

Structural characterisation of a medieval bell tower: First historical, experimental and numerical investigations

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ABSTRACT: The location of historical constructions, together with their complex configurations, often makes it difficult to apply dynamic actions in order to study their structural responses. In this context, a fortunate case is represented by bell towers, in which the motion of one or more of the bells present can be exploited to produce distinct dynamic actions, even quite strong ones, all of which can however be evaluated analytically and measured experimentally with quite high accuracy. Thus, it is possible to compare the experimental dynamic behaviour of such a tower with that predicted by discrete models.

The present paper reports on some first results of applying such a procedure to the "Matilde" bell tower in San Miniato (Pisa). The tower, currently embedded within the Cathedral building, represents an interesting case both historically as well as structurally by virtue of the profound constructive modifications the original structure has undergone over the centuries.

1 INTRODUCTION

Historical masonry towers, in all their forms - bell towers, civic towers, tower-houses and the watch towers of city walls - are found throughout the entire Italian peninsula, where they represent a distinctive feature of many of its historical centres and, in some cases, its countryside. In Roman and medieval times, some have had great strategic and military importance. The great variety of uses reserved for masonry towers has translated into a considerable variety of constructive configurations. Their heights vary from the 60-70 meters of the 11th-13th-century towers built by the powerful political associations with defensive functions in mind (but also serving as symbols of the power and wealth of their aristocratic family owners) to the 20-30 meters of the tower houses, which were widely popular in medieval Pisa (12th-13th C) (Binda et al., 1997a). In this context, a particular case is represented by the bell towers built beginning in the late 8th century, when Pope Stephan II (752 - 757) had one built to hold three bells in front of the façade of St. Peter's basilica.

Evaluation of the structural safety of historical masonry towers is one of the central issues in the maintenance of the national and world-wide patrimony of architectural monuments. To grasp the import of this field of study, one need only consider the interest aroused the world over by the consolidation operations carried out on the Leaning Tower of Pisa, or to recall the sudden collapse of the bell tower of San Marco in Venice (1902), the Civic tower in Pavia (1989), the bell tower of the church of St. Magdalena in Goch (Germany) (1992), or the bell tower of the Foligno Town Hall during a 1996 earthquake. Their vertical structure places towers at significant risk, not only due to the high stresses acting at their base, but also because of their great susceptibility to the effects of thermal variations and, especially, dynamic actions consequent to events such as earthquakes, the motion of the bells themselves, the vibrations produced by traffic or, lastly, the wind. In particular, vertical loads can provoke crushing phenomena of the masonry, as in the case of the bell tower of the Cathedral of Monza (Binda et al., 1998), or yielding of the foundation terrain and therefore the additional actions produced by the resulting inclination, such as, for in-

stance, what happened to the Garisenda tower in Bologna (Binda et al., 1997b). The extensive cracking revealed in many structures (Binda et al., 1997b) moreover testifies to the degrading actions linked to thermal variations.

Evaluation of the state of damage is commonly performed through on-site mechanical tests – flat-jack tests, hardness tests, pull-out tests, penetration tests, etc. – while in the case of the innermost layers, there are a number of semi-destructive techniques, such as core borings or local demolition, as well as, non-destructive techniques, such as sound and ultrasound studies or radar and tomography investigation (Binda et al., 2000). Dynamic tests are sometimes used to analyse a structure's integrity and to check the evolution over time of any degradation or the effectiveness of any structural consolidation operations. In the case of bell towers, in particular, important data are engendered through analyses of the structural response to actions of a dynamic nature, whether they be environmental or forced vibrations, produced through vibrodynes or dynamic actuators or, alternatively, by exploiting the motion of the bells themselves (Ceravolo et al., 1999). The present work presents some first results from an analysis of the dynamic response of the bell tower of the Cathedral of San Miniato al Tedesco (Pisa, Italy), commonly known as the Tower of Matilde. The tower is particularly interesting because of the variety and complexity of the consolidation operations it has been subjected to over the centuries: originally built as a watch tower in the *superiori incastellatura* (upper wall) of the medieval city, it underwent numerous and substantial modifications and was finally incorporated as a bell tower into the Cathedral of *Santa Maria e San Genesisio* (figure 1).

The ringing of the tower's Roman type bells (figure 2), effected by an electromechanical motor, seems to have produced wide-spread cracking throughout the bell tower and in correspondence to the Cathedral's roofing.



Figure 1: Matilde's Tower, San Miniato (I).



Figure 2: the greatest bell of the tower.

2 THE MEDIEVAL BELL TOWER OF SAN MINIATO.

The tower of San Miniato is about 35 meters in height, with a rectangular cross section of about 10 by 7 meters. Dating back to the mid 12th century, the tower was initially a military fortification incorporated into the old city walls, today completely destroyed. Some sources attribute its construction to the emperor Henry VI, son of Frederick Barbarossa, who was present in San Miniato in the years 1184-1194 (Lotti, 1980). Other authors place its construction to even earlier, in 1172, the year in which the medieval town beyond the fortress was destroyed (Cristiani Testi,

1967). The appellative “*Tower of Matilde*” originated in the historical character, Matilde di Canossa, who according to some sources was born in San Miniato in 1046. Although this remains to be confirmed, the name by which the tower is known is certainly related to the powerful Canossa family, who dominated a vast area of Tuscany, including the present-day town of San Miniato. The Tower's original structure (Cristiani Testi, 1967) was substantially different from its current one; in fact, it most likely had a crenelated crown (figure 3) and certainly had three tiers of semi-circular arches of varying heights, single on the short sides and paired on the long sides. Today, the arches on the two lowest levels have been sealed off with masonry walls.

Regarding the tower's actual construction, the above-cited study (Cristiani Testi, 1967) attributes it to a *Magister* (an official responsible for the functions of designer and foreman), who was also responsible for the work on the adjoining church of Saint Maria.

In the 13th century the Tower of Matilde was incorporated, abutting one of the entryways, into the new fortified city walls built by order of Emperor Frederick II of Swabia.

The ogival arched barbicans, the trim of the semicircular windows, also ogival, and the square aedicule at the corners of the balcony - all probably the work of Sicilian-Islamic fabrication - were added during this period. A print from the 14th century clearly shows that at this time the Tower of Matilde (as indicated in the panel of figure 4) was still incorporated into the same city walls as in the time of Frederick and was clearly separate from the Church of Saint Maria.

