SUMMARY
For many years the Colosseum has been the object of interdisciplinary research aimed at assessing its structural behaviour and safety level. Studies of the historical documentation, in situ investigation and analytical modelling have allowed us to understand the partial collapse causes. In this paper we are too proposed some mathematical models, which are carried out to allow the safety assessment in present situation after the restoration works in the 19th century. The results are discussed and the weakness of the monument with regard to earthquakes is outlined. The zones requiring strengthening are studied in detail and some preliminary criteria of intervention are proposed.

1. THE HISTORY AND THE STRUCTURAL BEHAVIOUR
1.1 From 72 a.D. to 443 a.D. - The first centuries
1.1.1 The history
The Flavius Amphitheatre, whose construction lasted about ten years, was dedicated and opened to public in 80 a.D., during the reign of Titus (fig. 1). The works presumably started in 72 a.D. with the simultaneous construction of four similar portions separated on plan by the two principal axes of symmetry; this explains the speed of the realisation, some structural imperfections, such as the insufficient bearing length of the blocks in some zones, and the presumably weak connection of the four portions in correspondence to the two principal axes. The Colosseum was covered with a velarium whose documentation has been completely lost; a possible configuration designed on the basis of structural analysis, that we have carried out taking into account the characteristic of the hemp ropes that probably have been used.
The second peculiarity of the construction, is the site that has probably played an important role in the damages and collapses as we shall see later. The site was originally the lake of the Domus Aurea and was then partially filled to create the basis for the foundations; the result was heterogeneous soil and foundations (partially represented by a concrete bed) and an alteration of the hydrographic settlement. In the first centuries of the Colosseum's life, it was affected by fires, small earthquakes and floods. In the 217 a.D. a fire, caused by lightening, burnt the timber structures of the top roof; important fires also occurred in 250 a.D. and 321 a.D. It is important however to observe that, although the restoration works following the 217 fire were recorded as having taken place over five years, it is unlikely that they substantially affected the stone elements calcined by the fire or the masonry damaged by the flood. However, in spite of these events, the Colosseum maintained the original strength and soundness.
Fig. 1: View of Colosseum

Fig. 2: The Colosseum after the earthquake of 1349
1.1.2 Mathematical models related to the dead load

In order to verify the original safety levels a first finite element mathematical model considered the Colosseum in its original form. The static analysis has demonstrated that the original structure performed well, showing the skill and forethought of the ancient designers and builders. The structure appears to be over dimensioned, reaching a maximum compressive stress of only 24 KN/m² at the base of the travertine piers. Whilst the strength is of many hundreds KN/m².

The model has also shown that the arches inserted in the elliptical walls induce a horizontal thrust with a radial component corresponding to each pier, that is added to the thrust produced by the ambulatory vaults; the corresponding tensile hooping stresses are anyway lesser than 1 KN/m².

1.2 From 443 to 1703 - The Four Strong Earthquakes

1.2.1 The history

The first important event that effectively caused some destructions of the monument was the earthquake in 443 a.D., estimated of VIII-IX grade Mercalli Scale, with the epicentre in the roman region. Paolus Diaconus in his "Historia Romana" said that "tam terribili terremotu Roma concussa est, ut plurimae aedes eius et aedificia correant".

The Colosseum suffered damages on the sitting grades, in the arena and podium areas and at the attic level. It took three consulates to be restored and some of the works can still be seen at the top of the external wall, where a chaotic cyclopic masonry was set.

After this, news about the building seems to fade: the last ludo meeting is recorded in the year 523 a.D.; then, in the year 663 a.D., Constantinus III ordered the tearing down of the bronze plates of the attic. It is believed that in this period the locking stirrups among the blocks were also taken out: this spoiling caused the rupture of many blocks, thus locally modifying the stress state distribution. After the 5th century the Colosseum was not only disused and abandoned but also small earthquakes occurred and expanding plant roots opened existing cracks and created new ones. There was then a IX grade earthquake in May 801, with probable epicentre in the Abruzzese Appennino. The cyclopic order columns at the attic inner level fell into the cavea, causing huge damages in the inclined barrel vaults and arena; other less striking structural cracks also occurred. We believe that this is the date of the loss of continuity at the top of the elliptical walls. The next information about the monument is dated at the beginning of the 12th century. It was then inhabited by patrician families (Frangipane and Annibaldi) who fortified the two lower levels of the south-eastern side.

