EXPERIMENTAL EVALUATION OF THE COEFFICIENT OF RESTITUTION OF ROCKING STONE MASONRY FAÇADES

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Masonry façades are known to behave poorly under seismic actions being one of the main goals of worldwide research to simulate adequately the dynamic behaviour of these elements. The simulation of masonry façades as rigid bodies with dynamic characteristics is one of the possible solutions to assess the dynamic behaviour of existing structures. Taking this into account, an experimental test procedure is presented to assess one main parameter (coefficient of restitution) that influences the global response of a rocking system.

The test procedure is fully described concerning the experimental test apparatus and monitoring devices, as well as the obtained experimental results and their correlation to theoretical values. Finally, some conclusions are drawn regarding the efficiency of the test procedure and the obtained values for the coefficient of restitution.

**Keywords:** Coefficient of restitution, rocking, stone masonry, out-of-plane

INTRODUCTION

The history reveals that earthquakes and masonry structures do not cohabit well. Indeed, earthquakes are a major cause of damage and collapses of unreinforced masonry structures, as the lighthouse of Alexandria or the Colossus of Rhodes.

The behaviour of unreinforced masonry structures, especially the existing and historical ones, is not governed only by the in-plane behaviour of the masonry elements but it is strongly affected by the out-of-plane performance (affected by the connections between elements), leading to the formation of local mechanisms, such as those presented in Figure 1 and observed in L’Aquila 2009 post-earthquake surveys.

Due to their distributed mass and small redundancy, masonry façades and parapets are highly vulnerable to ground motions, especially when near-source effects as velocity pulses are contained in the earthquake frequency content.
The dynamic behaviour of these elements may be simulated as rigid elements’ rocking around the base, as considered by Giuffré (1993); Sorrentino et al. (2008); D’Ayala & Shi (2011). The model which represents this behaviour, firstly introduced by Housner (1963) for inverted pendulum type structures, is represented by the motion equation (1), based on the equilibrium of a rocking body as shown in Figure 2.

\[ I_0 \dot{\theta}(t) = -W R_0 \sin[\alpha_0 - \theta(t)] \]  

(1)

In the previous equation, \( I_0 \) stands for the rotational inertia, \( \dot{\theta} \) is the angular acceleration corresponding to the block rotation \( \theta(t) \), \( W \) is the block weight, while \( R_0 \) and \( \alpha_0 \) are geometrical parameters determined relative to the element mass centre.

In this model, the energy contained in the element is only dissipated by impacts at the base, being this dissipation represented by the so-called coefficient of restitution (\( r \)). This coefficient, dependent of the contact interface as tested by ElGawady et al. (2011), was firstly presented by Housner (1963) as the ratio of kinetic energy before and after the impact, and afterwards defined by the ratio of angular velocities (before and after the impact) by Aslam et al. (1978). Because the latter definition is commonly used in several works in this subject.
(e.g. Liberatore & Spera (2001); Makris & Konstantinidis (2003); Peña et al. (2007); Sorrentino et al. (2011)), it will be also used in the present paper.

For a rectangular element, this restitution coefficient is bounded by a theoretical upper limit \( r_{\text{max}} \), determined by the geometrical parameters of the block and defined by Equation (2).

\[
r_{\text{max}} = 1 - \frac{3}{2} \sin^2 \alpha_0
\]  

(2)

However, discrepancies between the theoretical value and experimental ones were obtained by several authors in tests of single blocks made of different materials (e.g. Priestley et al. (1978); Wong & Tso (1989); Peña et al. (2007), among others). In what concerns the previous works where single blocks were tested. Once again, differences were obtained between theoretical and experimental values, leading to a proposed ratio of \( r / r_{\text{max}} = 0.95 \).

In this work, a novel testing technique is proposed in order to infer the coefficient of restitution of a sack stone masonry façade, resorting to an equivalent block approach, described in the following section.

**EQUIVALENT BLOCK APPROACH**

The behaviour of a block rocking around the base is determined based on the mass and rotational inertia properties, which depend on its geometrical properties. If Equation (1) is carefully observed, an equivalent structure (where \( I_0, W, R_0 \) and \( \alpha_0 \) are taken similar to the original rocking block) is likely to reproduce correctly the dynamic behaviour of the block. This may be the case of a stone masonry façade rocking around its base (or first bed joint level), as illustrated in Figure 3 a), where an equivalent block with similar properties is adopted to simulate the behaviour of the full façade.