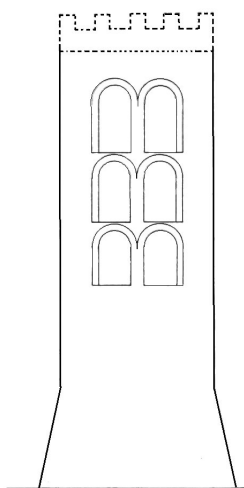


Figure 3: probably initial aspect of the tower.

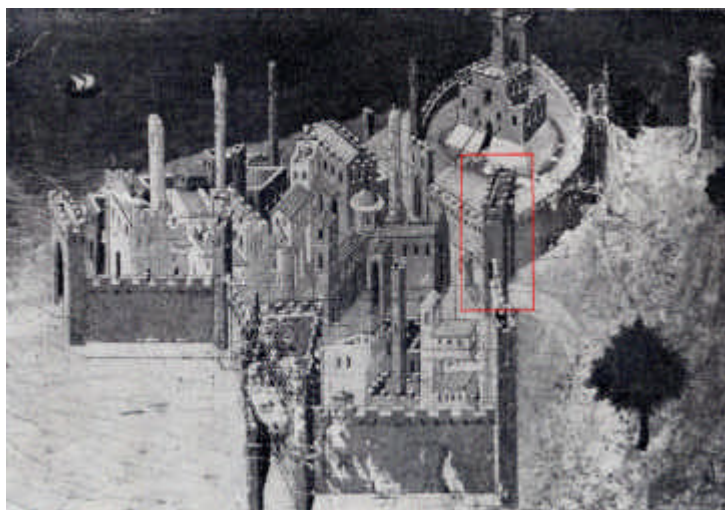


Figure 4: San Miniato in the XIV century.

The most structurally significant operations, however, were carried out at the end of the 15th century (1494 (Lotti, 1980), 1495-1498 (Cristiani Testi, 1967)), when the church of Santa Maria (named for Saint Genesius as well, when it became centre of the bishop's see) was enlarged to include the tower, which was thus transformed into its bell tower, situated above the apse of the nave. This restructuring involved demolition of the tower wall on the church side up to the height of the nave itself.

By 1438 a balance-wheel movement clock had already been set on the Tower (Lotti, 1980), and in 1497 the first bell was installed, while the other ones were added some years later. In 1623, the sacristies were built, closing the tower off laterally; demolition of the southern and eastern side scarps probably took place in this same period as well.

Over the course of the 18th and 19th centuries wide-ranging restoration work was carried out on the Cathedral, during which other structures were built abutting the base of the Tower.

The incorporation of the Tower of Matilde into the structure of the Church of Santa Maria is evident in both the general building plan in figure 5 and the North-South section shown in figure 6.

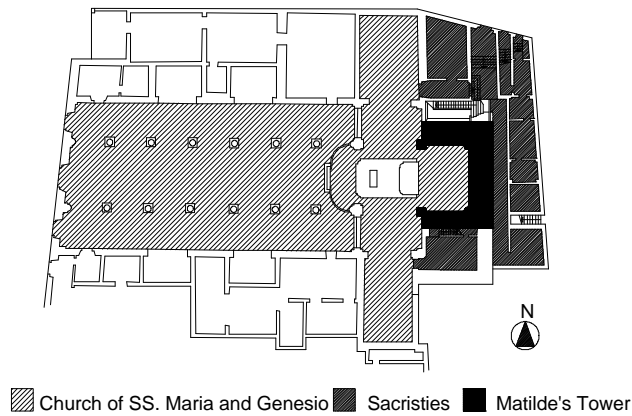


Figure 5: the plan of the church and of the bell tower.

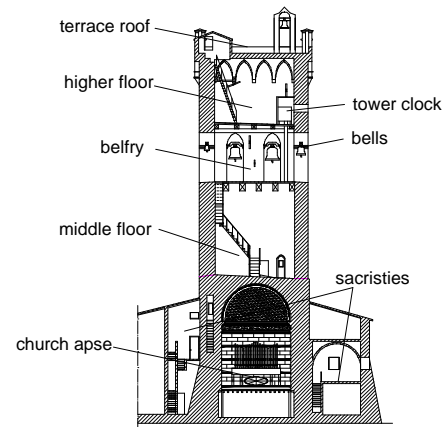


Figure 6: North-South cross section.

The cross-sectional dimensions of the tower vary from about 12.5 m x 8.2 m in its lower part (where originally it was probably even larger because of the scarp on all four sides), to 10.3 m x 7.3 m in the upper portions up to the crown, which is 11 m x 7.8 m. In the vertical (figure 6), the Tower is now divided into four floors: the first, corresponding to the cathedral apse, extends for a height of about 13 m and ends in a barrel vault with 6.5 m radius. The second floor, under the bells, is 9 m in height; it is accessed through a narrow staircase and ends in a masonry floor supported by cracking wooden beams reinforced with riveting. The third storey, about 5.2 m in height and closed off above by wooden planking, is the belfry, where 6 different-sized bells have been set in ogival openings. The fourth and last floor is about 6.2 in height; this contains the clockwork and ends with a vault supporting the flat roof.

It is likely that when (or immediately after) the tower's function was changed (from watch tower to bell tower incorporated in the Cathedral), the 30 cm -thick external walls were buttressed by another, considerably thicker wall (about 100 cm). This reinforcing wall is however arranged differently on the various walls and along the height of the tower. Moreover, the western wall bears two masonry arches, one ogival and the other semicircular.

The action of the bells must have aroused concern over the stability of the tower already during the 17th century. In effect, wide-spread cracking is visible along the masonry walls, while numerous stiffening chains have been placed both in the belfry and below the roof, some limited to a single masonry wall, others linking opposite walls.

