DEFINITION OF EQUIVALENT DAMPING FOR MASONRY STRUCTURES IN SUPPORT OF DISPLACEMENT BASED DESIGN

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In the last years, several researches have established the importance to correlate design limit
states to performance levels defined by displacement, in order to obtain a better control on
expected damage after an earthquake.
The displacement spectra proposed by code for damping values higher than the nominal value
of 5% of critical are generally obtained, as in the case of Eurocode 8 and the Italian Technical
Code (2008), by applying a scaling factor to the elastic response spectras 5% damped. This
factor is defined as a function of equivalent viscous
damping, which depends on hysteretic
energy dissipation.
The paper presents a procedure for determining the hysteretic damping for reinforced
masonry structures. The method makes use of the equivalent linear system, whose vibration
period is determined by nonlinear dynamic analysis on single degree of freedom idealized
system, by using synthetic spectrum compatible time histories. The analyses are performed
using an hysteretic model calibrated on experimental results obtained from cyclic shear
compression tests that were carried out by the University of Padova.

Keywords: masonry, displacement-based, energy dissipation, equivalent damping, dynamic analyses

INTRODUCTION

Force-based seismic design remains nowadays the most widespread method in the codes. In
spite of this, in the last decades (Moehle, 1992; Priestley, 1993), the awareness of how the
defformation, and not force, is the most meaningful parameter to quantify the expected damage
from an earthquake, has arisen in the scientific environment. Hence, in the following years,
several design methods based on this concept have been developed, called Displacement-
Based Design – DBD.
The method taken into account in this work is the Direct Displacement-Based Design
(DDBD), proposed by Priestley et al., (2007). It is based on the identification, in the initial
phase of design process, of a design displacement that ensures an acceptable damage level for
the considered seismic intensity. The method assumes that this displacement can be
determined without knowing the strength of structure. Another important concept of the
procedure is the equivalent structure, introduced by Gulkan & Sozen, (1974) and Shibata &
Sozen, (1976). This concept enables to represent the inelastic behaviour of a complex
structure through a single degree of freedom equivalent system. In such a way it is possible
the use of elastic displacement spectra, given by code, while taking into account the
deformation capacity of the real system.
DDBD method also needs the definition of equivalent viscous damping ($\xi_{eq}$). This parameter depends on the system capacity, when it is undergone to seismic action, to dissipate energy and it varies in function of structural typology. Once defined both design displacement and $\xi_{eq}$, it is possible to determine the effective period $T_{eff}$ of equivalent SDOF system by applying a reduction factor to the elastic displacement spectrum. This reduction factor is normally defined, as in the case of EN 1998-1: 2004. Eurocode 8, (2004) and DM 14/01/2008, (2008), utilizing the equivalent viscous damping.

Starting from effective period it is simple to compute the effective stiffness of equivalent SDOF system. Hence the design base shear is obtained by multiplying this stiffness for design displacement. The use of effective stiffness enables the evolving of inelastic forces related to given stiffness at each structural element.

In Italy, a research line (Linea 4) of the last RELUIS project has further developed the DDBD method for several structural typologies (Sullivan et al., (2009)). The results have been very heterogeneous thanks to the great difference among the initial states of art. Indeed, DDBD method is already developed in detail for some structural typologies (above all reinforced concrete structures), whereas for other typologies it is still at the beginning (for example retaining walls). The obtained results for the several typologies are collected in Linea IV final report (Calvi & Sullivan, (2009)).

In this context, the main aim of the research is to refine definition of energy dissipation in masonry structures so that the DDBD method can extend to such systems. In particular a procedure for the determination of equivalent viscous damping is presented. This procedure is composed by two phases. In the first it makes use of non-linear time-history (NLTH) analyses, by using an hysteretic model developed at University of Padova in order to reproduce the cyclic behaviour of reinforced masonry walls. In the latter, linear elastic analyses are performed on the equivalent system, in order to estimate equivalent viscous damping with the varying of displacement demand to the structure. Finally, the results of the analyses are presented and interpreted.

