Analysis and experiments of masonry arches

A.T. Vermeltfoort
Technische Universiteit Eindhoven, The Netherlands

ABSTRACT: In the Netherlands, almost all historical buildings are largely made of masonry. However, the arch as a form of construction has almost vanished in modern buildings. Arches are perfect as a test object for students because their behaviour can be understood, using simple applied mechanics. Form-finding techniques known from other disciplines can be used to design curved masonry structures. Graphical techniques and spreadsheet programs can also be used for analysis. However, to calculate bending and torsion moments FEM programs are convenient.

One rectangular and three skew 3-meter span arches were tested. The flexibility of the arch had considerable effects on measurements and load application. The position of the line of thrust can be 'observed' from the deformed arches, while the contact area in the cracks is only a few mm wide. Arches failed suddenly. Skew arches behaved in the same manner as rectangular arches, however warping effects should be studied further.

1. INTRODUCTION

In the Netherlands, almost all historical buildings are largely made of masonry with arches that span the existing openings. In modern buildings, steel or concrete structures are dominant. The arch as a form of construction has almost vanished and the lack of education in structural masonry was quite distinctly in relation to other materials. In Canada and England the same situation was recognised recently by Shrive (2001) and Roberts (2001).

At Dutch universities, education and research are strongly connected, Vermeltfoort (2000). Research is a substantial part of the curriculum. The interaction between structures and architecture is considered essential. The laboratory, named after the medieval Dutch researcher Pieter van Musschenbroek, has facilities for experimental research as well as numerical tools to simulate the behaviour of building elements and building materials

Students are quite free in picking topics for their projects in the second part of their study. Luckily, more and more students decide to do a 240-hour project in masonry. Students also have to work in outplacement projects preferably abroad. Prof. Harvey, University of Exeter, England, enabled the combination of the arch project at Eindhoven and work placement in Exeter, e.g. Boekel (2001) and Bertrand (1999). At present, Bill Harvey Associates are acting as consultants on bridges, Harvey (2001).

Arches are perfect as a test object: they are easy to handle, the required measuring techniques are not so complicated, and the behaviour of arches can be understood using simple applied mechanics. In spite of the fact that arches vanished as structural elements, knowledge of arch behaviour can come in handy in the design of concrete and steel frame structures, e.g. for three pinned frames. The form-finding principles used for masonry arches can be used also in the design of (lightweight) structures in other materials like tents and shells. The use of arches may be promoted by teaching about their behaviour and possibilities.

Senior students (from 3rd year on) work on individual projects. In experimental projects they work together with the laboratory staff. Instruction is given when needed. Experimental projects have more or less the same structure, which is as follows:

- Literature study on the theory of arches
- Experiments design of a test set up
building test set-up and specimens
testing / measuring
- Analysis (use of spread sheets e.g. Excell)
- Rapport (use of text editor e.g. Word) and presentation

Goal of the project was to offer a challenging educational environment to students. From the technical viewpoint the goal was to study the behaviour of rectangular and skew arches and the development of a test procedure. Experimental experience in testing arches will be used for the appraisal of (historical) buildings.

2. THEORETICAL ASPECTS OF MASONRY ARCHES

This chapter gives a short overview and discussion of some aspects of masonry arch behaviour that students should study thoroughly before starting the experiments. For their literature study students are referred to the lecture notes, Martens (1999), the books of Hendry (1987), Drysdale (1994), Heyman (1996) and O'Connor (1993) and, of course, to recent proceedings of masonry conferences.

2.1. Finding the ideal form of arches

In nature, openings are covered by curved structures such as the ceiling of caves. The first man made spans in brick were 'corbelled arches'. In corbelled arches, bed joints are horizontal, Drysdale (1994). For true arches, bed joints are 'perpendicular' to the thrust line, which ideally is in the centre of well-designed arches. Students become aware of the fact that structures should be shaped in a way that compression prevails, which often leads to a curved structure that follows the thrust line. The effect of weight on stability can easily be demonstrated by the overturning of a wall. The theoretical failure load for an equally distributed loaded arch is dominated by the compressive strength of the masonry as the full section is under almost equal compression. However, variable loads can cause a shift of the thrust line, causing cracking of the arch.

The changes in the position of the thrust line due to life loads are used to establish the thickness of the arch. In some cases the effects of back fill and surrounding masonry should be considered too.

