COMPARISON OF IN-PLANE SHEAR STRENGTH OF REINFORCED CONCRETE MASONRY WALLS

Hoque, Nusrat¹; Lissel, Shelley L.²

¹MSc Student, Dept. of Civil Engineering, University of Calgary, nusrat_hoque@yahoo.com.
²PhD, Associate Professor, Dept. of Civil Engineering, University of Calgary, sllissel@ucalgary.ca

Abstract: Masonry has long been used in construction practice due to its easy construction method, low cost and material availability. However, the performance of masonry, in particular unreinforced masonry, during earthquakes is quite poor and that makes it less popular in earthquake prone areas. Masonry is often used in construction of shear walls intended to resist seismic loading thus the in-plane shear strength has been a research interest for masonry researchers around the world in order to come to a safe but more realistic estimate of the strength. In masonry design codes in various countries, the in-plane shear strength modelled with different equations which tend to greatly overestimate the strength leading to inefficient and expensive construction. There is also no general agreement on the parameters that can affect the strength and this also reflects in the variability of assumed contribution of the different parameters. In this paper three design equations have been chosen for study. These equations are compared using data from various sources to determine which equations account for the contribution of various parameters and with how much accuracy. Subsequently, modifications to the most suitable equations have been proposed and also compared with the test results.

Keywords: In-plane shear, aspect ratio, design equations, grouted.

1. Introduction
Masonry structures exist and continue to be built all over the world. In areas prone to seismic activity, predicting the strength of masonry shear walls is of particular importance. Quite a lot of research has been carried out on reinforced masonry shear walls made of concrete block or clay brick in the United States, Canada, New Zealand, and Japan. Based on this research, various equations have been developed to predict the in-plane shear strength of masonry walls, mainly by statistical analysis of the test data. This paper aims to compare three of these equations for predicting shear strength of masonry walls using results from various sources and thus evaluate the effectiveness of each equation.

2. Shear Strength and Different Equations
The shear mechanism in masonry is quite a complex phenomenon. As a result, there have been numerous models developed by various masonry researchers. For practical purposes, when calculating the nominal shear strength, commonly the contribution from masonry, reinforcement and axial stress are calculated separately and simply added. In this paper, three equations for determining in-plane strength of masonry walls are evaluated to determine how effective they are in estimating strength by comparing the predicted strengths to test data...
reported by Shing et al (1990), Matasumara (Matsumura, 1988), Shedid et al (2008), Voon (Voon, 2007), Woodward and Rankin (1985), Sveinsson et al (1985), Tomazevic and Lutman (1988), Priestley (1976), Schultz (1996), Hidalgo et al (1978), and Chen et al. (1978). The selection of these equations is based on the criteria that two of these, namely the Anderson and Priestley (1992) equation and the NEHRP (1997) equation, are the basis of the equations in the Canadian and U.S masonry design codes respectively, and the Shing equation is the one most commonly used in research.

2.1 Shing equation (Shing et al, 1990)
Shing et al (1990) proposed an equation for predicting shear strength based on the test results from 22 walls. The proposed formula was also verified by the test results obtained from Sveinsson’s (Sveinsson et al, 1985) research. In the formulation of this equation, the shear strength depends mainly on the residual masonry strength and the amount of horizontal reinforcement. The residual masonry strength depends in turn on the amount of vertical reinforcement and compressive axial stress as these helps resist crack opening and thus increases the aggregate interlock force and the residual strength of the masonry. In evaluating the strength contribution of the horizontal reinforcement, Shing et al (1990) proposed that since diagonal cracks occur at approximately 45 degree angles, only the bars spread over the height of the wall will be activated by the 45 degree crack whereas the horizontal reinforcing bars in the top and bottom courses would not have adequate development length to develop tensile resistance. With these considerations, the equation takes on the following form.

\[ V_s = \left(0.166 + 0.0217 \rho_v f_y\right) \sqrt{f_m'} + 0.0217 \sqrt{f_m' \sigma + \frac{l-2d'}{L} \left(\frac{A_h f_yh}{L}\right)} \]  \hspace{1cm} (1)

