DEVELOPMENT OF A FIBER MODEL FOR LOAD BEARING MASONRY MEMBERS

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In the last few years, the possibility of being able to control damage based on the probability of occurrence of an earthquake and designing on the basis of different performance levels, has arose. Masonry is still a widespread construction system for low-rise residential buildings even in earthquake prone countries; hence masonry needs to develop these design concepts. Experimental tests were performed in recent years at the University of Padova on different masonry systems, both reinforced, and unreinforced with different joint types. The tests were aimed at characterizing the masonry behaviour under combined in-plane cyclic loading, and they were used to develop an analytical model that reproduces the experimental results. This model is a formulation of a fiber element and is cast in the general framework of the mixed method. It includes effects of shear deformation, diagonal shear failure mechanism and it follows the response in the post-peak phase. The model is able to interpret the performances of masonry panels linking them with limit states resulting from integration of cross-section equilibrium equations.

Keywords: unreinforced masonry, reinforced masonry, analytical modelling, displacement capacity.

INTRODUCTION

Finite Element modelling can provide powerful analysis tools for masonry structures; however, they are computationally demanding and a large number of user-specified parameters are needed for the constitutive behaviour definition. There are other approaches for modelling nonlinear behaviour of masonry structures, such as rigid models adopted by limit analysis, to estimate the ultimate load at collapse and the corresponding failure mechanism (Como e Grimaldi 1986, Abruzzese et al. 1992). Other approaches follow a bi-dimensional approximation of masonry piers or spandrels using strut-and-tie models e.g. (Calderoni et al., 1987 e 1989) or the more recent and sophisticated model from (Roca, 2006).

Bilinear constitutive laws are adopted for structural analysis methods, such as those developed by (Tomažević, 1978), and further refined in next years (Tomažević, 1999), based on the so-called storey-mechanism approach (POR method). Also based on simplified constitutive laws is the so-called macro-element discretization, which is based on an equivalent frame idealization of the structure (Magenes & Della Fontana, 1998) or (Lagomarsiino et al., 2007).

A different approach is proposed by (Benedetti & Steli, 2008). With a simple no-tension material and simple elastic perfectly-plastic rule for masonry and the usual hypothesis of straight cross sections, these authors present an explicit formulation for the shear-displacement curve of an URM pier, by integration of the curvature diagram. They also
extend it to the case of masonry with Fiber Reinforced Polymer (FRP) by introducing the hypothesis of an elastic–plastic resisting force (equal to the debonding value) of the FRP reinforcement up to crushing of masonry in compression (see Figure 1). This is a very simple model, that includes only an elastic shear deformation and is able to reproduce the behaviour until ultimate displacement, only accounting for a flexure failure. In any case, this work suggests that a cross sectional analysis approach can be developed also for masonry walls.

In general, if compared with finite element continuum models, the most important advantages of fiber element models are that they require a significantly reduced number of degrees of freedom and a significantly reduced number of user-defined constitutive parameters. In the nonlinear range, however, most conventional fiber models are limited to axial and flexural deformations, whereas the shear deformations are assumed to remain linear-elastic. Thus, these models are not effective in fully capturing the behaviour of structures where nonlinear shear deformations play a significant role such as RC or masonry non-slender walls.

Several fiber beam–column elements for Reinforced Concrete (RC) structures were developed in the last 20 years with capability of reproducing coupled axial force and flexural effects, e.g. (Spacone et al., 1996; Taucer et al., 1991). The fiber approach fits perfectly within the Euler–Bernoulli beam theory and caters for the accurate description of response of slender flexure-dominated members and full structures (Pinho & Elnashai, 2000). However, for structures with non-slender elements subjected to seismic loading, the coupling between shear, axial and bending action becomes important. Recent studies (Ceresa et al., 2009; Guedes & Pinto, 1997; Jiang & Kurama, 2010; Marini & Spacone, 2006; Marmo, 2008; Mostafaei & Vecchio, 2008; Petrangeli et al., 1999; Saritas & Filippou, 2009) attempted to overcome this limitation by introducing into the fiber approach the Timoshenko beam theory, or a generalized beam theory coupled with multi-axial constitutive laws for material.