2.1.1. Form finding aspects under dead load

Masonry only has a minor tensile strength in comparison with its compression strength. Bending strength depends on the amount of compression. Elements that can take compression only do not exist. When a compression load is applied, also some ability to take bending moments develops. However, elements that can take tension only do exist: a piece of wire or a chain. When strings are used to model structures, bending moment are neglected.

The ideal shape that an arch would take under its weight can be visualised by showing a wire, suspended between two points. The wire takes up a catenary shape under its own weight and is in pure tension. Imagine the shape turned upside down, and tension would convert into compression, giving the thrust line for this load situation.

In a catenary shaped arch, the weight is distributed evenly along the arch. It is easier to take the distribution of the weight (or permanent load) evenly distributed horizontally along the span which results in the chain taking the shape of a parabola. A circle segment, see fig. 1, also can approximate the catenary.

The catenary, the parabola and the circle segment of the tested arch can be represented in mathematical forms as follows:

catenary: $y = \cosh(x/1500-1)*921$ (1)
parabola: $y = ax^2$ $a = 1/4500$ (2)
circle segment: $y = \sqrt{(R^2-x^2)}$ $R = 2500$ mm (3)

These functions have their 'supports' (-1500,0) and (1500,0) and their 'top' (0,500) in common.

For gentle arches the differences in shapes between the three functions is relatively small, fig. 2. Better fits can be found by changing the parameters (catenary 1), or moving the parabola 7.16 mm downwards, see dotted line in fig. 2, or using the 'least squares' method.

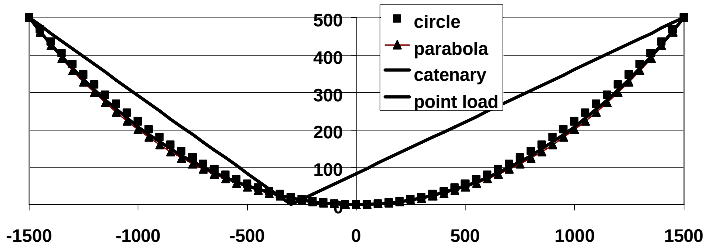

Figure 1 The shape of a catenary compared with circle segment and parabola

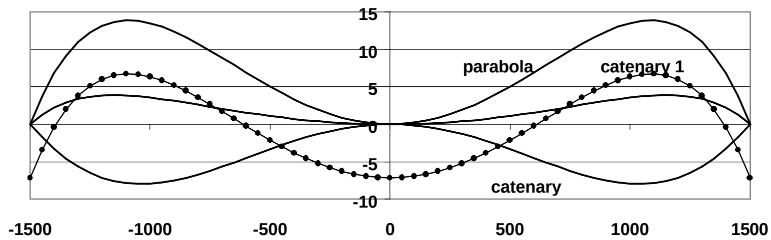

Figure 2 Differences (in mm) between catenary, parabola and circle segment

A distributed load can be abstracted to a number of concentrated loads and graphical solutions can be used to find the ideal shape of the arch. When the loads are less evenly distributed it is much more complicated to find the shape of the arch. Using the string model however gives the perfect shape instantly. Calculation of this shape is possible via the following iterative process. See fig. 3. The position of the supports are given and l_1 through l_{n+1} as the lengths of the strings between the loads P_1 through P_n. Estimate V_1 and H and calculate for each node from left to right: (i=1 to n) S_i, x_i, y_i, and V_{i+1}. Check whether the co-ordinates of the last point are approximately equal to those of the right hand support ($x_n \sim x_s$ and $y_n \sim y_s$). If this condition is not met satisfactorily, new values for V_1 and H should be estimated and the calculating process for the co-ordinates repeated. In this example, H is constant while the loads are vertical. By using a spreadsheet, engineering judgement and user interaction are integrated in the process.

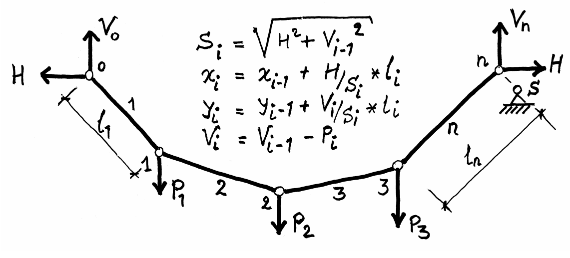

Figure 3 Finding the shape of a string with weights by calculation

Another, perhaps more sophisticated, manner of finding a form for a number of weights on a piece of string is the force density method, Schek, cited in Huisman (1994). The key in the force density method is that the designer chooses the ratio between the force in the string and the length of the string (S_i/l_i). Then, a set of equations can easily be formed. These equations represent equilibrium in all points where loads are applied and have the co-ordinates of these points as the unknown values.

