CHARLES BRIDGE IN PRAGUE

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SUMMARY

The last controversial reconstruction of the famous Charles bridge caused that the bridge has changed its structural behaviour. This resulted in excessive deflections and cracks. The aim of the present study was to understand the structural behaviour of the bridge using the finite element modelling in combination with a statistical simulation respecting different materials and their interactions. The variability of selected input parameters is taking into account - they are considered to be random variables. The statistical characteristics of a structural response were calculated based on the advanced Monte Carlo type simulation. The distribution functions of selected stresses and deflections are estimated providing a basis for confidence limits. The stochastic sensitivity analysis was also performed, which gives information about the influence of random variables on the structural response. Temperature load cases were analysed in order to assess the present stage and two suggested variants of the reconstruction. The results should be used for the evaluation of the proposed reconstruction and the selection of the most suitable way of repair works.

1. INTRODUCTION

The aim of this paper is to describe FEM modelling and probabilistic assessment procedures used for the analysis of the present stage and proposed reconstruction. The statistical approach is described in a way which can be generally useful for the statistical analysis of historical structures. The description of techniques used, the assumptions made and the interpretation of results are included. The techniques presented should contribute to the effort of correct assessment of safety of structures.

The safety can be considered to be a degree of assurance to maintain its required function and the capacity of resistance against deteriorating causes. Generally, safety levels should be distinguished and the safety should be evaluated at four basic stages: The planning stage, the design stage, the construction stage and the maintenance stage.
Considering historical bridges the last item - the maintenance stage, is the only remaining one for us to save our historical heritage for future generations. Our daily experience shows what would happen if either inspection or maintenance works were ignored. For a required functioning of a bridge the inspection and feasible maintenance treatment are inevitably necessary. Nowadays it is generally accepted that the so-called maintenance-free bridges do not exist. This applies, of course, for medieval masonry bridges, too.

Generally, the traditional methods of analysis treat the physical uncertainties of various parameters in a simple and efficient way. Minimum guaranteed values of these parameters, usually following the relevant standard or code of practice (e.g. strength, modulus, etc.) are used for the analysis. This is not applicable for some more complicated structures where the minimum (or maximum) guaranteed values may not produce the worse structural behaviour. Charles bridge is a typical example of such a structure. Due to the composition of different materials one cannot predict the structural behaviour under temperature loads.

A statistical approach provides us with a simple option how to deal with the above problem. If input parameters are significantly uncertain and structural behaviour unpredictable let the selected parameters be considered random variables. Then the treatment of structural system becomes feasible at higher quantitative level by efficient structural analysis as well as advanced stochastic procedures such as Monte Carlo type simulations. An acceptable determination of input parameters when analysing historical structures is often very limited. The statistical approach is, in some cases, the only way how to get a realistic prediction of structural behaviour.

2. THE HISTORY OF CHARLES BRIDGE

Charles bridge, Fig. 1, belongs to the most valuable historical monuments in the Czech Republic. It is probably the biggest tourist attraction in Prague and the Czechs are very proud of it.

The bridge was designed and built during the reign of King Charles IV by famous architect Petr Parléř and was finished in 1402. The bridge is 515.76 m long and the average width is 9.40 m. It has 16 arches with the span from 16.62 to 23.38 m supported by 17 piers. Charles bridge was originally called "Stone Bridge" and was built as the second stone bridge in Prague. The bridge was always the most important and busiest traffic route in the city and still creates an inseparable part of Prague’s panorama with the most impressive view of the Prague Castle. The Old Town Bridge Tower and many historical sculptures contribute to the outstanding general impression of the bridge.
The bridge was damaged many times during wars and by floods. The floods that partly damaged the bridge were in 1432, 1496, 1784 and the worse one, in 1890, completely destroyed two piers and three arches. The damage was considered a national disaster and the bridge was repaired in 1892. The damaged piers were founded on caissons and other piers were also secured by caissons. The damaged arches and piers were repaired using in most cases different building stones instead of the original ones. These repairs resulted, together with different technologies used, in a great diversity of the bridge.