![Figure 3: Out-of-plane rocking behaviour of a stone masonry façade: a) schematic representation; b) parameters involved in the rocking motion for a flexible interface; c) assumed complete section partialization and modified rotation axis](image)

However, stone masonry walls are usually made of mortar joints with limited compressive strength and null, or negligible, tensile strength. Thus, according to Figure 3-b and Figure 3-c, the rocking axis for this type of elements may not be placed at the block edge O, moving to a new position O’ due to the actual compressive strength of the mortar joint which shifts...
inwards the reaction force $W$ located at a distance $a_f$ from the edge. In that case, the dynamic equilibrium condition is modified and expressed by Equation (3), a procedure also suggested by Giuffrè (1993).

$$I_{O'}[\theta(t)] \cdot \dot{\theta}(t) = -W \cdot \left[ R_0 \sin[\alpha_0 - \theta(t)] - a_f[\theta(t)] \right]$$

wherein the terms $W$, $R_0$ and $\alpha_0$ are determined for undeformed position, while the parameter $I_{O'}$ (relative to the new axis position $O'$) depends on the rotation $\theta(t)$ and, therefore, is also time dependent.

By writing $p^2 = WR_0/I_{O'}$, Equation (3) may be simplified to Equation (4), where $p$ depends on the block rotation.

$$\dot{\theta}(t) = -p^2 \left[ \sin[\alpha_0 - \theta(t)] - \frac{a_f[\theta(t)]}{R_0} \right]$$

For a block equivalent to the original structure, having $h_{eq}$ and $t$, respectively for the height and thickness, the value of $p$ (representative of the dynamic characteristics of the block) may be obtained by Equation (5)

$$p = p[\theta(t)] = \sqrt{\frac{g R_0}{\frac{1}{12} t^2 + h_{eq}^2 + R[\theta(t)]^2}}$$

where, $R$ is the sole parameter related to the rotated block whose influence in the final result may be directly evaluated.

Resorting to Figure 3, the value of $R$ can be obtained directly as the distance from the block gravity centre to base rotation axis $O'$ located at a distance $a$ from the block edge. In order to simplify the computations, the section may be considered completely partialized for the rotated position, which yields the distance $a_f$ definition given by Equation (6), where $f_m$ stands for the masonry compressive strength and $l$ is the block width. Under the assumption of section partialization, it follows that the new rotation axis position is given by $a = 2 a_f$ and, using elementary geometrical relations, the distance $R$ may be obtained from Equation (7), where $y_{cg}$ is the height of the block gravity centre.

$$a_f(\theta) = \frac{W}{2 f_m t}$$

$$R[\theta(t)] = \sqrt{\frac{y_{cg}^2}{2} + \left[ \frac{t}{2} - a[\theta(t)] \right]^2}$$

The influence of $R$ and $I_{O'}$ in the final value of $p$ was evaluated in terms of the rocking block dynamic properties. Figure 4 presents that evaluation for slenderness ratios ($\lambda$) ranging from 1 to 8, where a negligible influence is observed for $\lambda > 4$. Indeed, masonry façades usually have slenderness ratios $\lambda > 4$, as for the case of the walls simulated by the experiments presented in
this paper, which means that using the constant $p_0$ value (relative to the block edge) can provide quite good approximations of the actual (variable) $p$ value.

Figure 4: Influence of the rotation axis position and of the block slenderness in the dynamic properties of the system for a flexible interface

Therefore, if the value of $p$ is correctly reproduced in an equivalent structure as well as the position of the rotation axis $a = 2a_f$ (Equation (6), considering full section partialization), that equivalent structure is suitable for simulating the behaviour of a masonry façades with similar inertia and geometrical parameters. This is the basis of the equivalent block approach which was adopted to perform the experiments reported in this paper.

SPECIMENS AND TEST SETUP

The performed testing campaign aimed at characterizing the coefficient of restitution of sack stone masonry façades for 2-sided rocking under free vibration. For this purpose, the simulation of the rocking behaviour of a masonry façade was made resorting to a small masonry portion (representative of the potentially cracked section at the wall base) rigidly connected to a very stiff top mass. This rigid mass was used to avoid the flexible behaviour of the wall and is the equivalent structure adopted to simulate the entire masonry façade sketched in Figure 3 a). As shown in Table 1, good agreement was obtained between the equivalent block and the real structure regarding the dynamic parameters $p$ and $\alpha_0$, which means that the free rocking response may be adequately reproduced by this approach.