**MASONRY SYSTEM**

At the University of Padova, different hysteretic models have been developed, in order to best reproduce the response in non-linear range, for both unreinforced masonry (URM) (da Porto et al., (2009a)) and reinforced masonry (RM), (da Porto et al., (2008) and da Porto et al., (2009b)). The model used in this work is based on that proposed by Tomaževič & Lutman, (1996), and it has been calibrated on the results of cyclic shear compression tests on RM walls.

The specimens were tested with a cantilever-type boundary condition, with fixed base and top free to rotate, by applying centered and constant vertical load and horizontal cyclic displacements. Eight panels have been tested, differentiated by aspect ratio (1.09 and 1.64) and vertical load applied (0.4 and 0.6 N/mm²), with the aim to force both shear and flexural failure mode, (Mosele et al., (2008) and Mosele, (2009)). The main results gained from tests (da Porto et al., (2011)), in terms of drift ($\Psi$) and failure modes, are summarized in Table 1. During experimental tests, the attainment of four limit states (LS), which idealize the behaviour of the masonry wall, was observed: flexural cracking ($d_f$, $H_f$), critical ($d_{cr}$, $H_{cr}$), maximum resistance ($d_{Hmax}$, $H_{Hmax}$) and maximum displacement ($d_{max}$, $H_{dmax}$). The first LS corresponds to the appearance of first cracking, usually the first horizontal cracks on bed joints at the bottom of the panels. The second LS corresponds to the first diagonally oriented cracks (shear failure) or to yielding of vertical bars (flexural failure).
Flexural behaviour is characterized by a deformation capacity greater than that of shear, not only for what concerns the ultimate capacity, but also compared to critical (drift range 0.53 - 0.73% versus 0.3%) and maximum resistance (drift range 1.3 - 1.6% versus 0.7 - 1.0%) LS. On the contrary, the first LS occurs at drift levels of about 0.1% for both failure modes.

Table 1 Main results obtained from shear compression tests in terms of drift at the limit states

<table>
<thead>
<tr>
<th>Panel</th>
<th>$\Psi_f$</th>
<th>$\Psi_{cr}$</th>
<th>$\Psi_{Hmax}$</th>
<th>$\Psi_{dmax}$</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sa04</td>
<td>0.09%</td>
<td>0.30%</td>
<td>1.04%</td>
<td>1.20%</td>
<td>Shear/Flexural</td>
</tr>
<tr>
<td>Ta04</td>
<td>0.07%</td>
<td>0.31%</td>
<td>0.81%</td>
<td>1.20%</td>
<td>Shear/Flexural</td>
</tr>
<tr>
<td>Sa06</td>
<td>0.08%</td>
<td>0.31%</td>
<td>0.74%</td>
<td>0.90%</td>
<td>Shear</td>
</tr>
<tr>
<td>Ta06</td>
<td>0.09%</td>
<td>0.31%</td>
<td>0.73%</td>
<td>0.75%</td>
<td>Shear</td>
</tr>
<tr>
<td>Sb04</td>
<td>0.08%</td>
<td>0.53%</td>
<td>1.61%</td>
<td>2.90%</td>
<td>Flexural</td>
</tr>
<tr>
<td>Tb04</td>
<td>0.11%</td>
<td>0.73%</td>
<td>1.48%</td>
<td>3.60%</td>
<td>Flexural</td>
</tr>
<tr>
<td>Sb06</td>
<td>0.08%</td>
<td>0.53%</td>
<td>1.27%</td>
<td>1.81%</td>
<td>Flexural</td>
</tr>
<tr>
<td>Tb06</td>
<td>0.09%</td>
<td>0.68%</td>
<td>1.26%</td>
<td>2.74%</td>
<td>Flexural</td>
</tr>
</tbody>
</table>

CYCLIC BEHAVIOUR MODELLING

The model is characterized by the definition of an envelope curve and loading-reloading rules. The envelope curve is obtained considering the four limit states (LS) surveyed during the experimental tests. The loading-reloading rules have been developed from observation of experimental behaviour. Generally, for both failure modes, the following observations can be done:

1. The loading phase can be divided in two parts: the first part, with low displacement and high stiffness, and a second part with lower stiffness. The former remains almost unchanged among different loading cycles, with only low decay of stiffness values; conversely, the second phase presents high stiffness decay. Furthermore, the transition between these two parts occurs when forces and displacements are close to the first limit state and decreases with the increase of cycle amplitudes.