The filling materials were removed and replaced by expanded clay concrete and a reinforced concrete slab without expansion joints during the last controversial reconstruction (1966-1975). The slab was anchored to the parapet walls and granite paving was put in cement mortar. These changes caused the bridge to become too rigid and resulted in several cracks mainly parallel to the lengthwise axis of the bridge. The cracks did not occur in joints only, but some of them appeared in the ashlars. Excessive deflections of the parapet walls were detected on several arches.

Figure 1: The views of Charles bridge.
The above mentioned damage together with the surface erosion of building stones initiated a new investigation supervised by the Charles Bridge Committee in 1994. Many bridge and structural engineers, architects, chemists, geologists, mineralogists, and state administrators were involved in a comprehensive preliminary investigation under several sub-committees that resulted in a proposal of new reconstruction of the bridge. The second author of this paper had a chance to participate as a member of the sub-committee Structural Analysis.

3. DETERMINISTIC COMPUTATIONAL MODEL

Finite element package ANSYS [1] was used as a basic tool for the present analysis. This commercial package offers a good selection of suitable 2D, 3D, and special elements. It is still practically impossible to analyse the whole structure like Charles bridge using 3D elements. 2D solid model of selected cross-sections (without modelling joints between different materials) was developed in the first step. Then the results were compared to the results of corresponding 3D model of a quarter of the arch. It was confirmed that the differences were quite significant.

2D models of selected cross-sections respecting joints between different materials were developed for the present analysis in three different variants corresponding to A) the current state and the state after the proposed reconstruction in the two variants - B) without expansion joints and C) with expansion joints. Arch number IX was selected for the analysis. Two cross-sections (on the top of the arch and on the foot of the arch) were analysed using 2D plane strain finite elements (Fig. 2).

Figure 2: FEM mesh of the bridge cross-section.
Three different load cases were considered in the analysis as a result of the study of meteorological data: I/ warm top surface and cold bottom surface, II/ warm surface and cold core, III/ cold surface and warm core. Respecting the collected meteorological data and previous analyses a non-stationary temperature field analysis was carried out to obtain possible temperature gradients in the structure under different conditions. The results were used as boundary conditions for the following stationary analyses. 2D temperature elements (PLANE55) were used to obtain the distribution of temperature field in the cross-section in the next step of the solution. The calculated temperatures (see Fig. 3 for the load case II) were converted into initial strains and then transformed into equivalent external loads for the structural analysis. This is the last part of the analysis that was carried out once, and was not a part of the stochastic repetitive analysis.

Figure 3: Temperature distribution for the load case II.

Figure 4: Materials of the bridge.
The temperature elements were changed to the corresponding structural plane strain elements (PLANE42), and then a standard structural analysis was performed one hundred times for various values of stochastic parameters. The bridge was built of sandstone, arenaceous marl, unknown materials and granite paving in sand. The current materials are sandstone, arenaceous marl, concrete slab anchored to the walls, expanded clay concrete and granite paving in mortar, Fig. 4. The proposed repair works comprise the removal of the concrete slab, expansion joints of expanded clay concrete and granite paving in sand. Different materials were connected using special contact elements (CONTAC12) which enable us to model friction, normal and tangential initial stiffness and possible gaps. These elements generally require Newton-Raphson iterations to find the correct status of contacts (opened or closed). Correct input of initial status avoided time consuming iterations to be done for all runs of stochastic analyses.

It was quite difficult to input correct coefficients of thermal expansion in the longitudinal direction and for expanded clay concrete with expansion joints. The longitudinal strains are zero for the plane strain analysis but the coefficient of the thermal expansion would produce unrealistic stresses in this direction. On the other hand, zero value of the coefficient of thermal expansion would not be correct for this analysis. As a compromise the coefficient of the thermal expansion in the longitudinal direction was decreased. It would also be difficult to model expansion joints. We assumed that the strains perpendicular to the expansion joints did not almost produce any stresses. So we again assumed the value of the coefficient of the thermal expansion to be decreased in this direction.

