space, and the width of a bridge is limited, it is necessary to undertake scaling to recover the lost load (Figure 4). This can be done by computing the load lost off a particular side of the bridge and restoring it as a triangular distribution tapering to nothing at the opposite edge. Accounting for the two edges adds a trapezium distribution.

It may seem appropriate to use the formula for distribution of a patch load, but this is unduly complex and offers no real benefit. A very close approximation to the result can be obtained using the line load formula with a slight additional travel. Setting the point load 150mm above the level at which the patch applies seems to produce a sensible result.

An Alternative Model for Longitudinal Distribution

Having adopted the Boussinesq distribution for transverse action, it seems appropraie to use something similar longitudinally. Here, though, the issues are more complex. All the distribution takes place in the fill and the fill is of varying depth. Where it might be possible to use the simple, semi-infinite half space rule for transverse distribution to a flat face, in the longitudinal direction the effect will be quite inadequate.

Distribution far from the centre line of the load remains very small for modest depths. However, there are three components of pressure applied and these cannot simply be used to act on the arch. These are vertical, horizontal and shear stresses. It is necessary to take the three components and rotate the axes to radial and tangential. The radial and shear stresses are then acting on the arch and the tangential stress in the soil is self equilibrating and therefore of no significance in the current work.

SKEW BRIDGES

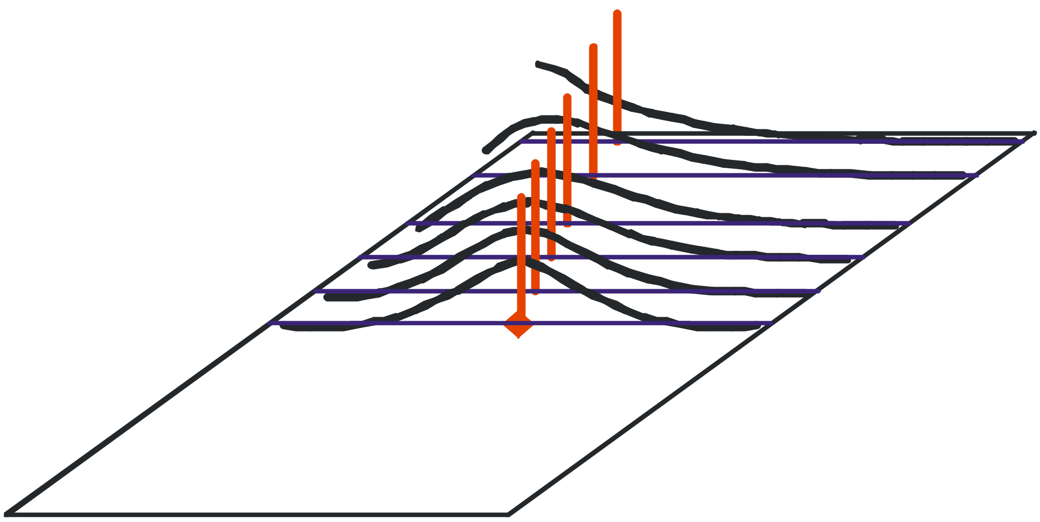

Figure 5 Distribution of live load thrust in half a skew bridge from FE analysis. Area under the curve is constant, Red bar shows centroid

The behaviour of skew bridges has always been a major issue in arch analysis. A simple finite element analysis was carried out using Oasys GSA and modelling the arch as elastic. This suggested that the distribution pattern ran on the square direction, but that thrust in the arch was then modified by the cut edges (Fig 5).