Furthermore, in plastic areas, decreasing shear strength of the element is noted, due to a decreasing or absent shear resistant mechanisms in the concrete. The influence of shear forces on the behaviour of the beams was modelled by (Priestley et al., 1994); such a model is based on a reduction of the shear strength depending on the local ductility, as expressed in terms of linear variation of the curvature. The (Cosenza et al., 2006) model represents an improvement of Priestley’s since it enables the sectional ductility at any step of the analysis to be directly determined and then evaluates the shear strength of those sections located in the plastic regions (see Figure 2).
In this context, the present work contributes to the development of a model for in-plane loaded masonry walls. In particular, a fiber model, which accounts for shear deformations and failure, and is focused on displacement capacity of masonry members, (directly related with performance levels), is presented. Experimental tests recently performed at the University of Padova on various masonry systems, were aimed at characterizing the behaviour of both reinforced and unreinforced masonry walls, with different horizontal and vertical joint types, under combined vertical and in-plane cyclic horizontal loads. The test results were used to calibrate and validate the analytical model which was then developed (Guidi, 2011). This model can reproduce the envelope curves of cyclic shear-compression tests and interprets the performances of walls relating them with limit states resulting from integration of cross-section equilibrium equations.

REFERENCE MASONRY SYSTEMS
Masonry systems were built with traditional materials but using innovative building solutions. Unreinforced masonry was made with thin layer joints (TM); with ordinary bed joint and interlocking system (tongue and groove) on the head joints (TG); and with ordinary bed joint and units with pocket for mortar infill (Po) (see Figure 3). The experimental and numerical results obtained on these masonry types are thoroughly described in (da Porto et al. 2009; 2010). The reinforced masonry system is based on the use of concentrated vertical reinforcement. Vertically perforated units are used for the reinforced confining columns, and special clay units, with horizontal holes and recesses for horizontal reinforcement, are used for the main portions of the masonry walls (see Figure 4). The construction system and the experimental results obtained on these masonry types are thoroughly described in (da Porto et al. 2011a; 2011b).

Figure 2. Bending-Shear interaction after Priestley et al.1994: failures mode (Cosenza et al., 2010).

Figure 3. The three unreinforced masonry unit types: TM TG and Po.
Figure 4. (a) Reinforced masonry system; details of horizontally perforated unit (b) and vertically perforated unit (c).

MODEL DEVELOPMENT: MOMENT-CURVATURE ANALYSIS
The developed model starts from Moment-Curvature analysis of panel with isostatic boundary conditions (fixed to the base) and considers masonry wall as an isotropic homogeneous material. Then, from pure flexural analysis, considering the contribution of vertical reinforcement bars at wall ends to the flexural strength and displacement (when they are present), the model adds shear deformation contribution. Hence, the model follows the wall behaviour in the post-peak branch of capacity curve, takes into account possible indirect tensile diagonal shear failure of panel and the strength contribution of horizontal reinforcement bars (when they are present), and approximates shear strength decay with increasing displacements.

The model is a formulation of a fiber element and is cast in the general framework of the mixed method. It is based on a non-linear algorithm that always maintains static equilibrium within the element and converges to a state that satisfies the element constitutive relation within a specified tolerance. The proposed solution algorithm is suitable for the analysis of the highly non-linear behaviour of softening members, such as reinforced and unreinforced masonry piers under varying axial load. The formulation of the wall element is based on the assumption of linear geometry. The element is subdivided into a discrete number of cross sections. Plane sections remain plane and normal to the longitudinal axis during the element deformation history. This hypothesis is acceptable for small deformations of elements composed of homogeneous materials, but it is also applied in studies which focus on hysteretic behaviour under large inelastic deformation reversals (Spacone et al., 1996).

The required inputs to build the moment and curvature functions are geometry, material properties and boundary conditions. These are respectively $L$, $H$, $t$ for length, height and thickness of wall (geometry), $f_{\text{cm}}$, $E_m$ and $G$ which are compressive strength, elastic and shear modulus of masonry (material properties), and $\sigma_0$ that is vertical compression stress (boundary condition). Obviously, in case that reinforced masonry is modelled, more input data are required, such as area of vertical and horizontal reinforcement bars ($A_v$ and $A_{vh}$ respectively); for vertical reinforcement bars, positioning with respect to the wall edge ($d'$); spacing between horizontal reinforcement bars ($s$), and steel material properties, such elastic moduli ($E_s$ and $E_{sh}$ respectively) and steel yield strengths ($f_{ys}$ and $f_{ysh}$).

For a generic cross-section, it is possible to find moment and curvature related to a specific strain (i.e. at the maximum compressed side) imposing equilibrium equations. By doing so, some limit states related to cross-section performances, although they are limited to flexural response, can be identified. Considering that shear deformations do not affect curvature, the previously calculated moment and curvature are still valid when shear deformation is added. Hence, the cantilever isostatic boundary conditions of the walls suggest that these cross-section limit states, related to the wall base section, will at least give the first indicators of the
entire wall behaviour. In addition, imposing that compression strain grows at the compressed toe, it is possible to control the whole moment curvature curves (in post-peak phase, too) by simply imposing equilibrium equations.