The deterministic procedure can be summarised in the following steps:

- Determination of material characteristics. Sources: tests, literature, previous projects, engineering judgement.
- Stationary temperature field analysis. Result: temperature distribution.
- Temperatures converted into initial strains. Done automatically in ANSYS.
- Structural analysis. Newton-Raphson iteration if necessary. Results: deflection, strain, stresses.

4. STATISTICAL APPROACH

4.1 General remarks

The reliability methods used for the probabilistic assessment of Charles bridge are described in this section. They have a general use and can easily be applied to a different computational model, to a different bridge, to a different historical structure. The authors believe that the so-called probabilistic risk assessment procedures will be applied in the field of historical structures more
often in the future, and the techniques described here represent one possible alternative how to deal with uncertainties of historical structures.

A statistical approach was utilised to model important basic material properties of Charles bridge that were considered as random variables. Their statistical characteristics were estimated and then used for statistical, sensitivity and probabilistic analyses. A powerful numerical simulation technique Latin Hypercube Sampling (LHS) [2], [3] in combination of FEM package ANSYS with different realisations of input random variables were used a hundred times for each load case and each variant, together nine hundred single structural analyses were performed. The aim was to obtain the scatter of stresses and deflections - to estimate their mean values and variances in selected nodes or elements and to use these results to evaluate the current state of the bridge and the proposed repair works.

The statistical analysis of Charles bridge has been performed in three steps: Firstly, Latin Hypercube samples were generated for random variables separately for all simulations without calculating structural response by FEM immediately. Then structural analysis of the bridge by ANSYS code was done a hundred times utilising once generated data according to LHS scheme. Selected response variables were saved in files. Finally, statistical sets are statistically evaluated, basic statistical characteristics of selected response variables are obtained. The triangle of the solution is sketched in Fig. 5.

Figure 5: Three basic steps of the analysis.
4.2 Input random variables

In probabilistic calculations the definition of the set of random variables is the primary task. It consists of two steps: Firstly, the choice of the parameters which will be considered stochastic and will be simulated as random variables. In our case of modelling thermal effect we consider only the most influencing material properties of the bridge - Young modulus of elasticity and the coefficient of thermal expansion for all materials used in the bridge structure to be random.

Secondly, every random variable $X$ should be described with the aid of the probability distribution function. Such a function is in many cases (two-parametric distributions) characterised by a particular type of distribution (e.g. a normal distribution or a uniform distribution), mean value $\mu(X)$ and standard deviation $\sigma(X)$. Very often the coefficient of variation $COV(X) = \sigma(X)/\mu(X)$ is used as a relative measure of variability.

If statistical data is available for all the variables, the statistical characteristics can be estimated and a theoretical model of probability distribution function can be assigned by well known procedures of mathematical statistics. In most cases there is scanty data available. The statistical properties will then have to be entirely estimated based on different assumptions.

The following sources for estimations (also for deterministic parameters involved) were used:

- in situ measurements
- information published in literature
- experience and opinion of experts
- engineering judgement and intuition

A synthesis of all information resulted in the estimation of limit values, $x_{high}$, $x_{low}$, above or below which the relevant variable will not be in all probability situated. For the normal distribution it is well known there is 95% probability that a variable will have a value situated between $\mu - 2\sigma$ and $\mu + 2\sigma$. Based on this assumption it is thus possible to estimate the mean value and the standard deviation:

$$\mu(X) = \frac{1}{2} \left( x_{high} + x_{low} \right)$$  \hspace{1cm} (1)

$$\sigma(X) = \frac{1}{4} \left( x_{high} - x_{low} \right)$$  \hspace{1cm} (2)

For a log-normal distribution the corresponding formulae are:

$$\mu(X) = \sqrt{x_{high} \cdot x_{low}}$$  \hspace{1cm} (3)

$$COV(X) = \frac{1}{4} \ln \left\{ \frac{x_{high}}{x_{low}} \right\}$$  \hspace{1cm} (4)
The choice between using the normal and log-normal distribution will depend on the physical nature of the random variable. Log-normal distribution is to be preferred for variables which are unable by nature to take on negative values. But it should be noted that for small coefficient of variation \( COV \leq 0.10 \) the difference between normal and log-normal distribution is negligible for practical purposes.