In Figure 5 the distribution of thrust is represented by a line which begins as a symmetrical bell with slightly down turned edges. The bell moves very slowly to the right as the thrust follows the skew. At the abutment, the result is a straight line (triangular) distribution, the bell element having run off the edge of the bridge

SPREADSHEET MODEL

A spreadsheet was constructed to explore the implications of this new distribution model. Compared to formal programming, spreadsheets are slow and somewhat limited, but for prototyping such software they are very powerful. Constructing the spreadsheet required the same processes as programming. An engineering model for the system was developed. A mathematical description of that model was set out. A scheme for handling the mathematics was described. A user interface was designed. The work was then constructed progressively. At every stage suitable testing schemes were incorporated to ensure that the outcomes were as anticipated.

A vital aspect of any such model is the user interface. The program must be capable of informing the user of the outcomes effectively and allowing him to respond appropriately. For visualisation, the aim was to construct a Zone of Thrust diagram (Harvey 1991) as the primary display of outcome. A very limited degree of automation was envisaged with the user required to steer the thrust through the structure. Conveniently, Excel provides a broad range of user input tools ideally suited to such applications.

The Boussinesq models described above provided the description of forces applied to the arch. These were interpreted as local values from the distribution as indicated in Figure 4. To analyse a complete bridge, facilities were provided for eight live loads, though initial development was completed with one. The whole live load calculation was completed on a single worksheet so that further loads could be added by the simple expedient of duplicating sheets.

The arch was divided in to 40 segments round the ring and analysis was possible at 40 long sections. Dead load forces were constant across the width, but each live load generated a different pressure regime.

Once the loading was computed, an initial thrust line was found for the dead loads and also, separately, for the live loads. Distributions were applied to the live load before the two were combined. The user was then able to adjust the reactions at one end of the bridge to achieve a suitable flow of thrust at a particular section, and then to pan across the width of the bridge to explore behaviour elsewhere.

EXAMPLE

A skew arch bridge was tested in April 2006. The results of this test fed into the construction of this model and it seems appropriate that an analysis of the bridge provide an example of use of the model.

Figure 6 A skew arch bridge

The bridge was segmental on the skew span, with a skew span of 12.19m and rise of 1.53m The square width was 12m and the ring thickness 500mm. There was 200mm of cover at the crown and the road surface was horizontal. There was room on the bridge to position 2 vehicles side by side and axle loads of 11 tonnes were considered with a load factor of 1.9 as specified in BD21 (Highways Agency 2004).