3 THE BELLS' ACTIONS TRANSMITTED TO THE TOWER.

3.1 *The bells.*

Throughout history, bronze bells have been extremely wide-spread man-made devices. Recent archaeological discoveries in various parts of Western Asia seem to confirm that the earliest *resonant bronze-work* was fabricated by the ancient Armenians. However, they were surely well-known in ancient China, Egypt, Greece and Rome, where they were used as signalling devices, ritual or magical articles, or markers for identifying domestic animals. The earliest Christian documents that speak of bells date back to the 6th C, and their use (for instance in signalling the passage of time) spread quickly to Italy, Spain, Gallia and Britain. In the 15th century, bronze bells began to take on the characteristic shape that has remained nearly unchanged up to today. In the 1500s, Flemish craftsmen were able to fashion bells so well “tuned” that they could be sounded together to play true musical compositions. In Italy, the most famous bell founders were the masters from Lucca, Pisa and Florence.

Even today, there are many different ways of assembling and actuating bells; of the main types, we recall the *Ambrosiano* (figure 7), the system *a sbalzo* or *alla Romana* (figure 2), the *Genovese* or *a tastiera* and others typical of certain Italian cities and regions, such as *Verona*, *Bologna*, *Friuli*, etc.

In modern systems, the bell is set in motion by a motor through a pinion-chain-wheel system: depending on the system adopted, the motor operates the bell in either one or both directions to guarantee periodic motion starting from an initial position, which in some cases is upside-down, hence the term, the “drinking-glass” position.

The wide variety of shapes, assembly and actuating methods, together with the dissipating effects of friction, preclude any precise predictions of the actions transmitted to the tower by the bells in motion. Thus, it is necessary to resort to direct measurements. In the case of the tower of Matilde, the heaviest of the bells has been subjected to an accurate geometric survey: its weight has been determined and its motion reconstructed experimentally through accelerometer measurements. As mentioned in the introduction, the "Matilde" Tower of the San Miniato Cathedral bears bells “alla Romano o a sbalzo”: they have no counterweights; their centres of rotation are approximately at the height of their couplings, and they never reach the “drinking-glass” position, but move through an arc of about 90° in both directions. Strong thrusts are thereby exercised on the supporting structures, and the rotational velocity of the bells themselves is also quite high (figure 8).

Table 1 presents the values of its volume and the principal inertial moments calculated with respect to the central reference in figure 9. The bell's weight, equal to about 1710 daN, has been deduced by assuming a value of 8434 daN/m³ for the specific weight of bronze. Its weight has also been determined indirectly by measuring the tensile stress in a steel cable sustaining the bell in a raised position. The angle of inclination of the bell's axis, **b**, was measured with an accelerometer placed in correspondence to the axis of rotation; the normal stress in the cable was then measured by a load cell, while the cable's angle δ of inclination from the horizontal was calculated by measuring the lengths *a*, *b* and *c* (figures 10 and 11). Repeated measurements, effected by lifting the bell to different angles, furnished values of the weight varying from 1616 to 1743 daN.



Figure 7: the ambrosiano system.



Figure 8: the greatest bell during its motus.

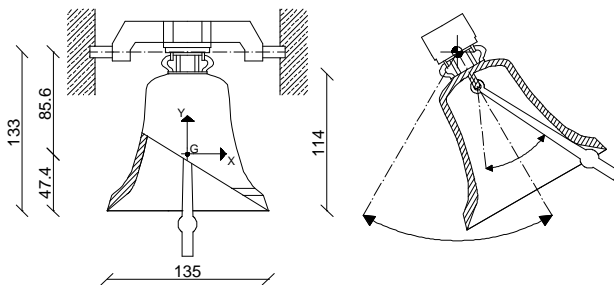


Figure 9: the greatest bell.

Table 1 : properties of the greatest bell.

Volume	$V_c = 0.2028 \text{ m}^3$
Moments of Inertia	$M_{xx} = 0.0441 \text{ m}^4$
	$M_{yy} = 0.0379 \text{ m}^4$
	$M_{zz} = 0.0441 \text{ m}^4$



Figure 10: weight measurements.

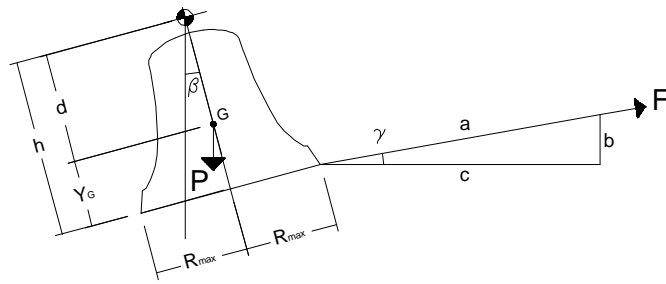


Figure 11: scheme of the weight measurement procedure.

3.2 The motion of the bell: experimental measures and analytical models.

The motion of the greatest bell was measured by recording the accelerations measured in the 4 accelerometers (HBM 200) positioned as illustrated schematically in figure 12.

The values of angle β , the velocity and the angular acceleration deduced from the accelerometer outputs have been compared with the corresponding quantities relative to the undamped free motion of a pendulum under conditions of large oscillations. In this case, $\beta(t)$, the angle of rotation of the bell, is a solution to the second-order non-linear differential equation

$$J \ddot{\beta}(t) + m g d \sin \beta(t) = 0 \tag{1}$$

in which t indicates time, J is the inertial moment of the bell's mass with respect to its axis of rotation, m its mass, g the acceleration of gravity, and d the distance of the centroid from the axis of rotation. The solution can clearly be expressed in terms of elliptical functions, with

$$\beta(t) = 2 \arcsin \left[\chi \operatorname{sn} \left(\sqrt{\frac{g}{l_r}} t \right) \right] \tag{2}$$

in which $l_r = J/(m d)$ is the reduced length of the equivalent pendulum, sn is Jacobi's elliptical function (Smirnov, 1964),

$$\chi^2 = \bar{\omega}^2 l_r / 4g \tag{3}$$

where

$$\bar{\omega} = \sqrt{\omega_0^2 + \frac{2g}{l_r} (1 - \cos \beta_0)}, \tag{4}$$

and ω_0 and β_0 are the initial values of the angular velocity and the angle of rotation, respectively.

From figures 13, 14 and 15, it is evident that the frequency of the undamped free motion relative to the same maximum angular amplitude is significantly different from that actually measured in true motion.