The above assumptions resulted in a choice of the type of distribution, the mean value and the standard deviation of the problem random variables. Random variables \( X_1, X_2, ..., X_{10} \) considered here are listed in Table 1. The mean values and standard deviations adopted are those values which were rated as providing the best estimates of the type indicated above. The normal probability distribution was considered to model Young modulus of elasticity \( E \) and the log-normal distribution for the coefficients of thermal expansion \( \lambda \). The statistical correlation among random variables was not considered here due to the lack of sufficient information. For variants B) and C) the table of random variables without concrete slab was used.

<table>
<thead>
<tr>
<th>Material</th>
<th>( X_i )</th>
<th>( \bar{x}_{\text{low}} )</th>
<th>( \bar{x}_{\text{high}} )</th>
<th>( \mu(X) )</th>
<th>( \sigma(X) )</th>
<th>( COV(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sandstone</td>
<td>( E = X_1 )</td>
<td>5.0</td>
<td>15.0</td>
<td>10.0</td>
<td>2.50</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>( \lambda = X_2 )</td>
<td>0.7</td>
<td>1.0</td>
<td>0.84</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>arenaceous marl</td>
<td>( E = X_3 )</td>
<td>3.0</td>
<td>8.0</td>
<td>5.5</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>( \lambda = X_4 )</td>
<td>0.7</td>
<td>1.0</td>
<td>0.84</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>concrete slab</td>
<td>( E = X_5 )</td>
<td>18.0</td>
<td>28.0</td>
<td>23.0</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>( \lambda = X_6 )</td>
<td>1.0</td>
<td>1.2</td>
<td>1.10</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>expanded clay</td>
<td>( E = X_7 )</td>
<td>8.0</td>
<td>12.0</td>
<td>10.0</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>concrete</td>
<td>( \lambda = X_8 )</td>
<td>0.4</td>
<td>0.9</td>
<td>0.60</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>granite paving</td>
<td>( E = X_9 )</td>
<td>4.0</td>
<td>8.0</td>
<td>6.00</td>
<td>1.00</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( \lambda = X_{10} )</td>
<td>1.0</td>
<td>1.2</td>
<td>1.10</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 1: Random variables \( (E \text{ in Gpa, } \lambda \text{ in } \text{C}^{-1} \times 10^{-5}) \).

### 4.3 Statistical analysis

Response function \( g(X) \) (FEM computational model) of random variables \( X = X_1, X_2, ..., X_{10} \) gives us response variables (e.g. deflections, stresses). The response variables are arranged into vector \( Z \):

\[
Z = g(X)
\]

The aim of the statistical analysis is the estimation of basic statistical parameters of response variables, e.g. mean values and variances. It can
easily be done by Monte Carlo simulation, by repetitive calculations of the structure. In case of simple Monte Carlo simulation such a process is time-consuming.

A special type of numerical simulation Latin Hypercube Sampling enables us to use a small number of simulation (a hundred) for satisfactory good estimates of basic statistical parameters [2], [3]. The probability distribution functions for all random variables are divided into $N$ equivalent intervals ($N$ is a number of simulations), the centroids of intervals are then used in simulation process. This means that the range of the probability distribution function $F(X_i)$ of each random variable $X_i$ is divided into $N$ intervals of equal probability $1/N$, see Fig. 6. The representative parameters of variables are selected randomly based on random permutations of integers $1, 2, \ldots, i, \ldots, N$. Every interval of each variable must be used only once during the simulation. A table of random permutations can be used conveniently, each row of such a table belongs to the specific simulation and the column corresponds to one of the input random variables [3].

Figure 6: The division of the distribution function into intervals.

4.4 Sensitivity analysis

An important task in the structural reliability analysis is to determine the significance of random variables - how they influence a response function of a specific problem. There are many different approaches of sensitivity analysis, a summary of present methods is given in [4].