Unfortunately there is no iconography available of this period, with the exception of a commemorative medal of Ludwig the Bavarian (year 1328) that shows the Colosseum behind other buildings with the upper row still intact; the occasion of the representation, the commemoration of the new emperor, and the small scale of the picture explain the lack of reliability. Minor earthquakes occurred in 1255, 1287, 1300, 1321, 1348.

Even if the pope Innocenzo IV ordered the destruction of the fortifications, this must have been more or less how the Colosseum was in 1349 when another destructive earthquake occurred in Rome with VIII-IX grade. The most famous witness, Petrarca, said "magna cum partem collapsat". This earthquake, the most destructive in central
Italy, with epicentrum in the Umbro-Abbruzzese, produced the complete failure of the two external cylindrical walls on the Celio side of the Colosseum. After the earthquake of 1349 the Colosseum, reduced to a ruin, was abandoned and in 1362 the heap of rubble, especially from the two external walls, constituted a small hill, named "Coxa Colisei" (fig. 2). The use of the materials (marble and brick) was a bone of contention between the roman people, the Frangipane family and the representatives of Pope Urbano VIII, as it was required for the construction of some roman palaces and St. Peter's Basilica.

It seems, anyway, that the removals were usually from the fallen portions; there is no evidence of demolition except for limited ones authorised by the popes. Some "spontaneous collapses" are recorded in 1646 and 1689; the cause of these phenomena that cannot be directly related to particularly events, nor to seismic actions, will be explained later.

In 1703 the fourth and last strong earthquake occurred; the origin was once again in the Appennino region and created wide destructions in the city of L'Aquila. Although the energy characteristics were similar to those of 1349 earthquake, the damages were much less extensive, as just one arch is recorded as having collapsed in the outer wall and three arches in the second wall; it appears therefore to be a reverse in the trend of the increase of damages and failures in the succession of the earthquake of 443-801-1349. We have not at present complete data to explain this different behaviour, one possible explanation however can lie in the different characteristics of the soil beneath the foundations; therefore, once the structures on the weaker bed have collapsed the remaining have a greater resistance. Deeper analysis and investigations are presently in progress.

1.2.2 Mathematical models related to seismic actions

The theoretical behaviour of the Colosseum under the effect of seismic action has been analysed starting with a global elastic finite element model taking into account the different characteristics of the soil.

To quantify the seismic action to be used in the mathematical analysis, two different methods, which gave similar results, have been considered:

a) the historical data about seismic events recorded in the area; it can be seen that an earthquake of VIII to IX grade, in M.S. scale, has a return period of about 500 hundred years;

b) studies on seismic characteristics of the area; some studies assessed that a Richter intensity of 6.68 can be attached to a period of 400 to 500 years and therefore a ground acceleration of 0.05 to 0.06g can be deduced. Evaluating the amplification of the masonry structure up to 2.5 to 3 times, a design value of 0.15g, is obtained, very close to the value 0.16 stated in the Italian Seismic Code for the 3rd category area, this is the value that we have adopted.

The models indicate that the zone of higher stresses is in the attic wall, and in particular in the zone of the minor axis towards the Colle Celio; this is the area where the first breach in the wall may have occurred. In fact it is here that the maximum horizontal stress (up to 8 Kg/cm², figs. 3, 4), due to seismic action in the x-direction, is reached. This level of tension cannot be sustained by the friction present in the horizontal dry joints of the blocks because of the low vertical load.

Similar results have been reached with the seismic action in y-direction. A detailed examination of the results of the seismic analysis allows us to consider that the attic wall could reach a critical condition near the minor axis (on the Celio side) with a large range of possible earthquake directions as opposed to the small range of earthquake directions which could affect the wall around the major axis.
Fig. 3: Tensile horizontal stresses due to the seismic action in the X-direction corresponding to the 3nd mode

