NUMERICAL ANALYSIS OF UNREINFORCED FLANGED WALLS SUBJECTED TO BIAXIAL BENDING

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Masonry walls of a building are subjected to horizontal loading caused by seismic or wind actions besides of the vertical loading. This horizontal loading may act in a different direction of the orthogonal axes of symmetry of the structural elements generating biaxial bending. Walls in masonry buildings usually have transversal walls, also called flanges. Those flanges increase the stiffness of the structural system and significantly improve the lateral capacity of the buildings mainly in case of biaxial bending. However, it is well known that these improvements only happen if the connection of walls is ensured. This work addresses the influence of flanges in the behavior of unreinforced masonry walls subjected to biaxial bending. A numerical study using a 3D-model is performed with the DIANA® software program based on the Finite Element Method. A parametrical analysis is carried out in order to define the influence of some parameters on the behavior of masonry walls with flanges subjected to biaxial bending, such as geometry wall, boundary conditions and the angle of horizontal loading application. The results indicated that flanges have considerable influence on the behavior of masonry walls under flexure and shear in case of the biaxial bending.

Keywords: connection, interlocking, flange, shear, flexure

INTRODUCTION

Masonry walls were mainly designed to resist gravity loads, however horizontal loads, induced by earthquakes and wind, generate severe forces in these walls. These actions act on a building in any direction submitting the structural elements to biaxial bending. In general masonry walls present the flexural strength in the longitudinal axis higher than the flexural strength in the transversal axis due to its small thickness related to its length. The consideration of flanges on masonry walls leads to an increase of both flexural strength making them closer. Masonry walls with flanges have already studied by some researchers (Yoshimura et al., 2003; Modena et al., 2004). However, these types of specimens need special testing apparatus. Therefore, single walls are the most commonly used system for in-plane and out-of-plane testing. Thus, considerable research works have focused on analyzing the behavior of single masonry walls (Shing et al., 1989; Tomaževič, 1999; Voon and Ingham, 2006; Haach et al., 2010).
A number of drawbacks occurs in experimental analysis since the test setups are usually complex (the real boundary conditions are hard to be known and represented), experimental setups are generally expensive, and results are sometimes scarce and limited to the conditions in which they have been obtained. Complementarily to experimental analysis, numerical modeling of masonry walls under horizontal loads contributes to increasing knowledge about their behavior, once it is validated variations of parameters that can influence the in-plane behavior can be analyzed. Flexural strength and deformation can be accurately evaluated by means of the simple flexure theory used for reinforced concrete structural elements. However, shear resisting mechanisms are considerably more complex. Thus, the consideration of axial force and biaxial bending further increases the complexity of the shear behavior of masonry walls. Thus, this work aims to study the influence of flanges on the behavior of unreinforced masonry walls subjected to biaxial bending through numerical modeling indicating some points to be experimentally analyzed in the future.

EXPERIMENTAL DETAILS
The numerical model used in this study was validated from the experimental result of in-plane test performed on an unreinforced concrete block masonry wall. Due to the absence of experimental results of flanged walls, rectangular walls were used for the validation of the numerical model. The experimental test consisted of an in-plane cyclic test on a cantilever wall following the typical test setup, shown in Figure 1, used for masonry walls under combined vertical and horizontal load (Vasconcelos and Lourenço, 2009).

The testing procedure was divided into two phases. First, the vertical load was applied at a rate of 0.25kN/s up to a vertical stress equal to 0.56 MPa, which was kept constant during the test. Next, horizontal displacements were imposed to the walls until failure. The cyclic tests were carried out under displacement control at a rate of 70 μm/s by means of an external LVDT connected to the horizontal actuator. Dimensions of the tested masonry walls were 1206mm x 800mm x 100mm. Hollow concrete units of 201mm(length) x 93mm(thickness) x 100mm(height) in half-scale were considered in the experimental test. These units have two cells of 60mm x 70mm and one small cell in the middle of the unit of 15mm x 70mm. The percentage of holes in the block is of about 46%, which, according to Eurocode 6 (2005), indicates that the units belong to Group 2. Reinforced concrete beams were placed at the bottom (280 mm x 280 mm x 1400 mm) and at the top (280 mm x 280 mm x 1200 mm) of the walls in order to ensure a uniform distribution of the applied vertical and horizontal loads.