2.1.2. Form finding aspects under variable loads

In many if not all practical cases, the weight of a masonry arch dominates. Fortunately, masonry material (essential clay) is available in large quantities. Also sand is often used as a back filling. Other kinds of loading can be incorporated in the cable model by adding equivalent weights.

Consequently, more than one thrust line is possible and all lines of thrust have to be contained within the arch. The lines of thrust for non-symmetric loads like wind or axle loads deviate from the ideal line of thrust and cause arch failure quite easily. The thrust line of a load in one point load deviates most from the ideal thrust line, fig. 1. Arches do not fail because of their insufficient material strength but on the fact the thrust line moved too far. The differences in the position of the thrust lines give an indication of the necessary width of the arch. According to O'connor (1993), pag. 181, the arch used in the experiments with a thickness of 10 cm and a span of 3 meter is relatively thick.

2.1.3. Remarks

The horizontal components of the reaction forces have to be carried by a supporting structure, like the foundation at ground level. When walls or columns support the arch the horizontal thrust must be taken by a tension element (steel). As steel was scarcely available in the past, the steel tendons often have a small section in historical buildings. The effect of moving of the supports should be considered too.

Bending moments are neglected in this kind of modelling. This is acceptable because the bending capacity is small after cracking of a section.

3. EXPERIMENTS

One rectangular and one skew arch were tested by Tom Bertrand (2000). Helen Kok (2001) tested two skew arches. Laboratory staff members assisted in building of the arch, the application of measuring tools and collecting data.

The main goal, besides offering a challenging learn-environment was to study differences in behaviour between skew and rectangular arches and to obtain experience in testing this type of structures.

3.1. Dimensions, shape and materials used

All four arches had a clear span of 3 m, an inner radius of 2.5 m and a sagitta of 0.5 m. Thickness was equal to the width of the brick (approximately 100 mm). Joint thickness was 12 mm. The arches had 51 layers, and a width of 6 bricks, (1.25 m). The formwork was the same for all four arches. In skew arches the longer side was parallel to the diagonal in their ground plan of 3 by 1.2 m, fig. 4.

The test-arches were build with Rijswaard soft mud bricks and 1:2:9 mortar. Brick compressive strength was 27 N/mm^2; mortar compressive strength was 2.5 N/mm^2. In all cases the bed joints were parallel to the support except in the last skew arch where they were perpendicular to the longer edges.

3.2. Experimental details

The arches were supported via L shaped steel profiles and ball bearings on a steel frame. The ball bearings reduced the friction in the support and consequently the tensile elements took the full horizontal thrust, fig. 6. In the L profile, bricks cut in shape were used to ease the force transition between arch and support. The tension bars were positioned in the centre of the arch and the support. In test R1 the tension bars where connected to the frame at one side, in the other tests they connected both supports leading to a more symmetric situation.

During construction of the arch the supports were connected rigidly to the frame. Forces in the tension bars, the horizontal displacements of the supports and the deflections of the arch under the loads were measured. The load was applied via rectangular hollow sections as 'line' loads, 600 mm centre to centre, using jacks, figure 5.

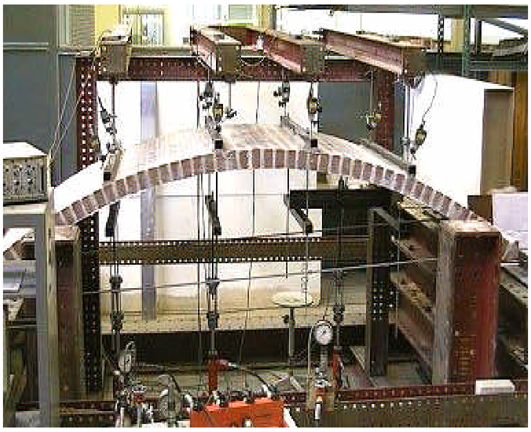

Figure 4 A skew arch resting on its formwork.