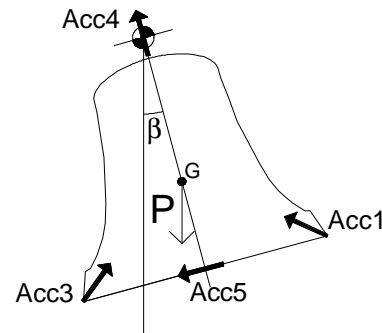


Figure 12: accelerometer arrangement on the bell.

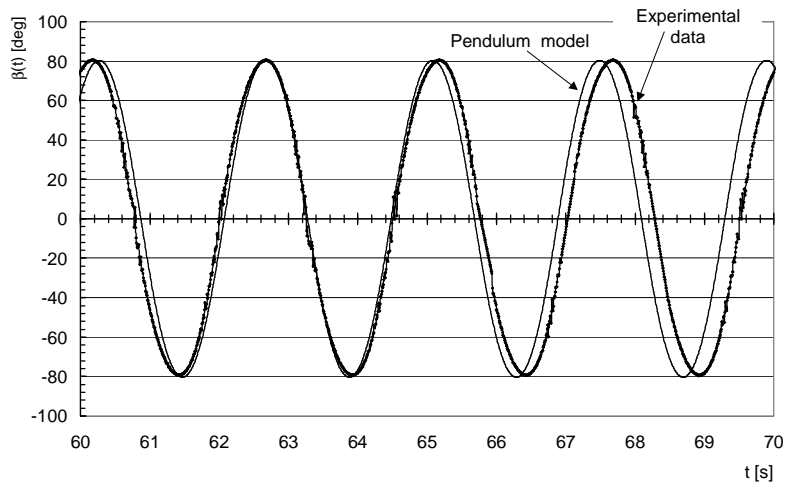


Figure 13: the behaviour of the function $\beta(t)$.

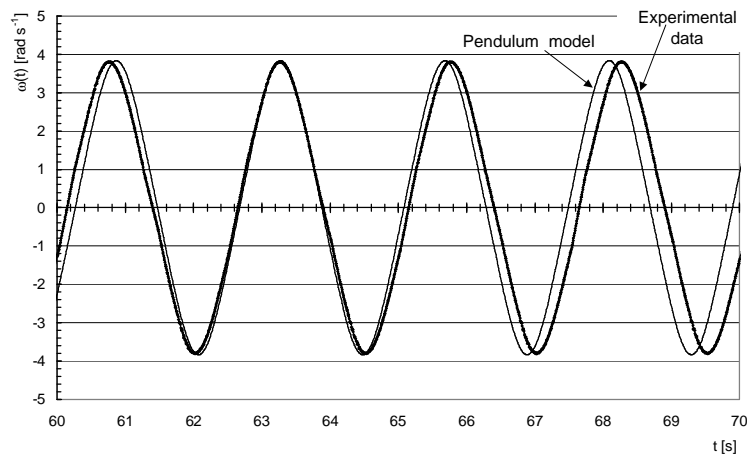


Figure 14: the behaviour of the function $\omega(t)$.

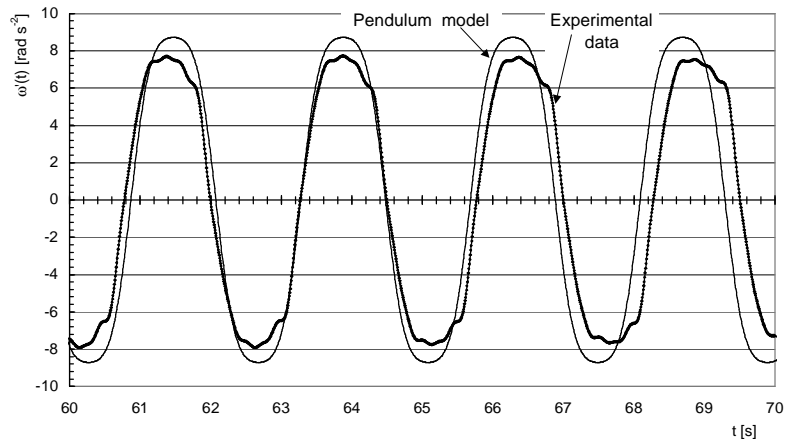


Figure 15: the behaviour of the function $\omega'(t)$.

The differences between the actual motion of the bells (partly forced and, in any event, damped as the result of the ever-present frictional forces) and the free motion corresponding to the same maximum angular amplitude are however not particularly great. Its true motion can instead be described highly accurately by replacing the actual bell with another one, identical except for the presence of an additional mass m_1 , situated along the bell's axis at an opportune distance d_1 from

the rotation centre. The additional mass m_1 and distance d_1 must be chosen in such way as to make the duration of the measured period coincide with that calculated for the free motion. In our case, m_1 and d_1 must be chosen so that

$$\frac{m d + m_1 d_1}{I + m_1 d_1^2} = 0.835 \text{ m}^{-1}. \quad (5)$$

Figure 16 show the value pairs (m_1, d_1) that satisfy relation (5). The negative values of additional mass indicated in the figure are clearly intended to express the mass to be subtracted from the bell in order to satisfy (5). An interesting finding is that a critical value of the distance exists ($d_{1,cr} = 1.1976$ meters), in correspondence to which the value of the additional mass diverges. Figures 17, 18, 19 and 20 show the behaviour of $\beta(t)$, $\omega(t)$ and the horizontal $H(t)$ and vertical $V(t)$ actions, respectively due to the actual motion, deduced from the accelerometer measurements, and the equivalent motion resulting from the *additional mass model*. Finally, figure 21 shows the frequency content of the horizontal action transmitted to the tower for both the experimental data and the analytical solution of the *additional mass model*, respectively: in both cases, the analysis has been conducted using the same sampling frequency. The good resulting fit suggests that accurate descriptions of the bells' motion and the effective consequent actions can likely be obtained in other experimental situations as well by applying an analogous model in which the bell, weighted with an opportune additional mass, is subjected to free motion with the same eigenperiod and the same maximum angular amplitude.