### Table 1. Properties of the equivalent block and comparison with the original structure

<table>
<thead>
<tr>
<th>System</th>
<th>Weight [kN]</th>
<th>Height [m]</th>
<th>Rotational inertia [kg m$^2$]</th>
<th>$R_0$ [m]</th>
<th>$p$ [rad/s]</th>
<th>$\alpha_0$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>48.2</td>
<td>2.7</td>
<td>15426</td>
<td>1.535</td>
<td>2.19</td>
<td>0.213</td>
</tr>
<tr>
<td>FR1</td>
<td>51.2</td>
<td>2.6</td>
<td>16107</td>
<td>1.525</td>
<td>2.20</td>
<td>0.215</td>
</tr>
<tr>
<td>FR2</td>
<td>52.7</td>
<td>2.7</td>
<td>17652</td>
<td>1.574</td>
<td>2.17</td>
<td>0.208</td>
</tr>
</tbody>
</table>

The complete specimen is presented in Figure 5, showing the adopted set-up for out-of-plane pulling the equivalent block and suddenly releasing it at a predefined rotation level ($\theta_0$), in order to evaluate the free rocking response with that initial rotation.
The initial rotation was given by a hydraulic actuator connected to the block through a thin high-strength steel bar, which was suddenly cut to trigger the free rocking experiments made on two similar specimens (FR1 and FR2) tested for 4 different initial rotation levels $\theta_0$.

Investigating the effects of repetition in the restitution coefficient was also one of the aims of the tests. For this purpose, several repetitions were performed for the same rotation level, as well as some final repetitions at the end of the tests. However, due to limitations in the paper length, this information is not presented here. In total, 36 tests were made as listed in Table 2.

![Figure 5: Specimen test under free rocking motion: a) test apparatus; b) pullout scheme](image)

Table 2. Sequence of the experimental tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\theta_0/\alpha$</th>
<th>Test name</th>
<th>Tests number</th>
<th>Tests</th>
<th>Total number of tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1</td>
<td>0.15</td>
<td>L1</td>
<td>1, 2, 3, 4, 5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>L2</td>
<td>1, 2, 3, 4, 5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>L3</td>
<td>1, 2, 3, 4, 5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>L4</td>
<td>1, 2, 3</td>
<td>3</td>
<td>18 + 3 repetitions at the end</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>L3</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>L2</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>L1</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FR2</td>
<td>0.35</td>
<td>L3</td>
<td>1, 2, 3</td>
<td>3</td>
<td>9 + 3 repetitions + 3</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>L2</td>
<td>1, 2, 3</td>
<td>3</td>
<td>9 + 3 repetitions + 3</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>L1</td>
<td>1, 2, 3</td>
<td>3</td>
<td>9 + 3 repetitions + 3</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>L3</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>L2</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>L1</td>
<td>R</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>L4</td>
<td>1, 2, 3</td>
<td>3</td>
<td>9 + 3 repetitions + 3</td>
</tr>
</tbody>
</table>

(*) Data not acquired due to acquisition problems

The data was acquired resorting to 24 LVDTs (Linear Voltage Displacement Transducers), 5 wire transducers (potentiometers), 8 accelerometers and 2 bi-axial tiltmeters, in accordance to Figure 6. The data acquisition was made with National Instruments (NI) hardware at 4000 Hz sampling frequency, while the acquisition software was developed by the authors within the LabVIEW platform (also form NI). The initial rotation ($\theta_0$) was read using the tiltmeters, while the rotation time histories were obtained through the wire transducers adopting a procedure similar to Mouzakis et al. (2002) due to the poor dynamic response of the tiltmeters.
EXPERIMENTAL RESULTS AND DATA INTERPRETATION

The test setup used permitted obtaining the correct dynamic rocking behaviour of the tested specimens, as presented in Figure 7. It should be referred that some discrepancies were observed in some of the experiments, as evidenced in Figure 7 for L3-1. However, the scatter was reduced and the results were very satisfactory. The angular velocity time histories were obtained by differentiating the rotation time histories.

It is worth emphasizing that this experimental campaign had two main objectives: i) evaluation of the developed test setup efficiency for simulating the free rocking behaviour of masonry façades; ii) determination of the restitution coefficient of free rocking masonry façades.