**Figure 1: Test setup used in experiment.**
The displacements of the wall under cyclic loading were measured by means of a set of LVDTs. A more detailed overview of the experimental result can be found in (Haach et al., 2010).

**NUMERICAL MODELLING**

The numerical model applied to study masonry walls under in-plane loading was defined using the DIANA® software program. The micro-modeling approach was chosen for the modeling since it includes all the basic failure mechanisms that characterize masonry, enabling the detailed representation of resisting mechanisms of the walls. In the numerical analysis only monotonic loading was considered. Newton-Raphson iteration procedure was used with a displacement control and an energetic convergence criterion with a tolerance of $10^{-2}$. Firstly, the validation of the numerical model was carried out based on the aforementioned experimental result. The second phase included a parametric analysis performed on the masonry walls subjected to biaxial bending using units with real dimensions (400mm x 200mm x 200mm). The boundary conditions assume a central role on the lateral behavior of masonry walls, as they govern the preponderant failure mechanism of the walls under horizontal cyclic loading. Two different boundary conditions were considered to evaluate their influence on the behavior of the concrete block masonry walls: cantilever walls and fixed ends walls. Haach et al. (2010) observed flexure failure for cantilever walls and shear failure for fixed end walls when this aspect ratio and normalized axial stress were assumed. Therefore, behavior of masonry walls can be observed in the case of flexure and shear failure modes. Unreinforced masonry walls with H-section and different flange lengths were considered in parametrical analysis. The walls were 2800mm in height and 2800mm in length. The length of four flanges was used in this study: 1000mm, 1800mm, 2600mm and 3400mm. An aspect ratio of $h/L = 1$ and normalized axial stresses of $\sigma/f_a = 0.10$ and $\sigma/f_a = 0.20$ were considered for cantilever and fixed-ends walls, respectively, in parametric analysis, based on the results of Haach et al. (2010). Where $h$ is the height of the wall, $L$ is the length of the wall, $\sigma$ is the pre-compression and $f_a$ is the compressive strength of masonry.

Different of the experiments, monotonic loads were applied to the specimens in numerical modelling. As in the case of the experiment, the axial load was applied to the first step and it was kept constant. After that, horizontal displacements were imposed on the walls until failure. Horizontal displacements were applied with 5 different angles of inclination related to the transversal axis of the masonry wall as shown in Figure 2.

![Figure 2: Geometry of walls and angles of inclination of the horizontal loading.](image)

As in the case of the experimental tests, a concrete beam was also modeled at the top of the walls and the lateral loading was applied at the mid height of the concrete beam. The bottom concrete beam was not included since its consideration does not influence the masonry wall
behavior. In cantilever walls, the continuum elements representing the masonry units located at the base of the wall were connected to the interface elements which were fully fixed in order to represent the fixed base of the masonry walls. The upper beam was connected to the wall through interface elements modeled with linear behavior and infinite stiffness to simulate a perfect bond between these two elements, as observed in the experimental test. In the case of fixed end walls, the top concrete beam had all degrees of freedom fully restrained.

The mesh was composed of continuum and interface elements to represent, respectively, the masonry units and the masonry joints, see Figure 3. In the case of the units, four-node isoparametric plane curved shell elements with Gauss integration scheme were adopted (Q20SH - DIANA®). Each masonry unit was modeled with two continuum elements. Potential vertical cracks of the units were introduced at mid-length of the units. The joints were lumped into the concrete units and the unit-mortar interface was represented by an interface element (N6IF - DIANA®) between two nodes in a three-dimensional configuration.