2. The unloading phase can be subdivided in three parts: the first characterized by a high value of stiffness which determines the width of cycle and the dissipated energy, the second in which stiffness is almost the same as in the second loading phase. Finally, the third phase where stiffness increases again and remains constant in the subsequent loading phase of the following loading cycle. The latter increase of stiffness happens when forces are similar to those at the stiffness change during the loading phase, giving the typical S form of hysteresis cycles.

Starting from these observations, the construction of the hysteresis loops has been based on the definition of four symmetrical points (A, B, C, D) (Figure 1). These points are found by means of two coefficients: $C_1$ and $C_2$, which are calculated by imposing the equality of the input energy and the dissipated energy between experimental and modelled loops.

The system is linear elastic until the displacements are smaller than the first limit state. Non-linear inelastic behaviour starts beyond this level of displacement. For this reason, the modelling of hysteretic cycles begins at the first cycle that goes beyond the elastic limit. Point A is placed on the first branch of the skeleton curve, namely the linear elastic phase. Its ordinate is expressed as a function of the maximum resistance ($H_{max}$) using the coefficient $C_1$. 
Point B represents the starting of unloading phase. The slope of the first unloading branch $K_{B-C}$ varies linearly in function of the current cycle width, from the values of elastic stiffness until the tangent stiffness at the second branch of envelope curve. The ordinate of point C is expressed as a function of point B by means of $C_2$ coefficient. Point D has the same ordinate of point A, and is found by imposing the equality of slope $K_{C-D}$ and $K_{A-B}$.

This scheme implies that point A remains constant and the amplitude of branch B-C increases until the attainment of maximum resistance, and then it decreases. These characteristics are in contrast with the experimental observations mentioned before. Indeed, with the increasing of cycle amplitude point A should move down and the cycle should open more. In order to remove this inconsistency the parameter $Z$ has been introduced. This parameter varies from one, for small displacements, and then decreases linearly. This parameter is utilized as correction factor in the calculation of $H_A$ and $H_C$, and allows to obtain modelled cycles more similar to the experimental ones, above all for what concern the input and dissipated energy.

The model is strongly based on obtained results by experimental tests, for what concern both the definition of envelope curve and the determination of coefficients $C_1$ and $C_2$, from which depend the reliability of the model. For this reason, the following analyses are related to reinforced masonry panels with the same aspect ratio and vertical load conditions of the shear compression tests that were carried out.

**PROCEDURE FOR THE DETERMINATION OF DAMPING**

The procedure is based on the difference, in terms of displacement, between non-linear analysis and linear analysis (Dwairi et al., 2007). It is possible to distinguish two phases: in the first phase nonlinear time history (NLTH) analyses are carried out using the hysteretic model above described. In the second phase, through an equivalent elastic system, equivalent viscous damping is defined.

For each time history considered, the following steps are performed:

1. Definition of target displacement. Hysteretic model considers the system response as elastic until the achievement of first limit state. Hence, target displacements are placed from first limit state until the ultimate displacement capacity. $n$ equal-spaced points subdivide the non-linear part of envelope curve (it was chosen $n = 7$).

2. Search of PGA multiplier factor. For each target displacement, NLTH analyses using the 10 synthetic time histories are carried out. Elastic damping is taken close to zero. These analyses are repeated scaling the TH using a multiplier factor of PGA until the maximum...
displacement achieved by NLTH is equal to target displacement with a specified tolerance. This is an iterative procedure where, at each iteration, the PGA is updated from previous one taking into account the difference between the actual maximum and the target displacements.

3. Determination of secant stiffness and effective period. When target displacement is achieved, the multiplier factor of PGA is known. Thus, the corresponding secant stiffness can be calculated. This is found in relation to the point, on the envelope curve, at target displacement (Figure 2). Knowing the secant stiffness it is easy to obtain the effective period:

\[ T_{\text{eff}} = 2\pi \cdot \left( \frac{m}{K_s} \right) \]  

4. Definition of Equivalent Linear System. The linear elastic equivalent system is defined through effective period (or the corresponding secant stiffness) given from previous step and from elastic component of damping equal to that one used in point 2. On this system is carried out a TH scaled using PGA in point 2. Maximum displacement obtained from this analysis represents the linear response of equivalent system for the considered elastic damping.