Where
\[ d' = \text{depth to vertical reinforcement from the edge of the wall in the cross-section.} \]
\[ \rho_v = \text{Vertical reinforcement ratio.} \]
\[ f_y = \text{Yield Strength of Vertical Steel.} \]
\[ f_m' = \text{Compressive Strength of Masonry.} \]
\[ \sigma = \text{Vertical Stress.} \]
\[ A_h = \text{Area of Horizontal Reinforcement.} \]
\[ L = \text{Length of the Wall.} \]
\[ f_yh = \text{Yield Strength of Horizontal reinforcement.} \]
\[ t = \text{Thickness of wall.} \]

2.2 Anderson and Priestley (1992)
Here there are three proposed modes of failure; namely diagonal shear, sliding shear, and flexure. Anderson and Priestley (1992) outlined formulae for each mode of failure containing various coefficients for various parameters. These coefficients were then evaluated by fitting them to test data obtained from three sources: Sveinsson et al (1985), Shing et al (1990) and Matasumara (Matsumura, 1988). According to their results, the contribution from shear steel in masonry is half that of that of reinforced concrete structures and surprisingly the vertical reinforcement had no effect on strength. It was also concluded that since the predictive equation did not appear to fit one set of data better than the other there may not be any dependency of wall shear strength on aspect ratio.
\[ V = C_{ap} k \sqrt{f_{m}} + 0.25 \sigma + 0.5 A_s f_{ys} \frac{d}{s A_n} \]  \hspace{1cm} (2)

Where

- \( C_{ap} = 0.24 \) for concrete
- \( = 0.12 \) for clay Brick
- \( k = 1 \) for a flexural ductility ratio of up to 2 and then decreases linearly to zero at a ductility ratio of 4.
- \( s = \text{Spacing of Horizontal Reinforcement} \)
- \( A_n = \text{Net cross-sectional Area} \)

### 2.3 NEHRP (1997)

The National Earthquake Hazard Reduction Program (NEHRP) encourages U.S design and building practices addressing earthquake hazards so that the resulting damage may be minimized. The NEHRP equation is similar to the Anderson and Priestley equation with the exception of a modification to account for the aspect ratio and then the equation becomes:

\[ V = 0.083 \left[ 4 - 1.75 \frac{M}{W} \right] \sqrt{f_{m}} + 0.25 \sigma + 0.5 A_s f_{ys} \frac{d}{s A_n} \]  \hspace{1cm} (3)

### 3. Comparison Between Equations

To compare the equations, data for which a change in only one parameter is evident was chosen from a comprehensive database of masonry wall test results (the database has not been included here in the paper due to space limitation).

#### 3.1 Effect of axial stress:

Woodward and Rankin (1985) performed testing on walls which had no reinforcement at all. The only variables were axial stress and aspect ratio. The experimental results for three walls having the same aspect ratio with varying axial stress were compared to the predicted value obtained from the three equations as shown in Figure 1.

![Figure 1: Comparison of Axial Stress and Strength for the test results of Woodward and Rankin (1985)](image)
In all cases the axial stress was calculated based on the gross area of the wall. Figure 1 shows that the predicted strengths from Shing’s equation were closest to the actual strength; however, all of the equations show the general trend of increasing shear strength with axial stress that is exhibited by the test results in this range of axial stress. So the axial stress term used in Shing’s equation can be regarded as most effective in predicting actual strength contribution from axial stress.

A similar comparison was made using data obtained from testing at the University of Calgary (Dickie and Lissel, 2011) as shown in Figure 2. Figure 1 and 2 show the same trend; in both cases the NEHRP equation overestimates the value more than any of the other equations.