Figure 5 Overview of the test set-up, load-positions 1, 2, 3 and 4 from left to right

The arches were loaded and unloaded two or three times with equal forces (± 4 x 5 kN each). In the last cycle, the load at position 2 was increased. It was the intention that the other loads remained equal. However, due to the arch deformations, the forces in the three other jacks varied. In tests R1 and S1 three jacks were used. The load of one jack was applied via a load distribution beam. In tests S2 and S3 four jacks were used. In S2 they each had their own pump. In test S3 the three jacks that were supposed to give the same load were connected hydraulically, leading to a smoother operation.

Figure 6 Details of the supports and load introduction

3.3. Results

Some important results are presented in Table 1.

Table 1 Overview of arches tested and results

	type	deflexion at position 2		failure load		
		4 equal loads	at failure	kN	kN	kN
R1	rectangular	9.0 kN/mm	55 mm	5.9	40.7	2*9.1
S1	skew //	16.9 kN/mm	--	--	--	--
S2	skew //	9.0 kN/mm		12.6	26.0	2 x 2.3
S3	skew ⊥	10.0 kN/mm	50 mm	8.8	22.88	2 x 8.8

bed joints parallel to support, ⊥ bed joints perpendicular to edge = 21.8° to supports.

In the first skew arch, S1, the jacks did not function properly due to errors in the hydraulic system. That was the reason to alter the number of jacks and pumps for the next tests.

The stiffness under four equal loads was approximately the same for all tests, except S1. The higher value of S1 perhaps has been caused by the ball-bearing supports that did not move freely in this test. The skew arches failed under smaller loads than the rectangular arch. Not only the intended force at position 2 but also all three others did vary except in test S3 where the jacks were connected hydraulically.

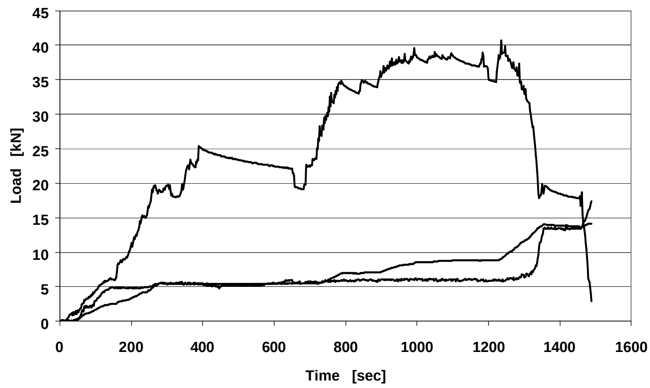

Figure 7 Load versus time for test R1, final load cycle

3.3.1. Load deflection diagram

The load deflection diagram for arch R1 under 4 X 5 kN load is presented in fig. 7.

A crack already developed at load position 2. During unloading this crack closed. During the increase of load 2 the force in jack 3 decreased. The arch took an other form, downwards at position 2 and upward at position 3 and 4. For measurements, forces were gradually increased to keep the arch more or less in the same position, e.g. between 400 and 600 sec. Ripples in fig. 7 were caused by manual operation of the jacks. This kind of behaviour, also observed in the other tests, is caused by the unintended changes in loading. When one jack is operated to change its load, the arch 'moves' and the other loads are effected. In the last test, S3, this effect was smallest.

In fig. 8 deplacements under equal loads are presented. It can be observed that at already some deformation differences developed, perhaps due to differences between in the supports.

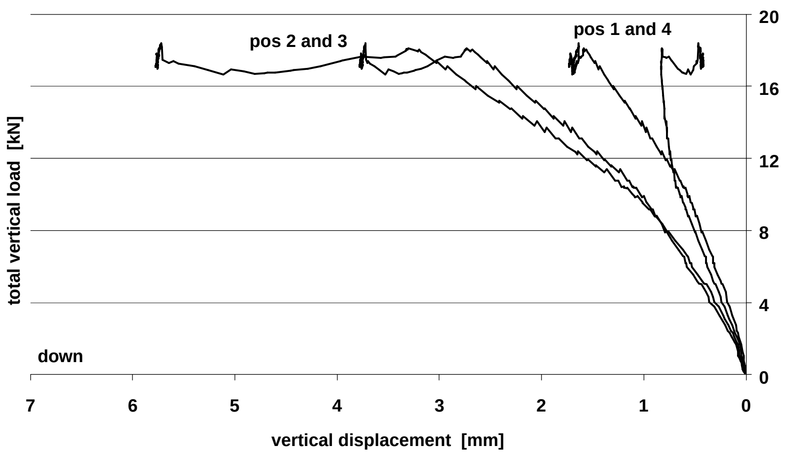

Figure 8 Vertical displacements versus the total of the four equal loads (down is positive).