LHS simulation can be efficiently used to obtain such an information. The sensitivity analysis is obtained as an additional result. This provides us with the information about the importance of basic random variables. Dominating and nondominating variables can be distinguished using certain sensitivity
measures. Here two different types of sensitivity were used: Nonparametric rank-order correlation and sensitivity in the form of variation coefficients.

The relative effect of each basic variable on the structural response can be measured using the partial correlation coefficient between each basic input variable and the response variable. The method is based on the assumption that the random variable which influences the response variable most considerably (either in a positive or negative sense) will have a higher correlation coefficient than other variables. In case of a very weak influence the correlation coefficient will be quite close to zero. In case of Latin Hypercube Sampling this kind of sensitivity analysis can be performed almost directly without any particular additional computational effort. Because the model for structural response is generally nonlinear, a nonparametric rank-order correlation is utilized. The key concept of nonparametric correlation is this: Instead of the actual numerical values we consider the values of its rank among all other values in the sample, that is 1, 2, ..., N. Then the resulting list of numbers will be drawn from a perfectly known probability distribution function—the integers are uniformly distributed. In case of Latin Hypercube Sampling the representative values of random variables cannot be identical, therefore there is no necessity to consider mid-ranking. The nonparametric correlation is more robust than the linear correlation, more resistant to defects in data and also distribution independent. Therefore it is particularly suitable for the sensitivity analysis based on Latin Hypercube Sampling.

As a measure of nonparametric correlation we use the statistic called Kendall’s tau. It uses only the relative ordering of ranks: higher in rank, lower in rank, or the same in rank. Since it uses a weak property of data, Kendall’s tau can be considered as a very robust. For a detailed description of calculation, see [4], here we present only a symbolic formulae. Kendall’s tau is the function of ranks \( q_j \) (the rank of a representative value of the random variable \( X_j \) in an ordered sample of \( N \) simulated values used in the \( j \)-th simulation which is equivalent to the integers in the table of random permutations in the LHS method) and \( p_i \) (the rank in ordered sample of the response variable obtained by the \( j \)-th run of the simulation process):

\[
\tau_j = \tau(q_j, p_i), j=1,2, ..., N
\]

In this way the correlation coefficient \( \tau, \in (-1,1) \) can easily be obtained for an arbitrary random variable and we can compare them. The greater absolute value of \( \tau \), for a variable \( X_j \), the greater influence has this variable on the structural response. An advantage of this approach is the fact that a sensitivity measure for all random variables can be obtained directly within one simulation analysis.

Another method for the sensitivity analysis can be applied also based on LHS method. The method is based on the comparison of variation coefficients of random variables and a response variable. Let us designate the partial
coefficient of variation $COV(Z_i)$ for the case in which the random variable $X_i$ is the only one treated as random in the simulation process. $COV(X_i)$ is the coefficient of variation of this variable. Other basic variables are kept at their mean values. The partial sensitivity factor $\alpha_i$ for the random variable $X_i$ may be defined as:

$$\alpha_i = \frac{COV(Z_i)}{COV(X_i)} \quad (7)$$

Such factors express the relative influence of individual variables on the variability of structural response. If all $n$ random variables are considered to be random, the following approximate formulae may be written:

$$COV(Z) = \left[ \sum_{i=1}^{n} (\alpha_i COV(X_i))^2 \right]^{1/2} \quad (8)$$

It can be seen that the absolute influence is represented by the value of $COV(Z_i)^2$. Such sensitivity may easily be depicted using pie charts. The absolute influence of variable $X_i$ on the variability of response variable $COV(Z)$ is best presented in percentage terms.

In case of sensitivity analysis in the terms of $COV$ the simulation process has to be repeated $n$-times. This fact represents a great disadvantage for reliability problems with a large number of random variables and time-consuming FEM calculation of structural response. For such cases, as modelling Charles bridge, Latin Hypercube response approximation can be efficiently used [5].