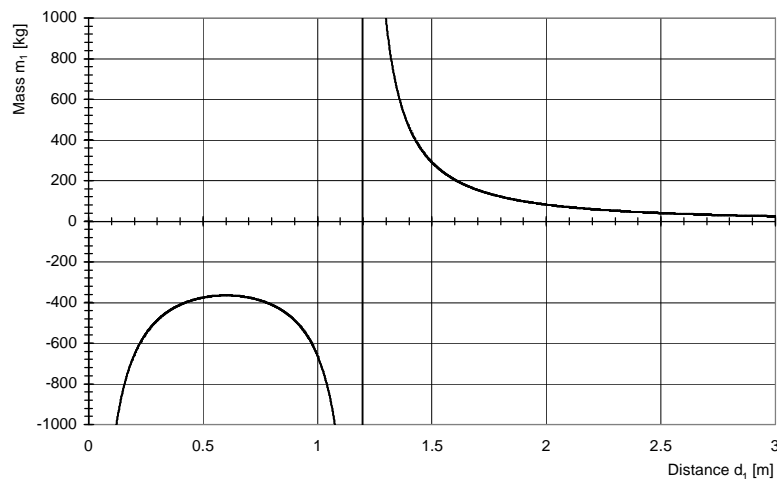


Figure 16: value of the additional mass vs. its distance from the rotation centre.

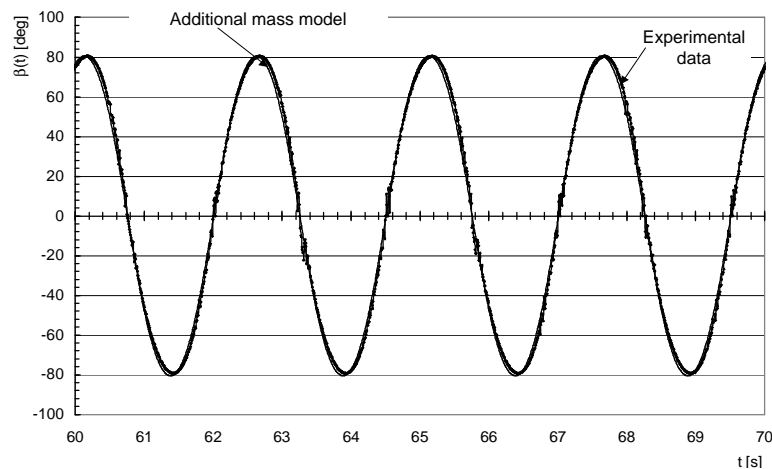


Figure 17: the behaviour of the function $\beta(t)$, (*additional mass model*).

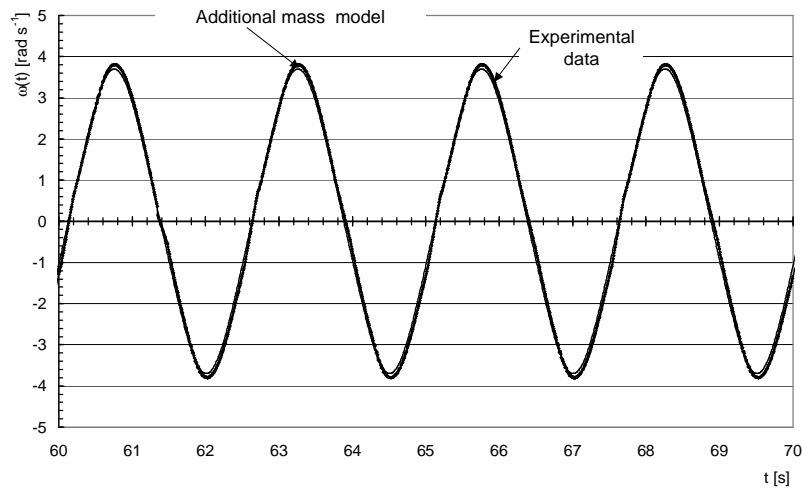


Figure 18: the behaviour of the function $\omega(t)$, (*additional mass model*).

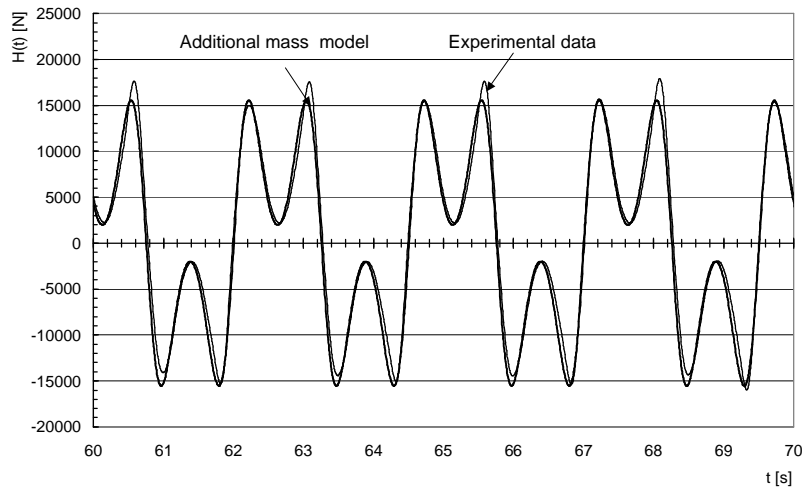


Figure 19: the behaviour of the function $H(t)$, (*additional mass model*).

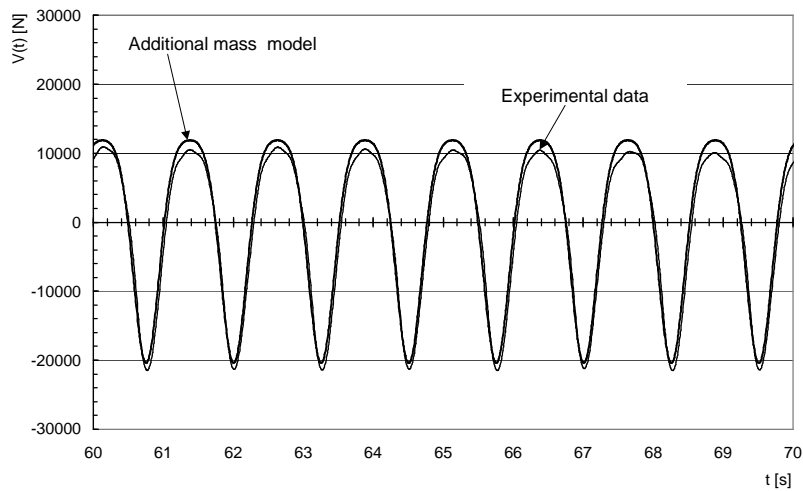


Figure 20: the behaviour of the function $V(t)$, (*additional mass model*).