![Figure 3: Mesh of finite elements: (a) example of mesh applied to the masonry walls and (b) Elements used in 2D-numerical modeling (DIANA®)](image)

In the micro modeling approach all the constituent materials of the masonry walls, with distinct mechanical properties, are independently described. Different material models were used to represent the behavior of the concrete at the top beam, concrete masonry units, vertical and horizontal joints and the potential cracks in the middle of the units. The mechanical properties used in the description of the material models were obtained from experimental tests carried out on materials and masonry assemblages (Haach et al., 2010). Isotropic elasticity was adopted for the upper concrete beam since the stresses developed in this element are very small and thus linear stress-strains relationship is valid. An elastic modulus equal to 30 GPa was used in the concrete beams, corresponding to a concrete with a compressive strength of about 30 MPa.

An interface cap model with modern plasticity concepts proposed by Lourenço and Rots (1997), and further enhanced by Van Zijl (2004), was used for interface elements describing the masonry joints. The interface material model is appropriate to simulate fracture, frictional slip as well as crushing along the material interfaces, which are the possible failure modes of the masonry unit-mortar interfaces. Among the mechanical properties used for defining the yield functions in tension, compression and shear of the unit-mortar interfaces are the normal and transversal stiffness of bed joints ($k_n = 11$ N/mm² and $k_s = 48$ N/mm², respectively). The normal stiffness was defined by fitting the numerical to the experimental results obtained in the masonry walls. The shear stiffness was obtained with the results of the shear tests
performed on triplet specimens to characterize the shear behavior of concrete unit-mortar interface (Haach et al., 2010). The yield function with exponential softening for the tension cut-off model requires knowing the tensile bond strength of bed joints ($f_i = 0.33$ MPa) and the mode I fracture energy ($G_{fI} = 0.017$ N/mm). The tensile-bond strength was obtained from the experimental results of the flexural tests on masonry carried out in the direction parallel to the bed joints (Haach et al., 2010). Due to the difficulty of obtaining mode I fracture energy of the unit-mortar interface, this mechanical property was defined by fitting the numerical to the experimental results obtained in the masonry walls. The shear behavior of the unit-mortar interfaces is represented by the Coulomb failure criterion. This function can be defined by knowing the cohesion ($c = 0.42$ MPa), friction coefficient ($\mu = 0.49$), the dilatancy coefficient ($\tan \psi = 0.52$), and the shear fracture energy ($G_{fII} = 0.1$ N/mm). In order to capture cohesion softening and friction softening, the residual friction coefficient ($\mu_{res} = 0.43$) should be obtained. All the parameters were obtained from the tests performed on triplet specimens (Haach et al., 2010). In the model, the dilatancy is considered to be dependent on the normal confining stress and on the shear slipping. Thus, for the correct definition of the dilatancy, the confining normal stress at which the dilatancy becomes zero ($\sigma_u = 1.35$ MPa) and the dilatancy shear slip degradation coefficient ($\delta = 1.64$), were also obtained by experimental analysis. Vertical and horizontal mortar joints were represented by the same material model using the same values of the properties. According to Lourenço and Rots (1997), it is useful to model potential cracks in units in order to avoid an overestimation of the collapse load and of the stiffness. Thus, potential cracks placed at the middle length of units were considered by using interface elements with a discrete cracking model. High stiffness should be considered for this interfaces, according to Lourenço and Rots (1997) ($k_n = 10^6$ N/mm$^3$ and $k_s = 10^6$ N/mm$^3$, respectively). In addition, an exponential softening behavior was adopted for the tensile behavior of these interfaces with the tensile bond strength ($f_i = 3.19$ MPa) obtained in uniaxial tensile tests performed on the concrete units (Haach et al., 2010) and the mode I fracture energy ($G_{fI} = 0.06$ N/mm) obtained from the experimental results obtained by Mohamad (2007) in concrete blocks with a similar composition of raw materials. The constitutive law for discrete cracking in DIANA® is based on a total deformation theory, which expresses the tractions as a function of the total relative displacements. The non-linear behavior of the concrete masonry units was represented by a Total Strain Crack Model based on a fixed stress-strain law concept available in the commercial software program DIANA®. It describes the tensile and compressive behavior of the material with one stress-strain relationship in a coordinate system that is fixed upon crack initiation. Exponential and parabolic constitutive laws were used to describe the tensile and compressive behavior of concrete masonry units, respectively. The mechanical properties needed to describe this material model are the elastic modulus of concrete units ($E = 9.57$ GPa), the Poisson’s ratio of concrete units ($\nu = 0.20$), the tensile strength of concrete units ($f_{tu} = 3.19$ MPa), the fracture energy of units under tension ($G_{fI} = 0.06$ N/mm) and the shear retention factor ($\beta = 0.01$). Due to the impossibility of obtaining the post-peak tension behavior of the three cell concrete units, the values of fracture energy in tension were obtained from the experimental results obtained by Mohamad (2007) in concrete blocks with a similar composition of raw materials. In this study, the compressive strength of units and fracture energy of units under compression were considered equal to the compressive strength of masonry and fracture energy of masonry under compression ($f_{cu} = 5.95$ MPa and $G_{fI} = 5.00$ N/mm, respectively). The shear behavior during cracking was described through a shear retention model defined by a constant value.
VALIDATION OF NUMERICAL MODEL
A numerical modeling only makes sense if it corresponds to the real model. Therefore, the first step of the numerical analysis comprises the calibration of the numerical model defined for the masonry shear walls, which is achieved from the comparison between experimental and numerical results. This enables to use a reliable model for the envisaged parametric study. The comparison between the cyclic force-displacement diagram, obtained in the experimental test with the numerical monotonic envelope, reveals that a reasonable agreement was attained between both approaches, see Figure 4.