The aim is to approximate response function $g(X)$ in order to avoid a time-consuming solution of the system of linear algebraic equations. This can be done by Taylor’s series around the expansion point. Neglecting higher order terms a second-order polynomial can be considered in standard space of random variables $Y_i = \left( X_i - \mu(X_i) \right) / \sigma(X_i)$, where mean values are expansion points:

$$\bar{g}(Y) = a_0 + \sum_{i=1}^{n} b_i Y_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} Y_i Y_j \quad (9)$$

$b_i, c_{ij}$ are coefficients related to the value of partial derivatives and $a_0$ is the value of the response function at expansion point, $a_0 = g(\mu(X))$.

Based on this concept a multiple regression model can be used to determine coefficients. The most decisive task is the selection of interpolation
points and the way of perturbing them around the expansion point. The Latin Hypercube Sampling ensures that multidimensional space of basic variables is covered by generated points quite regularly with a small number of simulations in comparison with the simple Monte Carlo random sampling to reach a comparable accuracy, the procedure can be called the Latin Hypercube Response Approximation Method (LHRAM) [5]. This advantage can be used to obtain interpolation points for an efficient response approximation. Because tensor \( c_{ij} \) is symmetric, at least \( N = 2n + n(n-1)/2 \) points must be generated to obtain polynomial coefficients. Generally higher \( N \) can be used, which leads to the multiple regression model. In our case of Charles bridge analysis we use \( N = 100 \) which is greater than the minimum necessary \( N = 65 \).

4.5 Probabilistic analysis

Comparing structural response variable \( Z_j(X) \) with required values \( Z_{j,req} \) in a certain chosen range, one can estimate the complementary cumulative distribution function of response variable \( Z_j \):

\[
\Phi(Z_j) = p(Z_j(X) \geq Z_{j,req})
\]  

(10)

In this way the probability that the structural response variable (stress or deflection) will be greater than the prescribed value is defined. Confidence limits for response variables can be estimated. A concept of the probability distribution function in the complementary form is used in order to express "subjective failure".

Here the numerical calculation of failure probabilities for the estimation of the complementary cumulative distribution function was based on a curve fitting approach [3], [6]. The selection of the most suitable theoretical model for the probability distribution function of a response variable was first done comparing theoretical and empirical probability distributions obtained from the simulation. The most suitable theoretical model among competing ones was selected and then used for the calculation of probabilities.

5. NUMERICAL RESULTS

5.1 Deterministic results

Sorting the results we had to solve the usual problem of any FEM analysis: there were too many available output data so it was difficult to select the most interesting and important ones. We considered node deflections, stress components and principal stresses in the selected elements.
The following results should be understood as a sample of possibilities given by ANSYS package and are related to the top of the arch, load case II, variant A. Contours of principal tension stresses on deformed shape are shown in Fig. 7. Contours of principal compression stresses are displayed in Fig. 8. Gaps between different materials due to the temperature load appeared in both figures. Directions and magnitudes of principal stresses are illustrated in Fig. 9.

Figure 7: Principal tension stresses.

Figure 8: Principal compression stresses.
5.2 Statistical results

From all results obtained by the statistical simulation the results for the principal tension stress at position $\alpha$ (see Fig. 10), for the horizontal deflection at the top edge of the parapet wall (position $\beta$) and for the principal tension and compression stress at the position $\gamma$ is shown here.

(a) The principal tension stress at position $\alpha$

The complementary probability distribution functions for variants A, B and C in all load cases I, II and III are presented in Fig. 10. One can easily observe mean values and full probabilistic information showing confidence limits (5 and 95% percentiles). It can be seen that the principal tension stress significantly decreased in the case of variant C comparing to variant A. In case of variant B it happened only in the load case I.

The results of nonparametric sensitivity analysis in the form of Kendall's $\tau$ for load case II are presented in Fig. 11. The plus sign represents a positive influence and the minus sign a negative influence of a random variable on the principal stress, the higher the more important. The dominating variable of the current state is the coefficient of the thermal expansion of expanded clay concrete ($X_s$) - Fig. 11 A. This dominancy diminishes in the state after the proposed reconstruction in variant C where Kendall's $\tau$ for variables of sandstone and expanded clay concrete increased.

Figure 9: Principal stresses.
Figure 10: The principal tension stress (position $\alpha$) - the complementary distribution function, load cases I, II and III.