When the wall reaches maximum bending moment at the base cross-section (according to the moment-curvature function), the remaining portion of the wall is subject to lower moment. Then, in the next step, base-wall bending moment decreases following moment-curvature function (beginning of the softening branch). The other cross-sections have different behaviour, as moment decreases although it did not reach maximum value. Hence, the base section behaves following the found moment-curvature function and its curvature grows with the strength softening, whereas this does not happen for the rest of the wall. Model does not provide a loading-unloading stress-strain function, so it needs to approximate this phenomenon with one more hypothesis at moment-curvature level. An hypothesis that is accepted in literature (Benedetti et al., 1982; Del Piero, 1983) describes compression unloading for masonry as linear, and was used in this model.

The validity of the analytical results also depends on the accuracy of the material models. Since the present study is limited to the behaviour of masonry members, and the effect of bond-slip on reinforcement bars is neglected, only two material models are required: one for masonry and one for reinforcing steel (when there is any). The element formulation simplifies the task of material model selection to uniaxial behaviour, which is thoroughly studied and well established to date. The model does not take into account the effect of confinement by transverse reinforcement because this phenomenon did not occur for the reinforced masonry construction system which this study refers to (Mosele, 2009). This is true for reinforced clay masonry walls in general (Tomaževič and Lutman, 1988), since reinforcement bars are placed into horizontal joints and they are not able to develop the same confinement effect as stirrups in reinforced concrete members. Hence, shear deformation of masonry element does not take into account the contribution of horizontal reinforcements, until shear failure occurs.

Strength deterioration of masonry members under large deformation reversals largely depends on the capacity of masonry to sustain stresses in the strain range beyond achievement of maximum strength. This requires the use of a refined, although simple, material model. The model used in this study is shown in Figure 5. The monotonic envelope of masonry in compression is a 4 branch multi-linear curve (plus a residual strength which is intended only for numerical stability); and masonry contribution in tension is neglected. Although more detailed models have been published since the (Kent & Park, 1971) one, this approximation represents a good compromise between simplicity and accuracy.

**SHEAR CONTRIBUTION IN THE MODEL**

The non-linear behaviour of the proposed element derives in part from the material constitutive laws and in part from shear stiffness degradation effect. Therefore shear effects are also included, which is a reasonable approximation for normal height to depth ratios of the masonry member.

After the calculation of pure flexural response the model adds deflections induced by shear. The constant shear coefficient $\chi$ is $6/5=1.2$ for rectangular cross sections; the applied horizontal force is $F$ and the neutral axis position of cross-section measured from compressed end of section is $\beta$. Fixing the applied force ($F$), $\beta$ varies along the height of the wall, from the whole section length ($L$) in the upper part, that is still elastic, and decreases with increasing moments down to the base section. Hence, when masonry reaches the elastic limit ($\varepsilon_{el}$) and enters the non-linear part of the stress-strain curve, shear modulus reduces. Besides, since $G$ is an elastic modulus, it does not act in the portion of cross section that is not in the elastic range.
These two observations are translated in the following Eq. 1 and 2. The integration of Eq. 3 (that expresses angular deflection due to shear) along the height of wall, yields shear displacement contribution.

\[
\beta_{el}(M) = \begin{cases} 
\beta(M), & \frac{\epsilon_{mr}}{\epsilon_{mr}(M)} \geq \beta(M) \\
\frac{\epsilon_{mr}}{\epsilon_{mr}(M)}, & \frac{\epsilon_{mr}}{\epsilon_{mr}(M)} < \beta(M)
\end{cases}
\] 

(1)

\[
G_{red}(M) = G \cdot \frac{\beta_{el}(M)}{\beta(M)}
\] 

(2)

\[
\gamma(M) = \frac{e^{F}}{G_{red}(M)\epsilon} \cdot \frac{1}{\beta_{el}(M)}
\] 

(3)

Shear failure is evaluated considering masonry piers subjected to global mechanisms. Shear strength (V_r) is a sum of two contributions: masonry shear strength (V_m) and horizontal reinforcement (V_{sh}, for reinforced masonry only).