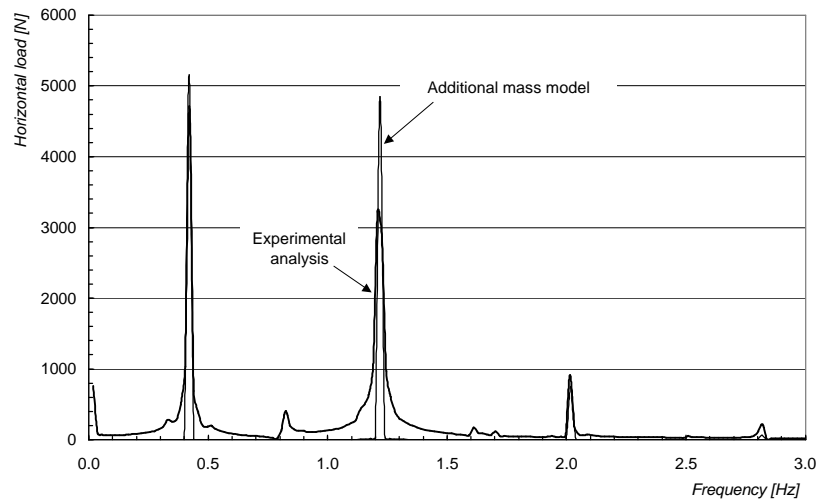


Figure 21: Fourier analysis of the horizontal load transmitted to the tower.

4 THE DYNAMIC BEHAVIOUR OF THE TOWER: SOME FIRST RESULTS

As seen in the foregoing, accurate description of the motion of the selected bell enables quite precise evaluation of the intensity of the horizontal $H(t)$ and vertical $V(t)$ dynamic actions exerted on the tower as a result of the bell's motion. A first, summary reconstruction of the actual forced motion of the bell tower is obtained through the recordings of the 4 accelerometers applied to the positions indicated in figure 22, in which the position of the line of action of the force $H(t)$ is also shown. Figures 23 and 24 show the frequency contents of the horizontal action $H(t)$ and acceleration $a_2(t)$ measured by the accelerometer labelled as number 2 in figure 22. Comparison of the two figures clearly reveals that the motion of the bell tower is, in a manner of speaking, "piloted" by the second fundamental frequency of the excitation force, $f_2 = 1.21$ Hz. It seems natural enough to expect that this frequency be close to one of the bell tower's first eigenfrequencies. With precisely the aim of verifying whether such a hypothesis is well-founded or not, we have developed a detailed numerical model of the bell tower using the finite-element calculation code ADINA 7.4. Figure 25 shows the mesh of the resulting numerical model, which contains 5,591 nodes and 4,002 library elements of the 3D-Solid Element type, to which linear-elastic behaviour has been attributed. Figures 26 and 27 show the first three calculated modal forms, corresponding respectively, to the first, second and third eigenfrequencies, all calculated under the assumptions of a material specific weight $\gamma = 18$ kN/m³, a Young's modulus, $E = 24,500$ N/mm² and a Poisson's coefficient $\nu = 0.2$. In the previously cited figures, 23 and 24, the two vertical lines correspond to the first two eigenfrequencies of the numerical model of the bell tower. The intensity of the measured accelerations and the considerable amplitude of the oscillations are due to the extreme proximity of the first eigenfrequency of the bell tower (as calculated through the numerical model) to the second fundamental frequency of the excitation force $H(t)$.

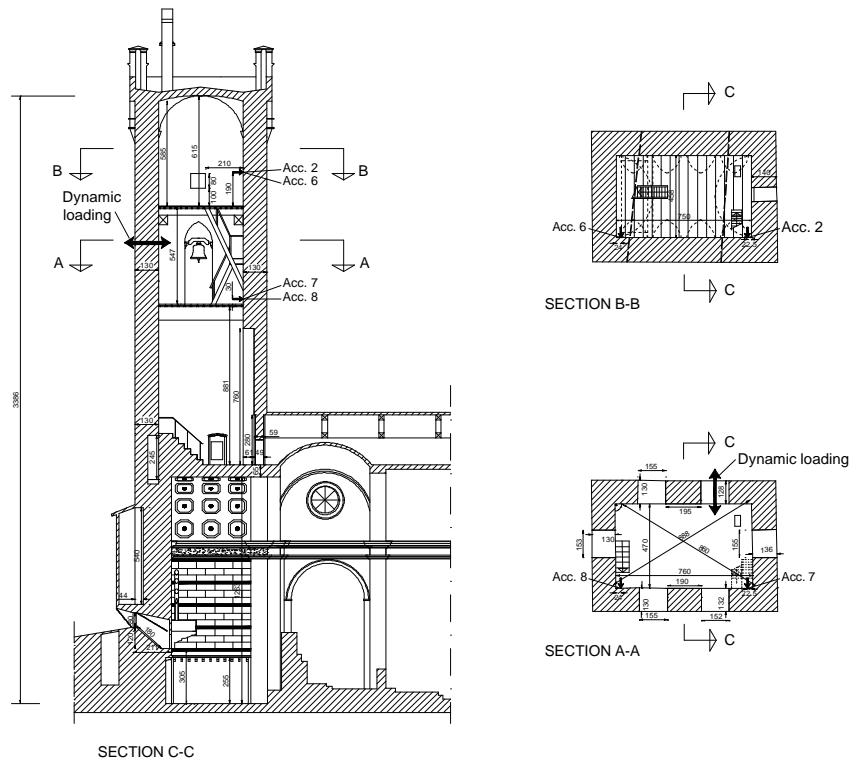


Figure 22: position of the accelerometers on the tower.