5. Search of equivalent viscous damping. Equivalent viscous damping (\(\xi_{\text{eq}}\)) means the value of damping which is able to make equal the displacement of equivalent linear system to target displacement. Hence, for each iteration, linear elastic analysis is repeated varying the damping value, starting from previous one and updating it on the basis of the difference between obtained and target displacement.

Elastic damping coefficient used in point 2 of procedure is close to zero because the aim is finding the damping component due to hysteretic dissipation. To avoid numerical problems the starting value of elastic damping was set at very low value. It was chosen a conventional value of 0.5% (one tenth of usual elastic damping component). The tolerance imposed in NLTH analyses to find the PGA was set to 2% of target displacement, while for linear elastic analyses was set to 1%.

![Figure 2 Determination of secant stiffness.](image-url)
SEISMIC INPUT
Dynamic analyses were carried out on 10 synthetic time-histories for each ground type considered. The time-histories were created in MATLAB™, at a sampling frequency of 100 Hz and a duration of 20.48 s. They are compatible with Eurocode 8 spectra, with a lower bound and upper bound of 10% of deviation between generated and code-prescribed spectra in the period range from 0.10 to 2.00 s (Figure 4).
Definition of the response spectra varies according to the different types of soils. The main five soil categories are: A, rock or other rock-like geological formation; B, very dense sand, gravel, or very stiff clay; C, medium-dense sand, gravel or medium stiff clay; D, loose-to-medium cohesionless soil or predominantly soft-to-firm cohesive soil; E, soil profile consisting of a surface alluvium layer.
The analyses were repeated for the two limit soil group classified, i.e. soil A and soil D. Despite soil E has a peak spectral acceleration higher than soil D, the latter shows a larger plateau that means a bigger seismic demand at medium-high periods (Figure 3).

RESULTS OF THE ANALYSES
For each modelled specimen (8) the procedure was repeated for each target displacement (7) and for each time history (10) generated for both ground type considered (A and D), giving a total number of 1120 analyses. Figure 5 shows the obtained results, divided for soil type. Each line in figures represents one experimental wall. Each one of seven dots composing the line is the mean value of 10 time histories.
Figure 6 presents the results in terms of equivalent viscous damping versus drift. Damping exhibits, from values close to zero (elastic component was set to 0.5%), an increasing trend of logarithmic type with the increasing of displacement. It can be noticed how, at the same drift level, a shear behaviour (red and orange curves) involves higher values of damping compared to flexural behaviour (blue and light blue curves). When shear failure occurs, it can be noted also that for displacement beyond maximum strength (drift 0.7÷1.0%) the damping shows a sudden increase, of about 50%, going from values around 10% until about 15%.
In Figure 6 the same results are showed, but displacement is non-dimensionalized to second limit state (dcr). This limit was chosen because it is representative of damage state in relation with failure mode. Indeed, as reported in Table 1, two drift levels can be considered at critical LS, 0.30% for shear behaviour and about twice this value for flexural behaviour (0.35÷0.7%). It can be noted that damping curves, related to this displacement ratio (DR), tend to be superimposed, for both failure modes and both soil types considered.
Furthermore, from results obtained for equivalent linear system, the effective period (Teff) of shear panels ranges from 0.1 s at elastic limit until about 0.35 s at ultimate displacement capacity. For flexural walls, indeed, this range is between 0.15 and 0.65 s. Taking into account the recommended spectra for ground types A and D, the period shift for all walls is included in plateau (TB < Teff < TC) with the exception of flexural walls under soil A (TC = 0.4 s). This explains why obtained curves for soil A and flexural behaviour (blue and light blue), for Displacement Ratio bigger than 2, tend to stabilize around 12%. Conversely, for soil D, whose spectrum is characterized by a wider plateau (TC = 0.8 s), it can be noted an always increasing trend, until values of about 20%.
Considering the same failure mode and same soil type, there are no sensitive differences between the two pre-load level applied: 0.4 N/mm², orange curves (shear) and light blue (flexure) and 0.6 N/mm², red curves (shear) and blue (flexure).
Figure 3 Elastic acceleration response spectra for the five soil categories.