![Strength vs. Axial Stress](image)

**Figure 2: Comparison of Axial Stress and Strength for the test results of Dickie and Lissel (2011)**

3.2 Effect of horizontal reinforcement: It is obvious from the test data of Voon (2007) shown in Figure 3 that increasing the horizontal reinforcement increases the shear strength up to a certain limit after which the shear strength remains constant. All of the equations captured this phenomenon quite well except for a slight decreasing trend after the upper limit is reached. Shing’s equation exhibits the largest variation in predicted values while the Anderson and Priestley equation consistently overpredicts the strength. Both the NEHRP and Anderson and Priestley equations show the same general relationship between shear strength and amount of horizontal reinforcement, the only difference being the term that takes into account the contribution of the masonry. The NEHRP equation accounts for the effect of aspect ratio as mentioned previously, which appears to make this equation more accurate in predicting the shear strength.
Figure 3: Comparison of Horizontal reinforcement ratio and Strength for the test result of Voon (2007)

So the contribution of horizontal reinforcement to in-plane strength appears to be approximately half the strength of the reinforcement provided. The NEHRP equation predicts a slightly larger value than the actual one which may be due to the term containing the contribution of masonry and aspect ratio.

3.3 Effect of Vertical Reinforcement: Neither the Anderson and Priestley equation nor the NEHRP equation contain a term that accounts for the vertical reinforcement, however the vertical reinforcement does have an influence on the strength as is apparent from the test results of Shedid et al (2008) shown in Figure 4. The slight increasing trend of the value for these two equations is due to the slight increase in horizontal reinforcement. Shing’s equation gives the largest variation in strength in this case. Since the contribution of horizontal reinforcement and aspect ratio are predicted most effectively by the NEHRP equation but the contribution of vertical reinforcement is only captured by Shing’s equation, then it stands to reason that adding this term to the NEHRP or Anderson and Priestley equations may give predicted values much closer to the actual ones; though this might be a bit overestimated as mentioned before due to the masonry term in the NEHRP equation.

Figure 4: Comparison of Strength and Vertical Reinforcement ratio.
3.4 Modified Equation: Combining these observations a proposed equation takes the following form:

\[
V = 0.083 \left( 4 - 1.75 \frac{M}{VL} \right) \sqrt{f_m} + 0.0217 (\rho_y f_{y,y} + \sigma) \sqrt{f_m} + 0.5 A_x f_{y,x} \frac{d}{sA_n} \ldots \ldots \ldots \ldots (4)
\]

If the aspect ratio remains small then this modified equation overestimates the shear strength, however it underestimates the results when the aspect ratio increases and is greater than 2. In those cases, further modification to the first part of the equation is necessary since this is the term that accounts for the masonry contribution. This term, in the NEHRP equation, also depends on the aspect ratio, so this requires further examination.

3.5 Effect of Aspect Ratio: To examine the effect of aspect ratio on shear strength, data was selected from Matasumara’s (Matsumura, 1988) experimental study with the same axial stress and horizontal reinforcement ratio and slightly different vertical reinforcement but varying aspect ratios. The curve fitting yields the following equation:

\[
V = 0.051 \left( 4.1 - \frac{M}{VL} \right) \sqrt{f_m} + 0.0217 (\rho_y f_{y,y} + \sigma) \sqrt{f_m} + 0.5 A_x f_{y,x} \frac{d}{sA_n} \ldots \ldots \ldots \ldots (5)
\]

![Figure 5: Strength Dependency on Aspect Ratio](image)

3.6 Effect of support conditions and grouting: In all the test data available, two types of support conditions and grouting were observed. The data contains test results for both single curvature (cantilever) and double curvature (fixed) bending support conditions as well as both partial and full grouting of masonry. None of the equations discussed herein; nor the proposed equation considers these conditions. Therefore, a comparison of the predicted shear strengths was carried out separating the data by type of support conditions and grouting.
4. Results
In-plane shear strength values were compiled from test results on 182 fully, partially grouted and ungrouted shear walls of both concrete and clay masonry walls selected from various sources. All experimental studies subjected walls to in-plane cyclic lateral loading and utilized displacement controlled testing. The majority of the walls were tested using fixed-fixed (double curvature) conditions but walls emulating cantilever (single curvature) conditions were also evaluated. The ratios of predicted to actual strength for walls tested under the fixed and cantilever conditions were determined for the different equations. The mean results have been tabulated in Table 1 where it is obvious that better correlation is achieved using the improved modified equation and this equation is more reliable in the case of walls tested under cantilever conditions than for the fixed condition case. It does, however, slightly underestimate the strength for the double curvature case. The Anderson and Priestley equation gives the most variable predicted values compared to any other equation. Similar calculations were carried out for the cases of fully grouted, partially grouted, and ungrouted walls, and the results are tabulated in Table 2 (mean ± S.D.).