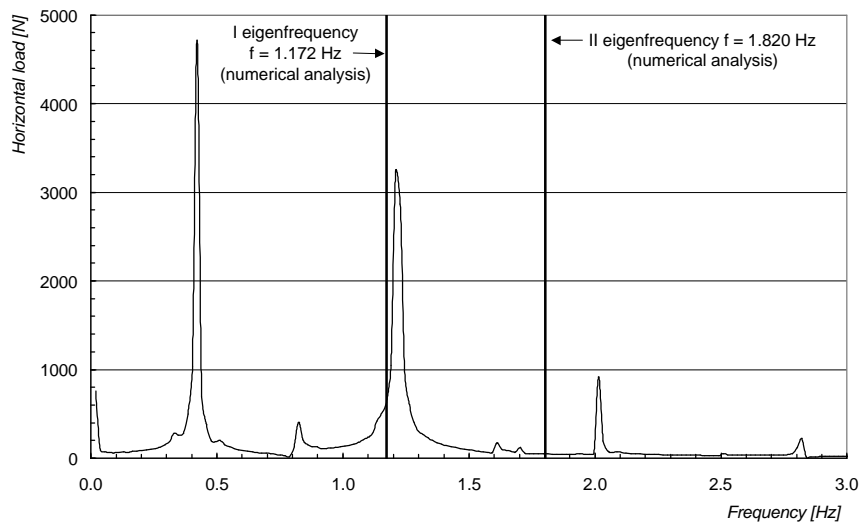


Figure 23: comparison between the frequency analysis of the horizontal force transmitted by the bell and the eigenfrequencies of the tower obtained by the numerical analysis.

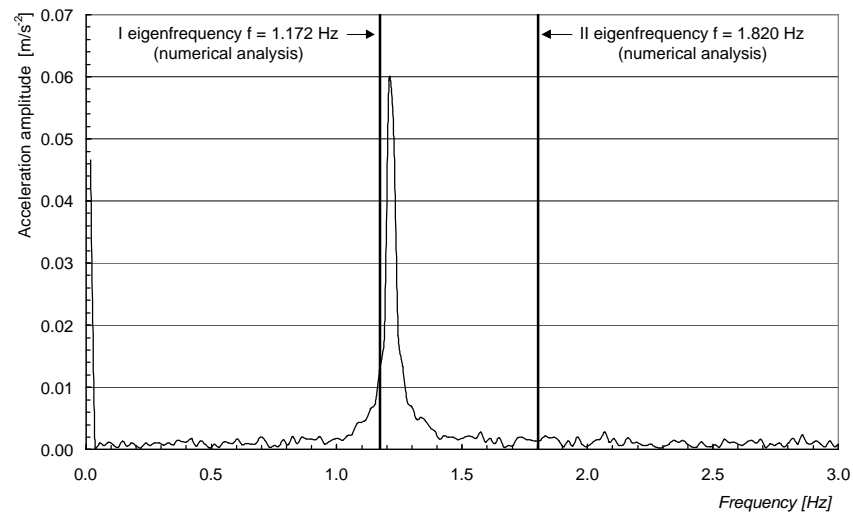


Figure 24: comparison between the frequency analysis of the acceleration recorded by acc. 2 and the eigenfrequencies of the tower obtained by the numerical analysis.

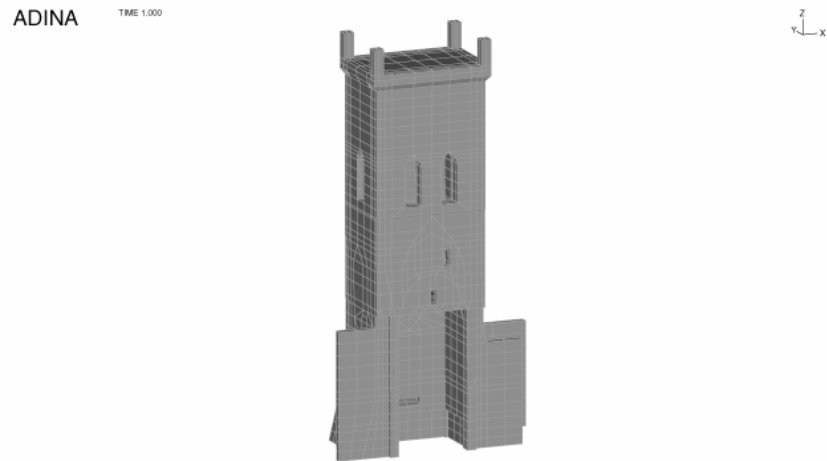


Figure 25: the mesh of the numerical model.

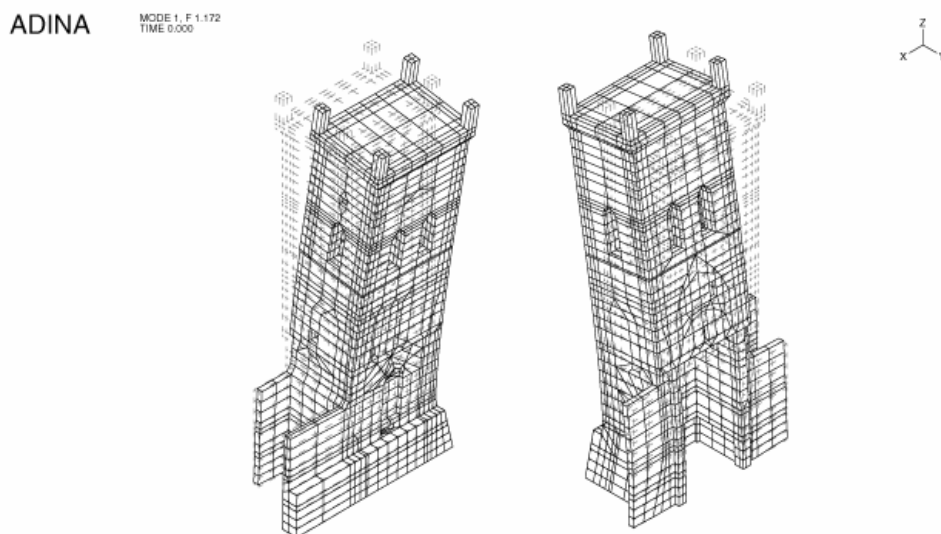


Figure 26: the 1st mode shape.