Figure 11: The principal tension stress (position $\alpha$) - nonparametric sensitivity, load case II in variants A and C.
The influence of variability of random variables on the coefficient of variation (COV) of the stress (0.53 - variant A, 0.17 - variant C) is expressed by pie charts in Fig. 12. Here the same trend observed in Fig. 11 can be interpreted as follows: The contribution of $X_a$ is 80% for variant A but only 14% for variant C.

(b) The horizontal deflection at the top edge of the parapet wall (position $\beta$)

The proposed reconstruction modelled by variants B, C resulted in a decrease of the horizontal deflection at the edge of the parapet wall. For load case I the situation is shown in Fig. 13. 90% confidence intervals are [mm]: A (0.449, 0.586), B (0.286, 0.490) and C (0.114, 0.175). It is obvious that variant C leads to better confidence intervals also in this case of the horizontal deflection. On the contrary to the principal stress in load case II the importance of the coefficient of the thermal expansion of expanded clay concrete did not decrease, Fig. 14 (COV 0.08 - variant A, 0.13 - variant C).
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![Graphs and figures]

Figure 14: The horizontal deflection at the top edge of the parapet wall (position \( \beta \)) - sensitivity in percentage of COV, load case I.

(c) The principal tension and compression stress at position \( \gamma \)

The complementary probability distribution functions of the principal tension and compression stresses for variants A, B and C in all load cases I, II and III are presented in Fig. 15 and Fig. 16.

![Graphs and figures]

Figure 15: The principal tension stress (position \( \gamma \)) - the complementary distribution function, load cases I, II and III.
A significant decrease of the principal tension stress for variant C can be observed for load cases I and II. Concerning principal compression stresses, variant C resulted in significantly lower values with small scatter in load cases I and III. For the warm surface and the cold core, load case II, it is opposite. But the shape and position of functions is similar, i.e. the coefficient of variation has approximately the same value for all variants (0.3).

Figure 16: The principal compression stress (position $\gamma$) - the complementary distribution function, load cases I, II and III.

6. CONCLUSIONS

It should be noted that the present statistical simulation was based on the 2D model that cannot produce accurate comprehensive results for this type of structures. The presented analysis was done for comparative purposes of thermal effect in different variants of the structure. Significant differences were found comparing structural deterministic results on 2D and 3D models [7]. 3D model should be used for more reliable results, but we realized that, bearing in mind contact elements, a total number of elements and a number of
analyses, requirements on hardware and the time for the development of the model would exceed our facilities. Although 3D model would produce more reliable results the 2D analysis was performed as an initial step and 3D analysis was left for future investigations.

It should also be mentioned that we did not work with random fields, with spatial variations of material properties. There are two main reasons to be stuck with the concept of random variables for the time being. Firstly, random fields require parameters difficult to obtain as the type of correlation function and correlation length. Secondly, the applicability of presented methods is easier and straightforward via an interface to standard computational FEM software. Random fields modelling requires a specialised software based on the stochastic finite element method and the separation of modelling process into two parts (deterministic computational model and statistical simulation) cannot be made so easily.

It was shown that more reliable results were obtained here by using the statistical approach rather than the classical deterministic analysis. The aim was to acquire more detailed knowledge of the behaviour in order to identify and prevent hazardous situations. These results might be beneficial for a better understanding of the structural behaviour and for the proposals of future reconstructions of Charles bridge.

Our deduction can be summarized into the following items:

• Stochastic results clearly indicated that significantly lower level of stresses (both magnitudes and scatters) appeared in variant C of proposed reconstruction (removed concrete slab, expanded clay concrete with expansion joints).
• Special contact elements are necessary for a realistic prediction of structural behaviour.
• FEM modelling in a combination with statistical simulation is a suitable tool for analyses of historical masonry structures.
• The variability of stresses and deflections appeared to be quite high (COV reached 0.5 in some cases).
• Distribution functions give us better insight and a basis for confidence limits.

7. ACKNOWLEDGEMENT

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8. REFERENCES