Figure 4 Elastic response spectra for the utilized time-histories and code recommended spectra for ground type A (left) and D (right).

Hence, all results have been summarized, distinguishing only between failure modes and ground types. In this case single values for each TH are used, with the aim of finding the best fitting function (Figure 7). Least square method on several type of function has been used: linear, polynomial and logarithmic. Generally, the latter gave the best results, in terms of coefficient of determination $R^2$. It can be immediately noticed that for soil A the curves are almost superimposed, although for low values of DR, less than one, the flexural curve (blue) is above that of shear behaviour (red). On the contrary, for soil D, the flexural curve is always above that of shear, probably due to the increasing trend of obtained values, as above mentioned. The obtained equations are the following:

\[ \xi_{eq} = 5.0 \cdot \ln(DR) + 7.0 \]  
\[ \xi_{eq} = 3.7 \cdot \ln(DR) + 7.9 \]  
\[ \xi_{eq} = 5.1 \cdot \ln(DR) + 7.1 \]  
\[ \xi_{eq} = 5.4 \cdot \ln(DR) + 9.6 \]

Equations (2) – (3) are referred to soil type A, (4) – (5) to soil D, (2) and (4) are for shear failure whereas (3) and (5) are for flexural failure.
Finally, the curves defined by these equations are plotted in Figure 8. Colour red is used for shear failure, blue for flexural failure, continuous line for soil A and dashed line for soil D. The curves are calculated starting from RS equal to 0.5, until values of 4 for shear panels and 6 for the flexural ones, coherently with the surveyed limits. As can be seen, there are no
differences between soil types, for what concerns shear behaviour. Conversely, flexural behaviour involves rather different values, according to soil type. Curves for soil D describes values about 25% higher than those related to soil A. For soil A, flexural behaviour involved values (from 5 to 15%) similar to those of shear behaviour on both A and D soil types. Since it is recommended to apply capacity design for RM structures, until further indications related to other soil types will be available, it is suggested to consider the curve for flexure (similar to those of shear) on soil A. This leads to underestimate the damping factor for soft soils, hence to a conservative design.

The damping curves in Figure 8 may be considered not to be valid for a complete multi-story structure, since they have been obtained from analyses of single walls. Indeed, as a further check of the validity of these results, analyses on MDOF structures are actually being carried out, to evaluate the influence of the various structural configurations on the values of equivalent damping factors. On the other hand, since no sensitive influence related to pre-load has been found, at least as regard this factor, the proposed damping values can be considered valid for the equivalent SDOF system structure. In addition, as indicated by Priestley et al., (2007), it is also possible to evaluate the global damping by the weighted average based on the energy dissipated by the several structural elements.

Figure 8 Comparison between the obtained regression functions.

CONCLUSIONS
In this paper a procedure for the determination of equivalent viscous damping is presented, with reference to displacement based design. The procedure has been applied to reinforced masonry system, tested at the University of Padova with shear compression tests. Non-linear dynamic analyses are performed, using spectrum compatible time-histories. The obtained results are differentiated by failure mode, shear and flexural, and by ground type. It is pointed out how the damping is better correlated to non-dimensionalized displacement, through critical limit displacement, rather than drift. In addition, for the definition of damping for masonry structures, the most sensitive parameters are the failure mode and the type of soil. On the contrary, although the investigated range is quite limited, it has not been observed any dependency to the applied pre-load level. The trend of damping versus DR can be considered of logarithmic type. For shear behaviour, there are no differences between soil types. Conversely, flexural behaviour involves different values, according to soil type. Considering the obtained results, and that for RM structures it is possible to apply capacity design, it is suggested that values of DR for damage and ultimate LSs can be 1 and 3 respectively, and equivalent viscous damping is taken as 8% and 12%.
The future developments of research concern the extension of study by the use of a fibre model (Guidi, 2011)) with which the monotonic envelope of reinforced masonry walls can be obtained. This model takes into account the effects of shear deformation, diagonal shear failure mechanism and it is able to follow response in post-peak phase. The goal is to extend the experimental results using parametric analyses, in order to calibrate simplified formulations for the determination of limit states. Hence will be possible to determine, through the described procedure, damping for various layouts of RM systems.

REFERENCES