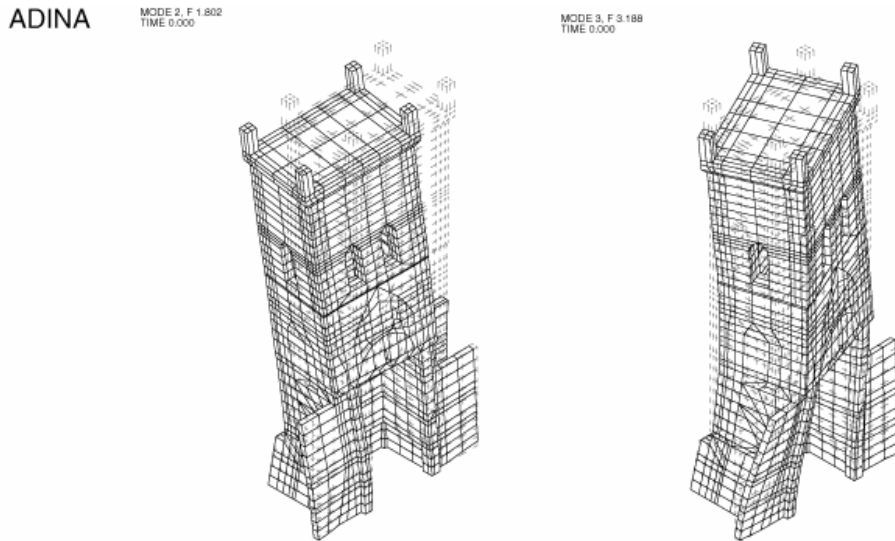


Figure 27: the 2nd and the 3rd mode shape.

We should emphasize that these first results refer to the isolated action of a single bell and are moreover based on a simple, purely linear elastic analysis, in which we have purposely ignored the material's obvious non-linearity. However, in an attempt to obtain confirmation, albeit partial, of the reliability of these results, we examined the dynamic behaviour of the numerical model when subjected to the horizontal action $H(t)$ alone. Figure 28 shows the comparison between the Fourier analyses of the accelerogram furnished by accelerometer labelled as n. 2 and of the corresponding output of the numerical model, calculated once again assuming Young modulus $E = 24500 \text{ N/mm}^2$ and a value of 0.025 for the damping ratio ξ (a recurrent value in the literature): the two responses, the experimental and numerical ones, coincide in frequency and differ in amplitude by no more than 5%.

It should however be noted that the response of the numerical model is strongly influenced by the choice of the two material constants, E and ξ . By way of example, figure 29 shows the frequency contents of the numerically calculated acceleration, $a_2(t)$, corresponding to three distinct choices for the value of Young's modulus. From this perspective, the good agreement obtained can be interpreted as a successful first attempt at identification of some average mechanical parameters of the bell tower.

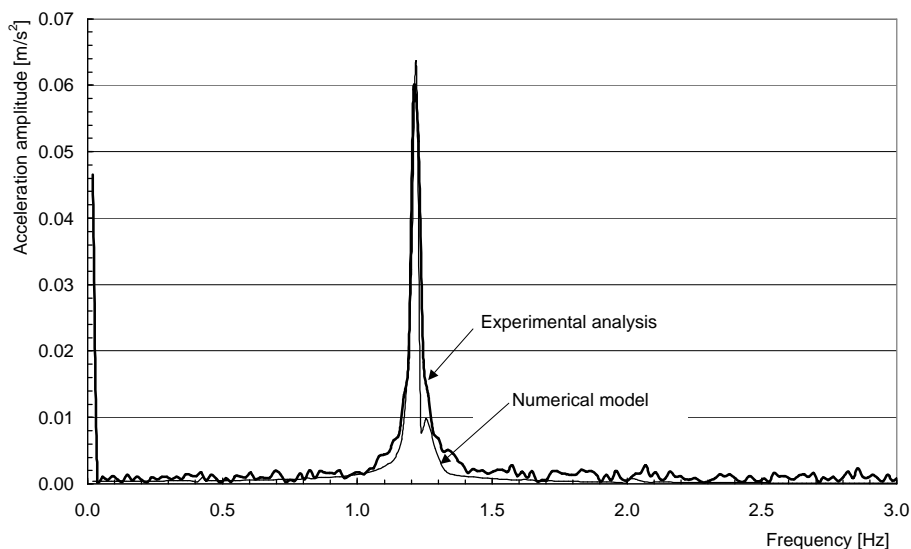


Figure 28: comparison between the Fourier analysis of the experimental recorded acceleration and the numerically evaluated one, accelerometer n. 2, $E = 24500 \text{ daN/mm}^2$.

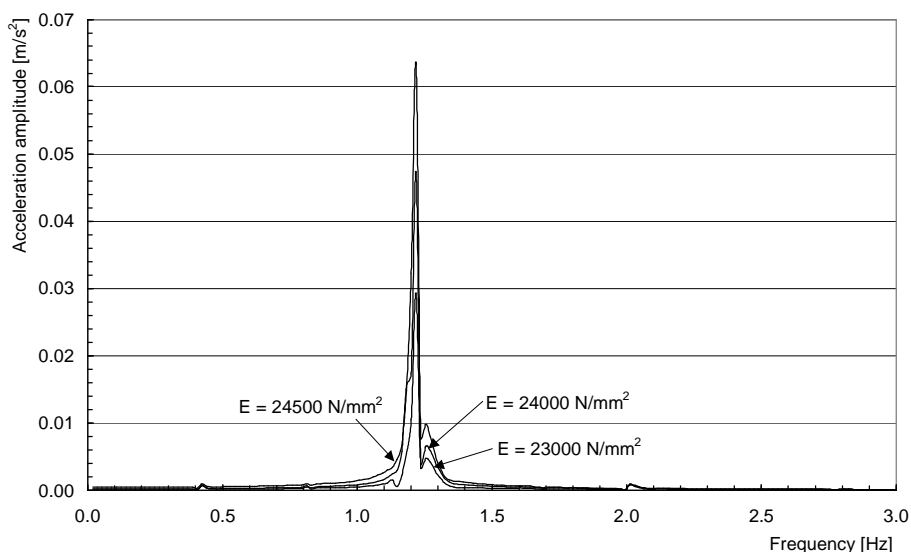


Figure 29: Fourier analysis of the acceleration numerically evaluated in correspondence of the position of accelerometer 2 varying the value of the Young modulus E.

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