The model follows a phenomenological approach that considers masonry contribution to shear strength as a tensile-induced failure that takes into account vertical load (Tomaževič & Lutman, 1988; Turnšek & Čačovič, 1971). The choice of using only diagonal tensile failure mechanism in shear is justified by the failure modes observed during the tests under consideration, and in others experimental works in literature (da Porto et al., 2009; Magenes et al., 2008; Tomaževič, 2009). In this model masonry shear strength is not constant, as for each bending moment value the eccentricity of load resultant (e) over the cross-section can be calculated according to formulations proposed in Eq. 4 (Bernardini et al., 1982a; Bernardini et al., 1982b). There, b is the tangential stress distribution factor that refers to the maximum stress value compared with the mean one. It varies between 1.5 for slender cantilever walls (H/L ≥ 1.5) and 1 for squat cantilever walls H/L ≤ 1. Hence, b factor is function of eccentricity (e = M/N) and indirectly function of bending moments.

Shear contribution due to horizontal reinforcements is given by Eq. 5. This formula calculates the number of stirrups across the diagonal crack (assumed to be 45° sloped starting from the effective length of the resisting section: d). Vertical spacing between horizontal reinforcement bars is named s and C_r is the reduction coefficient that takes into account efficiency of stress transfer among horizontal reinforcements and masonry (Anderson & Priestley, 1992; Mosele, 2009; Tomaževič & Lutman, 1997), where C_r is 0.6 in agreement with (DM 14/01/2008) and (Mosele, 2009).
When shear failure occurs, the model approximates empirically the post-peak load-displacement curve of the element following a criterion similar to that proposed by (Anderson & Priestley, 1992) and (Voon & Ingham, 2007). It has to be pointed out that the criteria presented by these authors are aimed at evaluating maximum shear strength for a given ductility, while the aim of our model is describing strength decay under increasing displacements (hence ductility in the broad sense), after maximum strength achievement.

The resulting force-displacement relation after shear failure is composed by two branches (see Figure 6). In the first branch, strength is constant while displacements increase, and in the second branch, a parabolic decay of strength (for increasing displacements) takes place, until a 20% strength decay, respect to maximum shear capacity, occurs. When the model shifts from the first to the second branch, displacements can be found with Eq. 6 and 7. There, $\Delta_0$ is the displacement point where constant strength ends when shear failure occurs. $\Delta_0$ was found limiting the $\Delta^*_0$ function between two constraints: $\Delta V_r$, i.e. displacement at which shear failure develops, and $\Delta M_{max}$, i.e. displacement corresponding to maximum flexural capacity. $\Delta^*_0$ function is characterized by two components. The first is a ratio that controls how much masonry shear capacity is far from flexural capacity (without horizontal reinforcement contribution) and how much “ductile” would be flexural post-peak response if shear failure would not be occurring. The second factor of $\Delta^*_0$ is a sum of displacements due to masonry shear strength and to horizontal reinforcement contribution (left over to $\Delta V_r$).

$$\Delta_0 = \left[ \frac{\Delta V_m}{\Delta M_{max}} \cdot \left( \frac{\Delta M_{max} - \Delta M_{ult}}{\Delta M_{ult}} \right) \right] \cdot \left[ \mu_{0a} \cdot \Delta V_m + \mu_{0b} \cdot (\Delta V_r - \Delta V_m) \right]$$  

$$\Delta_0 = \begin{cases} \Delta V_r, & \Delta_0 \leq \Delta V_r \\ \Delta^*_0, & \Delta_0 \leq \Delta M_{max} \\ \Delta V_m, & \Delta_0 > \Delta M_{max} \end{cases}$$  

$$\Delta^*_0 = \left[ \frac{\Delta V_m}{\Delta M_{max}} \cdot \left( \frac{\Delta M_{max} - \Delta M_{ult}}{\Delta M_{ult}} \right) \right] \cdot \left[ \mu_{1a} \cdot \Delta V_m + \mu_{1b} \cdot (\Delta_0 - \Delta V_m) \right]$$  

$$\Delta_1 = \begin{cases} 1.15 \cdot \Delta_0, & \Delta_1 \leq 1.15 \cdot \Delta_0 \\ 1.5 \cdot \Delta_0, & \Delta_1 \leq 2 \cdot \Delta M_{max} \end{cases}$$  