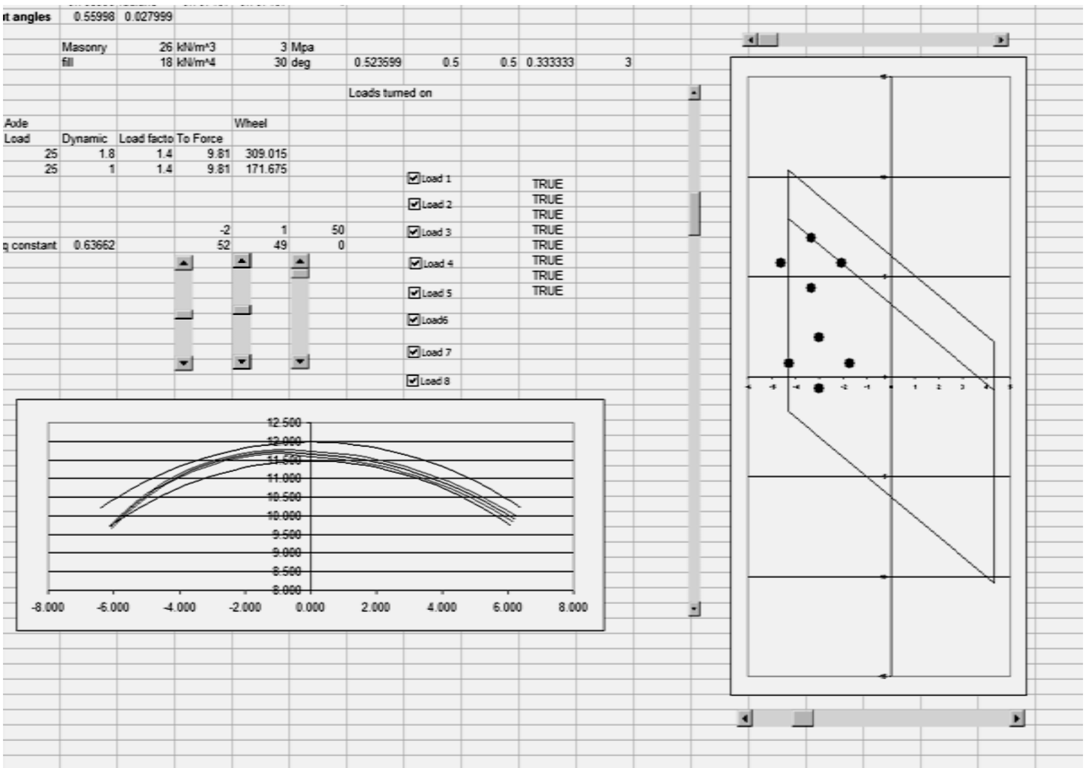

Figure 7 Working page of spreadsheet

The results of this analysis allow a measure of understanding of the differences between skew and square arches. The skew arch has a particularly shallow curve and as such is much less susceptible to mechanism type failure. There is, however, a concentration of stress at the load points which is not shown in other models and this may (in the limit) lead to local crushing or snap through or, more particularly, to very modest but irreversible local damage. Such damage is progressive and can lead to a bridge becoming unserviceable long before it reaches

failure. What is lacking, however, is a model for such failure which will allow a link between a new model for behaviour and a new assessment.

IMPLICATIONS FOR OTHER STRUCTURES

The flow of force in skew arches described here is particularly relevant when thinking about vaults and domes. Indeed, vaulted structures are also frequently encountered on railways, particularly in the under-croft of stations. Analysing such structures is a complex issue, especially when they have already suffered some damage. There is a long standing belief that force in vaults flows down the steepest slope to the rib and then down the rib to the support. A little thought will show that this cannot be the case because the rib would have to be extremely stiff before it could provide a stiffer load path than direct flow down the vault web.

Arch viaducts have proved to be difficult subjects for assessment. The effective strip models used for single spans grossly under predict capacity of multi span bridges. The distribution model described will allow a user to demonstrate much increased capacity in viaducts.

FUTURE DEVELOPMENTS

Bridges do fail because of the types of local action described above. The simple criteria of mechanism and crushing action do not address the implications of this and it may be necessary to consider rotation in joints. Such a criterion would have to be based on some form of elastic analysis of at least parts of the structure. Such analysis could be provided as an extension of what has been done here. The zone of thrust computed is demonstrably a lower bound solution provided the geometry of the arch does not change sufficiently to reduce the capacity. It is therefore useful to compute the deformation implied by the thrust line. This is a direct computation and so quite straightforward.

Bridges, and indeed other thrusting structures, suffer damage which is nearly always localised in some way. Local damage alters the possible flow of thrust and this has implications throughout the structure. A model is hypothesised in which the local computed thrust (both live and dead load) in a damaged area is reapplied and re-distributed as a negative force flow. This is a complex model and would require considerable effort in programming before it could be properly demonstrated. It seems unlikely that an effective prototype could be constructed entirely within a spreadsheet.

The author is, among other things, the proprietor of the Archie-M software which is used in a number of countries for arch bridge assessment. Incorporating the complex models described here into Archie-M is a major task but is in hand.

CONCLUSIONS

A new model for distribution of load in arch bridges has been described. The model is demonstrably better than any existing simple tool for assessing arch bridges.

A particular benefit of the new model is that it can deal explicitly with skew.

The distribution of load through viaducts both from span to span and through the piers is important in confirming the capacity of these structures and the model proposed is capable of yielding much improved results.

Damage in bridges and other structures can be modelled explicitly by using a negative thrust distribution to remove the effect of patches.

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