![Force-Displacement Diagram](image)

**Figure 4:** Comparison between experimental and numerical results (force-displacement diagram).

In terms of failure mode, the numerical modeling agrees reasonably well with the experimental result, in spite of the monotonic loading considered in the numerical modeling. As shown Figure 5, the numerical results represented the three main crack patterns developed during the experimental behavior of the wall, namely flexural cracking, diagonal cracking and crushing at the bottom of the wall. In the experimental test, after the diagonal crack and crushing at the bottom corner occurred, the upper part of the wall slid over the diagonal crack.

![Crack Patterns](image)

**(a)**

**Figure 5:** Comparison between experimental and numerical results (failure mode).

In the numerical modeling, some penetrations of the elements in the compressed corner were observed during the sliding. In general, the results of the numerical modeling showed a
reasonable agreement with the experimental result, meaning that it satisfactorily represents the lateral in-plane behavior of masonry walls. This indicates that the numerical model is adequate to proceed with the parametric study with accuracy.

RESULTS AND DISCUSSION OF PARAMETRICAL ANALYSIS

Most of the cantilever walls failed under flexure, see Figure 6a. Horizontal flexural cracks appeared at the first joint from the bottom due to increasing tensile stresses associated to the flexure of the wall. This damage basically depends on the tensile bond strength of the unit-mortar interface. With the increased of lateral displacement, the length of the horizontal cracks tended to increase, leading to the translation of the neutral axis and thus to the increase of the compressive stresses on the opposite side and also the crushing of masonry. On the other hand, most of fixed-ends masonry walls failed under shear, see Figure 6b. Horizontal flexural cracks also appeared at the first joint from the bottom, due to the increasing of tensile stresses associated to the flexure of the wall, however the failure occurred due to the opening of diagonal cracks before the crushing of masonry at the bottom of wall.