3.3.2. Shape of the arch at failure

As mentioned earlier, (chapter 2), deformations were small under symmetrical loads, Table 1 and fig. 8, while the thrust line almost coincides with the centreline of the arch. One larger load causes much larger deformations, fig. 9. The arch moved downward near load 2 and clearly upward near loads 1 and 3. Figure 10 shows the deformed arch at the end of the test. From the measured deformed geometry of the arch a kink at 600 mm from the left was observed, corresponding to the position of load 1.

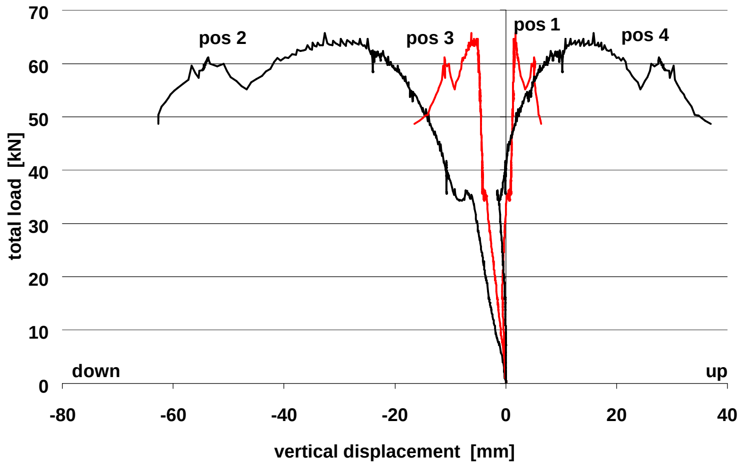

Figure 9 Vertical displacements versus the total of three equal and one larger load.

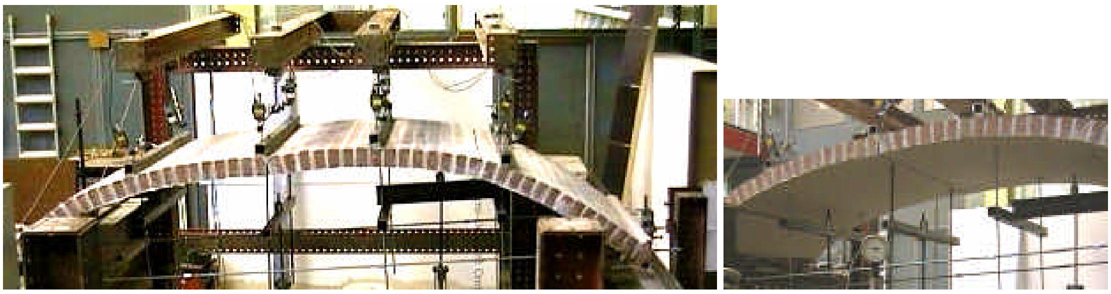

Figure 10 Deformed arch almost at the end of testing, and crack beneath largest load, position 2, test R1

For an arch four hinges are necessary to collapse. Two hinges developed due to rotation of the supports. Another hinge developed beneath the largest load and one developed more to the right hand side than expected, just left from load at position 4. Also cracks were observed near the right hand support. In the right hand part of the arch, more than the one crack needed to form a hinge developed. In that part, the line of thrust passed the edge of the arch over a considerable length.

3.4. Skew arches

Besides the development of a test and test procedure the differences between rectangular and skew arches were studied. Skewnes could affect such things as load position and ring thickness versus span. The loads were applied as line loads similar to the rectangular arch. However the load-lines were perpendicular to the length of the arch. Larger deformations than for the rectangular arch and some warping can be observed in fig. 11. In the last arch, S3, the cracks ran through the bed joints, perpendicular to the longest side, while in the other tests the cracks were mainly parallel to the supports. However, joint direction has no significant effect on strength.