Fig. 4: Tensile horizontal stresses due to the seismic action in the Y-direction
Another zone where the model shows a critical situation is in the first level of piers in the inner elliptical wall, near the major axis. Particularly with seismic action in the x-direction some piers' sections are partialised by the presence of tensile stresses and reach compression of about 50 KN/m². The tensile stresses anyway are not high enough to cause the visible damages of the first level piers and it is evident that a single earthquake could not have caused the ruin of a large portion of the monument. It is necessary to interpret the elastic and post-elastic behaviour of the Colosseum in its original configuration to understand the real phenomena. Referring to fig. 5, during an earthquake, while the vertical axial force "N" remains constant, there is a transfer of effort, from the stresses, "T", which characterise the circumferential behaviour, to the bending behaviour "M", in the pillars. This behaviour is schematically represented in fig. 6: in the elastic phase it is the circumferential tensile stresses in the elliptical walls which give the main contribution; it is only after they reach their peak value linked to the frictional limit of the horizontal joint, that larger deformations (qualitatively represented by δ in the diagram), and relevant bending moments are generated in the pillars. In conclusion, we pass progressively from an "N-T" behaviour to an "N-(T)-M" behaviour. It is important to note that, after the first earthquake, permanent deformations (and thus permanent bending moments) remain so that the global behaviour, even for low level seismic actions, becomes weaker and weaker; fig. 7, show the sinusoidal deformations, signs of the earthquakes on the external wall of the Coliseum.

This interpretation of the phenomena is in accordance with the historical documents that indicate that the two earthquakes of 443 and 801 caused only a loosening of the masonry in the attic level, as well as in the horizontal bands of the lower levels, with consequent sliding between the blocks, particularly in the zone of the minor axis on the Celio side. These sliding movements caused increments in the annular length and thus out of plumb. Moreover this initiated the vertical cracks which began in the attic wall and which were left unrepaired after the earthquake of 801. Despite the small entity of these damages, they compromised the efficiency of the two-dimensional behaviour of the attic wall, so that it was more easily damaged during successive minor earthquakes (1255, 1287, 1300, 1348). This weakness of the attic wall, which extended to the lower levels, explains the ruin of the large portion of the two outer elliptical walls near the minor axis on the Celio side during the earthquake of 1349.

An interesting question is whether the failure of the Celio side is mostly due to the weaker foundations. In effect the remaining side shows tensions that surpass the limit for the initiation of frictional movement (with the seismic action in y-direction in the mathematical model). Thus it is possible that the foundation factor was important not only during seismic actions (amplifying deformations and tensions on the Celio side as visible in the model for action in the x-direction), but also after and between those events, with differential settlement due to the effects of the out-of-plumb, especially on the Celio side, these aspects are the subject of deeper investigations.

1.3 From 1703 to 1979 - From the Ruin to the Restoration Works

After the 1703 earthquake the Colosseum remained abandoned and consequently suffered progressive deterioration. The 19th century is the century of the biggest work ever carried out on the Colosseum. Attention was initially focused on the lateral borders of the surviving façade, where the lack of continuity and thus the disappearance of circumferential stresses, created the most unfavourable situation, worsened by the dynamic actions that affected the existing boundary of the collapsed zones.
Fig. 5: Main stresses acting on the Colosseum

Fig. 6: Evolution of the main stresses after block friction overtake
Fig. 7: Signs of earthquakes (sinusoidal deformation) visible on the external wall

Fig. 8: Valadier’s abutment
The abutments of Stern (1805-1807) and Valadier (1822-1826) stopped the deterioration process and generated a new, adequate static situation (fig. 8); the same cannot be said for the dynamic behaviour as we shall discuss later. From 1831 to 1846 important works were carried out by Salvi with the reconstruction of the interior circumferential wall on the Celio side and the installation of a system of radial tie-bars in the alignments around the imperial entrance. Although these works were realised, and although no strong earthquake has occurred since 1703, a dangerous situation, very close to collapse, occurred in 1979 when we were asked by the Archaeological Superintendent of Rome to check some piers in the inner circumferential wall, near the Stern abutment. A deep process of cracking (fig. 9), was immediately evident, so that urgent shorage and intervention were required.

This event, which initiated the study summarised here, gave us the opportunity to highlight the fact that the earthquake produced not only direct collapses, but also started a process of deterioration related to the high stress levels reached, which created cracks and micro cracks sensitive to thermo-hygrometric conditions, and particularly to the effects of frost provoking an increment of internal stresses in the outer layer of the blocks, thus facilitating spalling.

This phenomenon justifies the numerous "spontaneous collapses" that have contributed to the large quantity of material that has been lost during the centuries; the fact that only the collapses corresponding to the earthquakes and few other situations are recorded by historians is not a surprise: for centuries the Colosseum has been an abandoned ruin, no one being interested in what happened to it. Fig. 10 shows from a qualitative point of view the evolution of the collapses and the weakening of the structure.