Table 1: Comparison of the ratio of predicted strength to test strength using different equations for different support conditions

<table>
<thead>
<tr>
<th>Equation</th>
<th>Ratio (Single curvature)</th>
<th>Ratio (Double curvature)</th>
<th>Standard Deviation (Single Curvature)</th>
<th>Standard Deviation (Double Curvature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shing et al</td>
<td>1.54</td>
<td>1.28</td>
<td>0.57</td>
<td>0.48</td>
</tr>
<tr>
<td>Anderson and Priestley</td>
<td>1.62</td>
<td>1.49</td>
<td>0.76</td>
<td>0.87</td>
</tr>
<tr>
<td>NEHRP</td>
<td>1.12</td>
<td>0.87</td>
<td>0.39</td>
<td>0.58</td>
</tr>
<tr>
<td>Improved (eqn.4)</td>
<td>1.00</td>
<td>0.94</td>
<td>0.37</td>
<td>0.49</td>
</tr>
<tr>
<td>Modified Improved (eqn. 5)</td>
<td>1.22</td>
<td>1.00</td>
<td>0.41</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Table 2: Comparison of the ratio of predicted strength to test strength using different equations for different grouting

<table>
<thead>
<tr>
<th>Equation</th>
<th>Ratio (Full grouting)</th>
<th>Ratio (Partial grouting)</th>
<th>Ratio (Ungrounded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shing et al</td>
<td>1.29±0.56</td>
<td>1.49±0.42</td>
<td>1.27±0.15</td>
</tr>
<tr>
<td>Anderson and Priestley</td>
<td>1.66±0.64</td>
<td>1.42±0.96</td>
<td>2.32±0.39</td>
</tr>
<tr>
<td>NEHRP</td>
<td>0.92±0.34</td>
<td>0.84±0.75</td>
<td>2.05±0.25</td>
</tr>
<tr>
<td>Improved (eqn.4)</td>
<td>0.80±0.33</td>
<td>1.20±0.56</td>
<td>1.45±0.19</td>
</tr>
<tr>
<td>Modified Improved (eqn.5)</td>
<td>0.97±0.39</td>
<td>1.18±0.37</td>
<td>1.24±0.17</td>
</tr>
</tbody>
</table>

5. Conclusions
The in-plane shear strength of masonry has been a research subject for a number of decades due to the lack of understanding of the behaviour of walls under in-plane load. Three equations were chosen from those available in the literature for the purpose of comparison. The comparisons made in this paper show that aspect ratio, horizontal reinforcement ratio, vertical reinforcement ratio, axial stress all tend to increase strength; but all the existing equations do not seem to account for the effect of all parameters properly. The different equations consider the contribution of different parameters almost equally or with slightly higher estimation than the test results. Therefore, an attempt has been made in this paper to find an optimum solution for in-plane strength determination based on the results of past research. A simple additive approach was used for this process by taking into consideration the contribution of several parameters affecting strength. A modified equation was proposed by combining different terms from the equations compared. The resulting equation was then compared with a wide range of test data and further improved. The final modified improved equation predicts the strength more accurately than the others, but is slightly non-conservative in some cases.

Acknowledgement: The authors wish to acknowledge the financial support provided by NSERC (Natural Science and Engineering Research Council), the Masonry Contractors Association of Alberta (Southern Region) and the Canadian Concrete Masonry Producers Association. In addition, special thanks to Jocelyn Dickie (Ph.D student) for her support.
References:


