COMPUTER SIMULATION OF LOAD-DEFORMATION CURVES FOR MASONRY WALLS IN PSEUDO-STATIC TESTS

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SUMMARY

A rigid-body-spring-model (RBSM) based simulation algorithm was proposed to simulate the behaviors of masonry wall specimens in pseudo-static tests. Incorporating several calculation instances, the simulation results were verified by available test records. It was indicated that the proposed algorithm is applicable in simulating the load-deformation curves of masonry walls suffering interface type of failure under cyclic lateral loading and can serve as an assistant tool of pseudo-static test to study seismic properties of masonry walls.

INTRODUCTION

Masonry is one of the oldest and once the most popular constructional materials, but using of masonry has been constrained in today's world, especially in seismic regions, due to the vulnerability of masonry structures under earthquake. Knowledge of seismic properties of masonry structures is important to reliable protection of architectural heritages and reasonable design of new buildings.

One of the important characters in seismic responses of masonry structures is that the connections between neighboring members are rather weak (Weng et al. 2000). The responses are often affected by the arrangement of shearing walls and their seismic properties which have been researched intensively (Magenes and Calvi 1997; D'Ayala and Speranza 2003). Pseudo-static test was widely used in these researches. However, when parameter study is necessary to get general knowledge of walls with varying geometry or made up of varying mortar and bricks, the test is often time and economic demanding. Computer simulation based

on numerical methods is an effective way to expand the results obtained through tests and make more general knowledge about seismic properties of structural members made up of masonry. But there are also difficulties to be overcome in implementation of computer simulation for masonry walls.

Firstly, continuity of material is a widely adopted assumption in numerical methods for structural analysis. But for blocky nature masonry, which deformation field is far from continuity because of the generation and development of cracks under loads, this assumption seems unreasonable. Secondly, mechanical properties of masonry are complicated and affected by many factors. Even under the condition of uniaxial loading, general mechanical model of masonry is not available yet. If hysteretic properties were needed to be considered, the existing models are seldom effective (Casolo and Pena 2007). Thirdly, the computer simulation of responses of masonry walls under pseudo-static load is heavily time-consuming. That makes it difficult to conduct parameter studies and the effectiveness of simulation is impaired.

The discrete element method (DEM) proposed by Cundall can be used to predict dynamic responses of assembly of rocks (Cundall 1971). Assumption of continuity is not incorporated explicitly in the method. Similar to the idea of DEM, Kawai have proposed a rigid-body-spring-model (RBSM) to simulate small deformation responses of structural members under loads (Kawai 1978; Qian and Zhang 1991). If responses of materials stepped into the range of large deformation, the RBSM should be adjusted. A RBSM based algorithm has been proposed by the authors to simulate the collapse responses of masonry structures under severe earthquake (Peng et al. 2004). In this paper, the algorithm is adjusted to simulate the load-deformation curves of masonry walls in pseudo-static tests.

FORMULATION

To illustrate the procedure of modeling simply and clearly, 2 dimensional figures in wall plane are used in the following sections. Considering two moving elements in Figure 1, and to stipulate that positive direction of translation is align with the coordinate axes and positive direction of rotation is determined by right-hand rule, the length of the connecting spring at original position can be calculated by

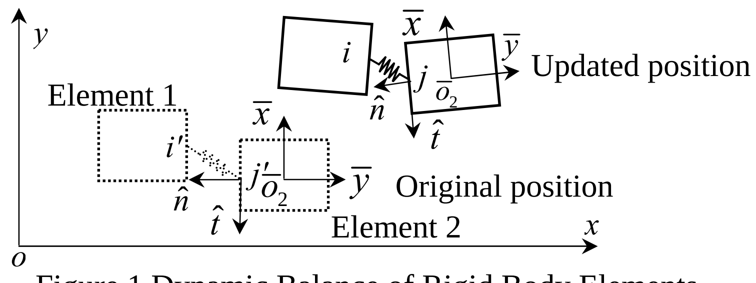


Figure 1 Dynamic Balance of Rigid Body Elements

$$\begin{cases} l'_{\hat{n}} = (\mathbf{R}_{i'} - \mathbf{R}_{j'}) \cdot \hat{\mathbf{n}} \\ l'_{\hat{t}} = (\mathbf{R}_{i'} - \mathbf{R}_{j'}) \cdot \hat{\mathbf{t}} \end{cases} \tag{1}$$

Where, $l'_{\hat{n}}$ and $l'_{\hat{t}}$ are lengths of the spring in local normal and tangent direction at original position, $\mathbf{R}_{i'}$ and $\mathbf{R}_{j'}$ are position vectors of point i' and j' respectively in global coordinate system, $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ are unit outer normal vector and unit tangent vector in global coordinate system of point j' respectively. And the length of the connecting spring at updated position can be calculated by

$$\begin{cases} l_{\hat{n}} = (\mathbf{R}_i - \mathbf{R}_j) \cdot \hat{\mathbf{n}} \\ l_{\hat{t}} = (\mathbf{R}_i - \mathbf{R}_j) \cdot \hat{\mathbf{t}} \end{cases} \tag{2}$$

Where, $l_{\hat{n}}$ and $l_{\hat{t}}$ are lengths of the spring in local normal and tangent direction at updated position, $\mathbf{R}_i$ and $\mathbf{R}_j$ are position vectors of point i and j respectively in global coordinate system.

If rotation of the elements is small and can be ignored, like in pseudo-static tests, equation 3 can be derived,

$$\begin{cases} \mathbf{R}_i = \mathbf{R}_{i'} + \mathbf{\Delta}_1 \\ \mathbf{R}_j = \mathbf{R}_{j'} + \mathbf{\Delta}_2 \end{cases} \tag{3}$$

Where, $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ are translation values of centroid of element 1 and element 2. And increments of the spring length are

$$\begin{cases} \Delta l_{\hat{n}} = l_{\hat{n}} - l'_{\hat{n}} \\ \Delta l_{\hat{t}} = l_{\hat{t}} - l'_{\hat{t}} \end{cases} \tag{4}$$

Where, $\Delta l_{\hat{n}}$ and $\Delta l_{\hat{t}}$ are increments of spring length in local normal and tangent direction. And the spring force can be determined by

$$\begin{cases} \mathbf{F} = \mathbf{F}_n + \mathbf{F}_t \\ \mathbf{F}_n = \mathbf{F}_{n'} + \Delta F_n \hat{\mathbf{n}} \\ \mathbf{F}_t = \mathbf{F}_{t'} + \Delta F_t \hat{\mathbf{t}} \end{cases} \tag{5}$$

Where, $\mathbf{F}_{n'}$ and $\mathbf{F}_{t'}$ are normal and tangent component of spring force at original position, $\mathbf{F}_n$ and $\mathbf{F}_t$ are normal and tangent component of spring force at updated position, ΔF_n and ΔF_t are scalar increment of normal and tangent component of spring force which are calculated according to constitutive model of masonry materials.

Ignoring the rotation, the dynamic equations of each rigid body element is

$$\begin{cases} m\ddot{x} + c_x \dot{x} = F_x \\ m\ddot{y} + c_y \dot{y} = F_y \\ m\ddot{z} + c_z \dot{z} = F_z \end{cases} \tag{6}$$

Where, $\ddot{x}, \ddot{y}$ and $\ddot{z}$ are acceleration responses of the element in directions of three axes of global coordinate system respectively, $\dot{x}, \dot{y}$ and $\dot{z}$ are velocity responses of the element in directions of three axes of global coordinate system respectively, m is mass of the element, c_x, c_y and c_z are damping coefficients, F_x, F_y and F_z are resultant forces applied on the element in directions of three axes of global coordinate system respectively, including all

spring forces which can be calculated by equation (5) and the gravity.

Using central difference method to reduce the order of dynamic equations will lead to

$$\begin{cases} \dot{x}\left(t+\Delta t/2\right)=\left[\dot{x}\left(t-\Delta t/2\right)\cdot\left(m-c_x\ \Delta t/2\right)+F_x\Delta t\right]\Big/\left(m+c_x\ \Delta t/2\right) \\ \dot{y}\left(t+\Delta t/2\right)=\left[\dot{y}\left(t-\Delta t/2\right)\cdot\left(m-c_y\ \Delta t/2\right)+F_y\Delta t\right]\Big/\left(m+c_y\ \Delta t/2\right) \\ \dot{z}\left(t+\Delta t/2\right)=\left[\dot{z}\left(t-\Delta t/2\right)\cdot\left(m-c_z\ \Delta t/2\right)+F_z\Delta t\right]\Big/\left(m+c_z\ \Delta t/2\right) \end{cases} \tag{7}$$

Where, Δt is the time interval, $\dot{x}(t+\Delta t/2), \dot{y}(t+\Delta t/2)$ and $\dot{z}(t+\Delta t/2)$ are velocity responses at the time $t+\Delta t/2$ in directions of three axes of global coordinate system respectively, $\dot{x}(t-\Delta t/2), \dot{y}(t-\Delta t/2)$ and $\dot{z}(t-\Delta t/2)$ are velocity responses at the time $t-\Delta t/2$ in directions of three axes of global coordinate system respectively.

Using central difference method to reduce the order of equation (7) will lead to

$$\begin{cases} x\left(t+\Delta t\right)=x\left(t\right)+\Delta x=x\left(t\right)+\dot{x}\left(t+\Delta t\right)\cdot\Delta t \\ y\left(t+\Delta t\right)=y\left(t\right)+\Delta y=y\left(t\right)+\dot{y}\left(t+\Delta t\right)\cdot\Delta t \\ z\left(t+\Delta t\right)=z\left(t\right)+\Delta z=z\left(t\right)+\dot{z}\left(t+\Delta t\right)\cdot\Delta t \end{cases} \tag{8}$$

Where, $x(t+\Delta t), y(t+\Delta t)$ and $z(t+\Delta t)$ are translation responses at the time $t+\Delta t$ in directions of three axes of global coordinate system respectively, $x(t), y(t)$ and $z(t)$ are translation responses at the time t in directions of three axes of global coordinate system respectively, $\Delta x, \Delta y$ and Δz are increments of translation responses during the time interval Δt in directions of three axes of global coordinate system respectively.

Now, if initial conditions of the target problem are given, the solution can performed according to the sequence shown in Figure 2 (Cundall 1971).

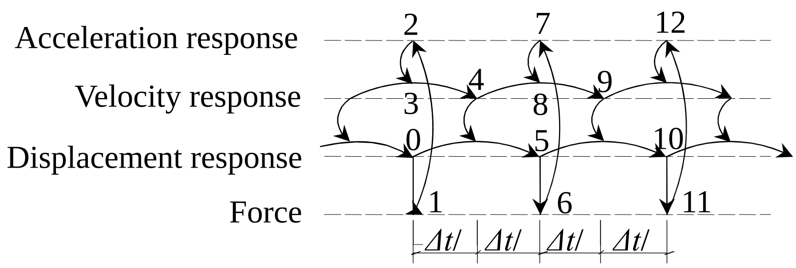


Figure 2 The Dynamic Relaxation Solution Procedure

MODELING OF MASONRY WALLS

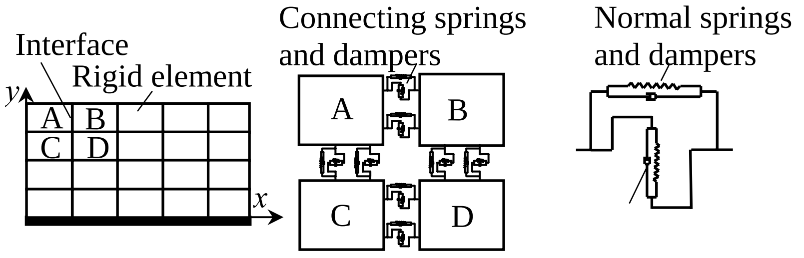
Meshing of the Wall

To decrease the number of degree of freedom, bricks/blocks and the mortar beds are smeared into one element. Masonry walls are meshed homogeneously using cuboid elements. And the resulting rigid element systems can crack at interfaces, even, the elements could depart from

each other if the cracks developed (Figure 3). Movement of the elements is controlled by the connecting springs. Normal strain and shearing strain of the springs are measured through relative displacement and geometry of two neighboring elements. Taking the elements in Figure 4 for example,

$$\varepsilon = -\frac{2|\Delta y|}{h_A + h_B}, \gamma = \frac{2|\Delta x|}{h_A + h_B} \tag{9}$$

Where, ε and γ and are normal and shear strain of the connecting springs, h_A, h_B are dimensions of the elements. And in Figure 4, O_A is the original centroid of element A, O_B is the original centroid of element B, and O_B' is the updated centroid of element B.



(a) Calculation model (b)Connection of elements (c)Connecting springs and dampers

Figure 3 Meshing of Masonry Wall

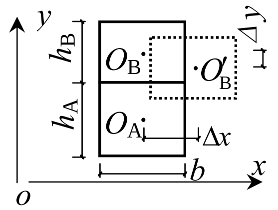
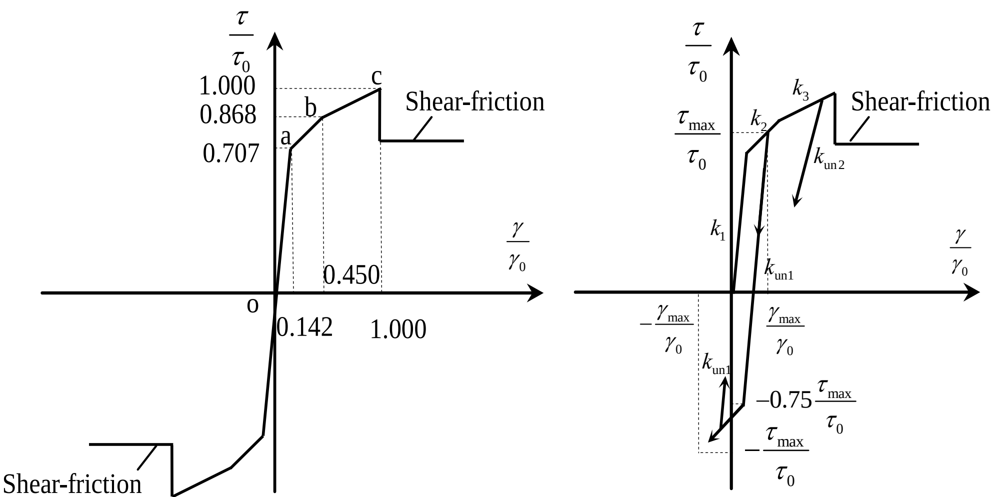


Figure 4 Calculation of Strains

Constitutive Model and Failure Criteria

Under cyclic lateral load, hysteretic behavior of the masonry material constructed with low strength mortar can be described by a multilinear model (Zheng 1988). To simulate the the slip between the blocks, the multilinear model is modified as Fig. 5. where τ_0 and γ_0 are the shearing strength and the corresponding shearing strain, τ and γ are shearing stress and corresponding shearing strain, γ_{max} and τ_{max} are maximum shearing strain reached of masonry under cyclic load and corresponding shearing stress, $k_i (i = 1 \sim 3)$ is loading



(a) Skeleton curve (b) Hysteretic rules

Figure 5 Constitutive Model of Masonry

stiffness, $k_{\text{un}i}(i=1\sim 2)$ is unloading stiffness. And $k_{\text{un}1}$ equals to k_1, $k_{\text{un}2}$ equals to the slope between point b and o. The increment of shearing spring force is

$$\begin{cases} \Delta F_t = k_i(\frac{\Delta\gamma}{\gamma_0})\tau_0 A, \ (i=1\sim 3), \text{for loading} \\ \Delta F_t = -k_{\text{un}i}(\frac{\Delta\gamma}{\gamma_0})\tau_0 A, \ (i=1\sim 2), \text{for unloading} \end{cases} \tag{10}$$

In vertical direction, the masonry material can be taken as elastic and increment of spring force can be calculated by the following equation

$$\Delta F_n = E(\frac{\Delta\varepsilon}{\varepsilon_0})\sigma_0 A \tag{11}$$

Where, σ_0 and ε_0 are the compressive strength and corresponding compressive strain, σ and ε are normal stress and corresponding normal strain, E is the elastic modulus of masonry material, and A stands for the area of the masonry section represented by the spring.

For masonry walls in structures the normal stress will result in frictional stress in shearing direction after the shearing springs break. The phenomena can be described by a horizontal branch in constitutive model of masonry. Meanwhile, normal springs are unbreakable under compression and will break under tension when the ultimate tensile strength is exceeded.

Damping Coefficients

Damping coefficients are set to the critical value for every element in every time interval,

$$c_x = \sqrt{\frac{\sum k_x}{m}} = \sqrt{\frac{|\mathbf{F}_x|/|\Delta x|}{m}}, c_y = \sqrt{\frac{\sum k_y}{m}} = \sqrt{\frac{|\mathbf{F}_y|/|\Delta y|}{m}}, c_z = \sqrt{\frac{\sum k_z}{m}} = \sqrt{\frac{|\mathbf{F}_z|/|\Delta z|}{m}} \tag{12}$$

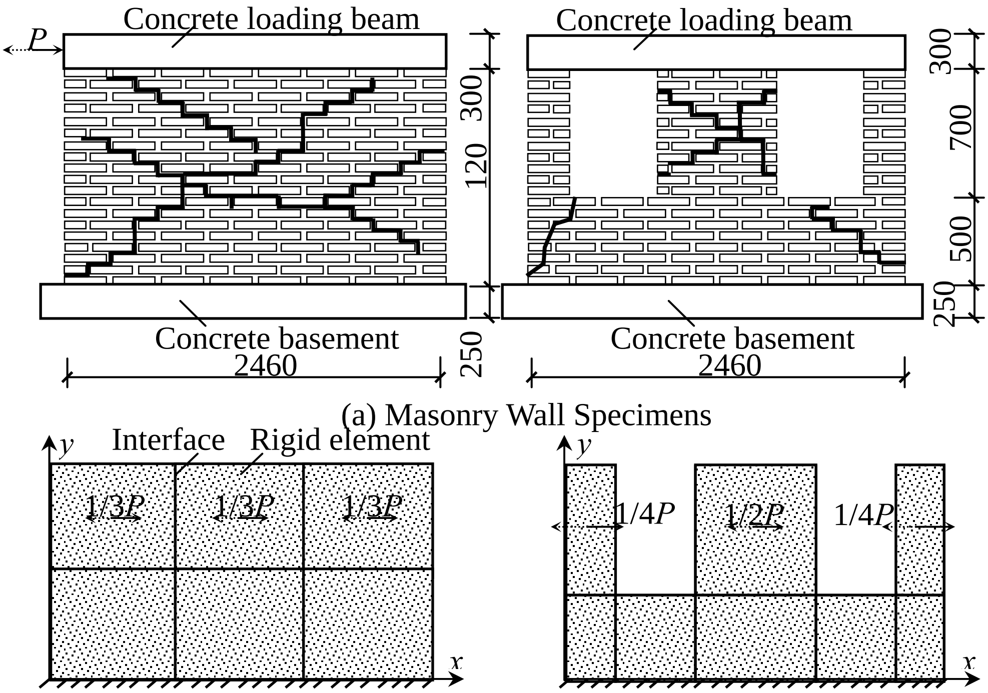
Where, $\sum k_x, \sum k_y$ and $\sum k_z$ are resultant stiffness of all springs in *x*, *y*, and *z* direction of global coordinate system. $\mathbf{F}_x, \mathbf{F}_y$ and $\mathbf{F}_z$ are components of resultant force at the beginning of the time interval in global coordinate system.

VERIFICATION OF THE PROPOSED ALGORITHM

To study seismic behavior of masonry walls constructed with low strength mortar, pseudo-static tests had been performed on wall specimens (Zheng 1988). To verify practicability of the proposed algorithm, tests on 4 specimens were simulated (Figure 6).

In the tests, cyclic loads were applied at a frequency of 0.05Hz. Interface cracks were generated near bottom corner of the specimens when the load was around 60~80% of its ultimate value. And slope of the load-deformation curve was changed accordingly. The interface cracks were connected each other to form step diagonal cracks along with increasing of the loads. The step diagonal cracks crossed each other and critical crack of X shape were then formed when the load increased to its ultimate value. In specimens with windows, cracks were also generated near bottom corners. Because strength of the mortar was low, few bricks were broken during the test, and interface cracks resulted in shear failure of these specimens

(Fig. 6a). From the failure mode shown in Fig. 6a, it can be seen that at least 6 rigid elements are needed to simulate the behavior of a masonry wall suffering interface type of failure under cyclic lateral load.



(a) Masonry Wall Specimens

(b) Calculation Models

Figure 6 Modeling of Masonry Wall Specimens in Pseudo-Static Tests

In the simulation, wall specimens with windows were mesh into 8 elements and the number of DOF was 16.Wall specimens without windows were mesh into 6 elements and the number of DOF was 12. Modulus of elasticity of the materials was 6.22×10^3MPa, tensile strength was 0.50MPa, shearing strength was 0.25MPa, and the shearing strain corresponding to the shearing strength was 0.002. Using a PC with a CPU of 2GHz and memory of 512MB, less the 1 hour are needed to complete the modeling and the calculation. However, the time used for modeling and calculation will increase if the meshing is refined. Efficiency graphic user interface (GUI) is helpful in saving time of modeling.

Shape of the load-deformation curves generated by tests and simulations is similar (Figure 7 and Figure 8, Where P is the lateral load and Δ is the lateral deformation measured at the top of the specimen). It is indicated that the proposed algorithm is suitable to masonry walls constructed with low strength mortar in which most of the cracks are generated in mortar beds. The agreement results from the proper constitutive model used, also from very small time interval used to insure stability of the calculation. For simulation of load-deformation curves of the specimens, the computation time can be saved by decreasing the number of elements for a single wall. But for simulation of crack development and failure modes of the specimens, more rigid elements need to be used for a single wall in the calculation. Of cause, it is time consuming.

CONCLUSIONS

1) Load-deformation curves of masonry walls under cyclic lateral loading can be well

simulated using the proposed algorithm.

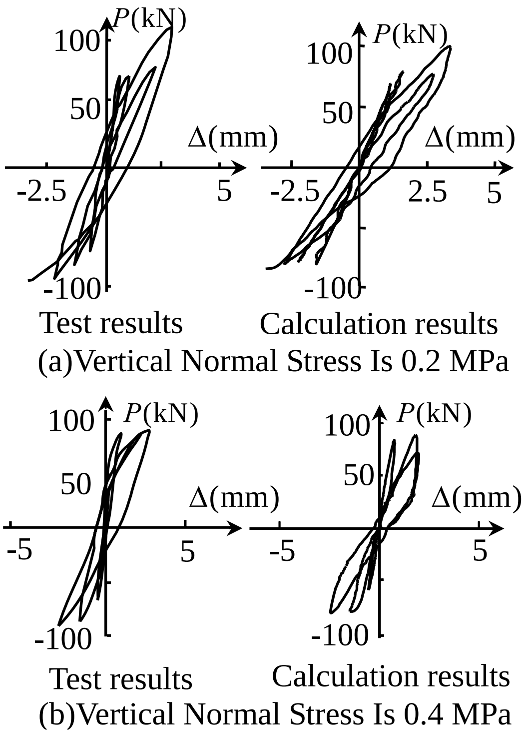


Figure 7 Results Comparisons for Wall Specimens without Window

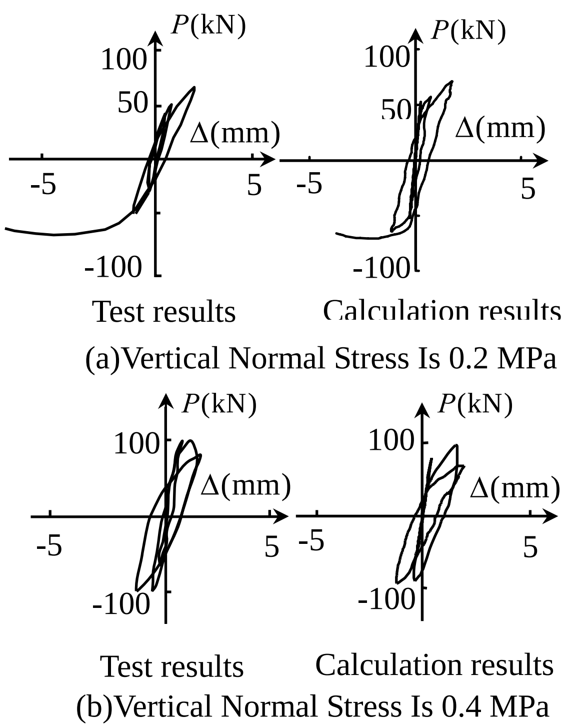


Figure 8 Results Comparisons for Wall Specimens with Windows

2) Demands on continuity of material can be relaxed in simulation of load-deformation curves for masonry walls under cyclic lateral loading.

3) The constitutive model of masonry materials constructed with low strength mortar under cyclic lateral loading can be described by the multilinear model.

4) Critical damping coefficients calculated in each time interval should be used in simulating pseudo-static tests.

5) To perform the simulation of other types of failure of wall specimens under cyclic lateral loading, intensive studies are still needed.

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