$\Delta_1$ is the ultimate displacement when shear failure occurs (Eq. 8), after the parabolic decay starting at $\Delta_0$. $\Delta_1$ was found limiting the $\Delta^*_1$ function (Eq. 9) between two constraints: $(1.15 \cdot \Delta_0)$, considered as a minimum multiplier factor according to experimental data, and $(2 \cdot \Delta M_{max})$, which appears as a reasonable limit for shear failure ductility. $\Delta^*_1$ has the same form of $\Delta^*_0$, but it is fitted with experimental results using different $\mu_{1a}$ and $\mu_{1b}$ coefficients that magnify respectively $\Delta V_m$ and the contribution of stirrups to displacement ($\Delta_0 - \Delta V_m$). The coefficients $\mu_{0a}$, $\mu_{0b}$, $\mu_{1a}$ and $\mu_{1b}$ were fitted using both reinforced and unreinforced masonry which had shear failure and their proposed values are respectively: $\mu_{0a} = 2.8$, $\mu_{0b} = 27.5$, $\mu_{1a} = 5.6$ and $\mu_{1b} = 31$.

**VALIDATION OF PROPOSED MODEL**

To validate the analytical model, a series of analyses to reproduce the experimentally observed data were done. The calibration process was quite easy thanks to the simplified hypotheses used in the model, and the resulting limited number of parameters needed. The
The model uses, obviously, the same geometrical and boundary conditions of experimental test set-up (namely \( L, H, t, \sigma_0 \)). The mechanical parameters result from experimental tests done. Starting from the experimental results, \( G \) was the only value to be calibrated, to catch the initial stiffness of the capacity curves.

The results for the three unreinforced masonry systems modelled are summarized in Figure 7. In that pictures, the colours refer to vertical compression level (0.17, 0.22, 0.27, 0.33% of masonry compressive strength); continuous lines represent the model results, whereas dashed lines are the envelope curves of cyclic shear compression tests experimentally done. The dots are the limit states reached by the base cross-section. The model was able to well reproduce initial elastic stiffness. Comparison of the various masonry types showed that their behaviour was similar. In general model highlight that for all masonry types the sequence of limit states is not constant with different vertical load. When low vertical stress ratio was applied, the first limit state is horizontal cracking at base, with reduction of cross-section that actually responds to compression. The response is almost linear until the second limit state, first non-linearity in masonry, was achieved for most stressed element in compression. On the contrary, for higher vertical stress ratios model come out from masonry stress-strain linear branch before the wall base cross-section show the first horizontal crack.

Figure 8 shows the model versus experimental load-displacement capacity curves of reinforced masonry walls. These are under a vertical compression load of \( \sigma_0 = 0.4 \) and 0.6 N/mm\(^2\) respectively. Graphs on the left show a wall with slenderness ratio \( H/L=1.09 \) and the graphs on the right show a flexural failure condition with the slenderness ratio \( H/L=1.63 \).

When flexural failure occurs, the sequence was similar but, before reaching \( F_{\text{max}} \), yielding of vertical reinforcements in tension (\( F_{\text{ys}} \)) also occurred. When shear failure occurs, the sequence of limit states was: horizontal cracking at base cross-section, first non-linearity in masonry (\( F_e \)), achievement of masonry shear strength (\( V_m \)), masonry yielding in compression (\( F_{ym} \)), maximum horizontal capacity (\( F_{\text{max}} \)) and maximum displacement (\( d_{\text{max}} \)).
For a complete description of model validation see (Guidi, 2011).

**CONCLUSIONS**

A model for masonry walls (S.D.o.F. systems) under in-plane vertical and horizontal forces, capable of reproducing experimental load-displacement capacity curves, considering nonlinear shear deformations and taking into account both flexural and shear failures, was developed. Reinforced masonry system and three types of load-bearing unreinforced masonry walls, made with perforated clay units and various types of head and bed joints, were modelled under in-plane cyclic loads reproducing experimental tests.

As expected, in general, flexural behaviour was fairly well reproduced by model achieving both strength and displacement showed in experimental tests, despite some inconsistencies for unreinforced masonries. However, they can partially be imputed to test variability of masonry specimens themself.

In general, modelled walls are greatly influenced by shear deformation and, obviously, shear deformation is controlled to a great extent by shear modulus (G). Despite this sensitivity, model was in good agreement with experimental G obtained from shear-compression tests and with lower bound of values provided by (Circolare 2/02/2009 n. 617 C.S.LL.PP., 2009). Shear strength formulation adopted was able to correctly forecast experimental walls subjected to shear failure and also, with acceptable approximation, their loads and displacements.

The model catches the achievement of various limit states, which represent the performances of masonry walls related to cross-sectional behaviour (e.g., when masonry piers reach compressive strength at the base section) or to the global wall behaviour (e.g., when walls reach their shear strength). It reproduces the initial stiffness and the correct sequence of various limit states and failure modes for all tested walls.
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