![Figure 6: Failure mode of masonry walls with horizontal displacements applies with 45° and flange length equal to 1800 mm: (a) Cantilever wall and (b) Fixed-ends wall.](image)

Cantilever masonry walls which horizontal displacements were applied with 0° presented out-of-plane flexure failure with some torsion of the flanges, see Figure 7a. In case of fixed-ends masonry walls, shear failure of the flanges occurred in all specimens except that wall with flange length equal to 1000mm, see Figure 7b. Only fixed-ends walls which the horizontal displacements were applied with 0° presented diagonal cracking in flanges. The increase of the length of flanges may change the failure mode independently of the direction of loading.

![Figure 7: Failure mode of masonry walls with horizontal displacements applies with 0° and flange length equal to 1800 mm: (a) Cantilever wall and (b) Fixed-ends wall.](image)
Results presented in Table 1 clearly shows that increasing the flange length, the flexure and shear strengths of masonry walls increases independently of the direction of loading. Lower is the angle of inclination of loading; higher is the increasing of the strength since the direction of loading tends to align with the longitudinal direction of flanges.

Table 1: Numerical Results

<table>
<thead>
<tr>
<th>Angle</th>
<th>Flange Length (mm)</th>
<th>Ultimate Load (kN)</th>
<th>Failure Mode</th>
<th>Angle</th>
<th>Flange Length (mm)</th>
<th>Ultimate Load (kN)</th>
<th>Failure Mode</th>
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The increase of the angle of inclination of horizontal loading and the flange length generated an enhancement on the stiffness of the masonry walls and a brittle behaviour for both boundary conditions, see Figure 8. In terms of strength, the increase of the angle of inclination of horizontal loading lead to an increase of the ultimate load for masonry walls which had failure mode of flexure while in case of walls which had failure mode of shear, a small reduction of the ultimate load was observed. A big difference is observed in the behaviour of specimen evaluated with the angle of inclination of horizontal loading equal to 0° and the others. This behaviour can be explained by the fact that in this case only flanges are contributing to the stiffness and strength of the wall.

Observing the distribution of stresses in masonry walls, flanges resisted a fraction of normal stresses generated by the horizontal loading, see Figure 9a. Thus, the increase of the flange length led to the increase of the ultimate load resisted by the wall. On the other hand, flanges showed small values of shear stresses for all flange length and angles of inclination of horizontal loading. This fact indicated that the flanges did not contribute to the shear strength of masonry walls, see Figure 9b.

Results indicated that in flexure, masonry walls with flanges behave as a monolithic element when subjected to biaxial bending and the failure mode is global. On the other hand, in shear,
flanges improves the stiffness of wall but the failure mode is localized and governed by the shear strength of the wall independently of the flanges length.

![Graph](image1)

**Figure 8:** Force vs. Displacement of masonry walls with flange length = 2600 mm: (a) Cantilever walls and (b) Fixed-ends walls.

![Graph](image2)

**Figure 9:** Stresses in flanges of fixed-ends masonry wall with horizontal displacements applies with 45° and flange length equal to 2600 mm (a) normal stress and (b) shear stress.

**CONCLUSION**

The behavior of masonry walls with flanges subjected to biaxial bending was evaluated through numerical modeling. From the numerical analysis, the main following conclusion can be drawn:

(a) The flange length increases the stiffness of masonry walls subjected to lateral loading applied in any direction and increases the lateral strength independently of boundary conditions;
(b) In cantilever walls flexure is preponderant, whereas in fixed-end walls, the shear failure prevails.
(c) The angle of inclination of horizontal loading influences the flexure strength of masonry walls. In case of shear strength, this angle seems to have a very small influence.
Finally, this paper did not present practical design implications in this moment. However, results pointed out some aspects and indicators in the behavior of flanged walls that should be experimentally confirmed in order to clarify the behavior of these structures and to allow the development of accurate design models.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the funding provided by FAPESP – Fundação de Amparo à Pesquisa do Estado de São Paulo (grant no 2009/10053-9).

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