For the first purpose, the obtained results can be compared to theoretical values obtained from the dynamic differential equation taking into account an initial rotation value $\phi_0 = \theta_0/\alpha$, presented initially by the classic rocking theory Housner (1963) without initial angular
velocity or acceleration. For this purpose, a comparison was made between the rocking period and the initial rotation given by Equation (8), being the results presented in Figure 8.

$$T = \frac{4}{p} \cosh^{-1} \left( \frac{1}{1-\varphi_0} \right)$$  \hspace{1cm} (8)

![Figure 8: Comparison between theoretical curve and experimental values](image)

The determination of the experimental rocking periods ($T$) was computed with the angular velocity peaks, while the peak rotation values ($\theta/\alpha$) were obtained directly from the rotation time histories.

From Figure 8, it is possible to conclude that the experimental test setup correctly simulated the rocking behaviour of a stone masonry façade under free vibration regime. Moreover, it should be referred that better correlation between the theoretical and experimental values can be obtained if the flexible interface is considered, making use of Equations (6) and (7) to determine the rotation point ($a$) and, subsequently, the geometrical value $\alpha$.

After validating the test setup, the determination of the coefficient of restitution ($r$) was pursued adopting two different approaches: the first relies exclusively on pure experimental evidence, using directly the velocity records, while the second is based on the classic rocking theory and the experimental rotation readings.

For the first mentioned case, Equation (9) was used where $\dot{\theta}_n$ is the peak velocity at impact $n$, while $\dot{\theta}_{n+2}$ is the peak velocity at impact $n + 2$.

$$r = \sqrt{\frac{\dot{\theta}_{n+2}}{\dot{\theta}_n}}$$ \hspace{1cm} (9)

Concerning the determination based on the classic rocking theory, commonly used by several authors in previous works, the restitution coefficient may be obtained directly from Equation (10), where $n$ stands for the $n$-th impact, while $\varphi_0$ is the initial rotation ($\theta_0/\alpha$) and $\varphi_n$ is the rotation at the $n$-th impact ($\theta_n/\alpha$)
\[ r = \sqrt{\frac{1 - (1 - |\phi_0|)^2}{1 - (1 - |\phi_0|)^2}} \]  

(10)

Taking into account both approaches for the computation of the restitution coefficient and the large amount of data available for such calculation, the coefficient of restitution was determined for each test level and for both specimens. The main results are presented in Figure 9.

Figure 9 shows that the values obtained exclusively from the experimental evidence are lower than those computed using the classic theory with measured rotations. Moreover, when compared to the maximum theoretical value \( r_{max} \), computed using Equation (2) with the values presented in Table 1, similar or even higher restitution coefficient values maybe obtained with the classic theory. In order to summarise the most relevant information, Table 3 presents a comparison between the obtained values through both approaches and the theoretical maximum value \( r_{max} \).

Table 3: Restitution coefficient values obtained in the free rocking tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Classic theory</th>
<th>Experimental evidence</th>
<th>Theoretical maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>( r/r_{max} )</td>
<td>Average</td>
</tr>
<tr>
<td>FR1</td>
<td>0.895</td>
<td>0.96</td>
<td>0.819</td>
</tr>
<tr>
<td>FR2</td>
<td>0.931</td>
<td>1.00</td>
<td>0.895</td>
</tr>
<tr>
<td>Average</td>
<td>0.913</td>
<td>0.98</td>
<td>0.857</td>
</tr>
</tbody>
</table>

Table 3, evidences that an average lower bound of 0.88 was obtained (FR1, from the exclusive experimental evidence), while 1.00 was the average upper limit output (computed from the classic theory). For this reason, the proposed value of \( r/r_{max} = 0.95 \) as proposed by Sorrentino et al. (2011) seems not advisable for this type of masonry and, therefore, a conservative value of \( r/r_{max} = 1.0 \) may be suggested. However, as shown in Figure 9, slightly...
higher values may be obtained, which means that \( r_{max} \) may not be seen as the maximum upper bound of the restitution coefficient.

**CONCLUSIONS**

This paper focused on the experimental determination of the restitution coefficient commonly used in the dynamic models which simulate the out-of-plane behaviour of masonry façades. For this purpose, a novel test setup based on the so-called equivalent block approach was developed, ensuring a proper rocking response of a stone masonry portion at a predetermined level. The method was validated against theoretical formulae, while the restitution coefficient was obtained resorting to both purely experimental information and to theoretical computation with experimental results. Finally, ratios of \( \frac{r}{r_{max}} \) = 0.88 and 1.00 were obtained as lower and upper bounds suggestions for the coefficient of restitution of sack stone masonry walls.

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