2. THE PRESENT SITUATION AND THE SAFETY EVALUATION

On the basis of what has been shown it is possible now to give a judgement of the present safety level by a critical interconnection of the information by means of three different processes: the direct observation, the mathematical analysis, the historical survey.

I - The direct observation - A carefully observation of the Colosseum highlights the main alterations that can be summarised as follows:
- the materials: deteriorations are present in many parts of the Monument, linked to the fire, weathering and to the effects of atmospheric attack, frost, etc. . . .; high stresses and cracks have accelerated the related phenomena;
- the geometry: alterations are particularly evident in the "earthquake signs", such as the frequent separations between the elliptical and radial walls (fig. 11), the sliding of the blocks in the walls in the vaults and the arches, the out-of-plumb of walls and pillars, the deformations of the facade (fig. 7).

This last aspect is of particular interest as the circumferential sinusoidal deformation, not always in phase from a level to another has also produce relative rotations about their own vertical axes and twisting effects in some pillars.
- the previous works: the collapses have deeply changed the original construction whose most evident aspects are the Stern and Valadier abutments, the works of Salvi, the strengthening of some elements (arches, pillars, . . . ) and the insertion of metal chains, (fig. 12) related to the out-of-plumb which progressively increased over time;

II - The Mathematical Models - Different models were made to take into account different hypothesis on the structural behaviour of the monument, due to different hypothetical efficiency of the structural elements.

The mathematical analysis indicates that the surviving facade, between the Stern and Valadier abutments as well as the abutments themselves, reach tensile stresses, initially
Fig. 9: Cracks and crushing discovered in 1979 in some pillars

THE EVOLUTION OF DAMAGES DUE TO EARTHQUAKES AND OTHER REDUCTIONS IN STRUCTURAL RESISTANCE OVER TIME.

Fig. 10: Qualitative evolution of failures and loss of material
Fig. 11: Separation between circumferential and radial walls

Fig. 12: Chains placed in the 19th century
in the circumferential direction, which overtake the friction between the blocks; this means that sliding between the travertine blocks can be produced, and as a consequence of the increased bending moment in the pillars, there is a reduction in the effective section of the pillars themselves that can lead to critical situations. The mathematical models also show that the current situation is weaker than the original situation due to the loss of circumferential continuity.

III - The historical survey strong earthquakes (VIII and IX degrees on the Mercalli scale) documented in Rome every 3-5 centuries, have always produced damage and collapses; besides they have initiated deterioration processes which have brought about collapses or dangerous situations decades or centuries after the event itself.

To interpret the lesson from history regarding the present situation we have to take into account three aspects:

- the increasing weakness of the structure after each earthquake in relation to the sliding between the blocks, the deformations, the out-of-plumb and the increasing stresses in the columns; from this point of view the present situation is worse than the situation in the past.

- the role of the soil and foundations; the zone where the characteristics are worse corresponds to the most damaged part of the monument; from this point of view the capacity of the surviving structures, especially the façade, appear better as they probably bear onto efficient foundations;

- the different geometry and the configuration of the actual structure, and in particular the façade including the Stern and Valadier abutments; this situation has never been tested by strong earthquakes so no specific information can be extracted from history.

3. PRELIMINARY CRITERIA OF INTERVENTION

The interaction between the different information mentioned above and their critical analysis, makes it possible to have a reliable picture of the situation and allows us to express some concern, especially with the prospect of the next strong earthquake.

The exceptional value of the monument requires however prudence in the judgements: deeper analysis and more sophisticated mathematical models are going to be developed, and we expect in few years to have acquired elements to reduce the uncertainties; it will be possible therefore to ensure an adequate margin of safety and durability, minimizing any alteration corresponding to the interventions.

The preliminary indications on the possible strategies for interventions on the façade can be synthesised as follows:

- to connect the external wall to the ambulatory vaults and the radial walls (after checking the effectiveness of the chains placed in the nineteenth century);

- to improve the circumferential resistance which is currently only provided by the friction between the blocks;

- to improve the transverse flexural resistance of the abutments where the circumferential collaboration is missing.

Some cables (steel or synthetic fibres), possibly prestressed should be useful for these purposes; as previous mentioned, however, these are just preliminary ideas to be verified and detailed on the basis of the new data that will be acquired, taking into account that, due to the dynamic nature of the problem, any possibility to dissipate part of the energy must be favoured.