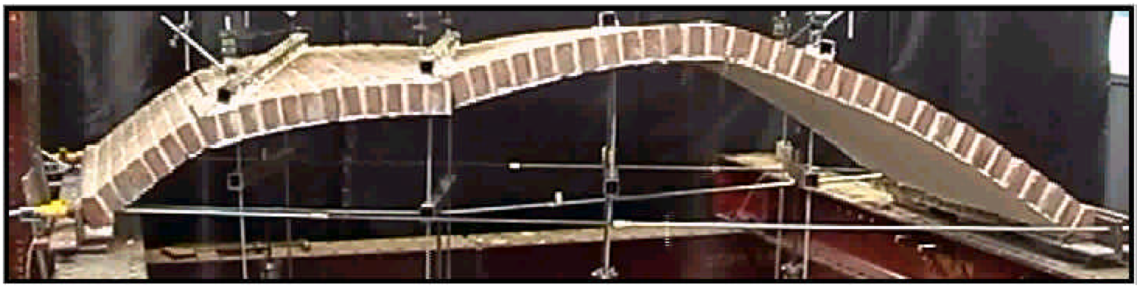

Figure 11 collapse of skew arch S2

4. ANALYSIS OF THE TESTED ARCHES

The ‘string’ and the graphical method discussed in chapter 2 assume hinged connections and flexible elements. Their use is easy to understand. In combination with modern software these methods can be very useful, Harvey, (2001). With other (fenite element) software, flexibility can be taken into account and bending moments established.

Analytically, the arch can be represented as a bend, statically-undetermined structure and using the equations for displacements and rotation the reaction forces can be established, Shrive (2000). In fact, the reactions (horizontal and vertical forces and moment) at one support are the unknowns. With given values (zero in most cases) for the horizontal and vertical displacements and rotation at the supports, the unknown forces can be calculated.

Using fenite element (FE) software, actually the same happens. The arch is distributed in a number of elements (10 in our case) and the loads in various cases applied. The load bearing capacity of the arch is controlled by the eccentricity of the normal force in critical sections, i.e. $e < 0.5\ h$. In other words, the position of the thrust line should be within the section of the arch.

As most software give their results as forces and moments (N, V, M) in a number of points, these results were fed into a spreadsheet and the ratio M/N for each point calculated and plotted perpendicular to the centre line of the arch.

Calculations were made for: a) own weight of the arch, b) four equal loads of 5 kN and c) loads of 5 kN, 15 kN, 5 kN and 5 kN respectively. Further, $A = 1200\ cm^2$, $W = 2880\ cm^3$, $I = 17280\ cm^4$ and $E = 3000\ N/mm^2$. Loads were applied as in the test. The lines of thrust for the cracked and uncracked arch show that only the situation with one larger load is critical in the section where the larger load was applied. Further, the already mentioned area where the line of thrust reached the edge of the arch was clearly visible.

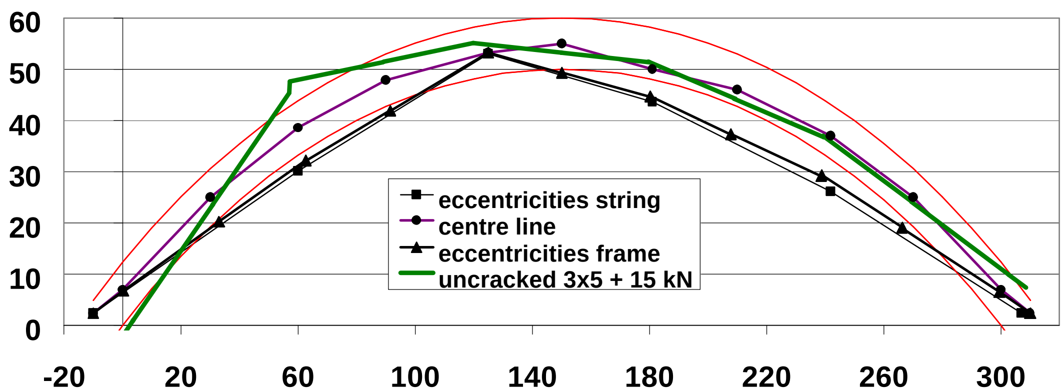

Figure 12 Eccentricities (or thrust line) in the three pinned arch

In the test, first two hinges developed near the supports and than a hinge beneath the largest load (position 2). For this 3 pinned situation a good resemblance was found using the ‘string’ method and the FE method for the largest loads observed in the test, see table 1. The calculated eccentricities representing the thrust line are plotted in fig. 12. The thrust line is outside the arch over a considerable area in the right hand part, where consequently more cracks were observed.

4.1.1. Skew arches

Numerically, skew arches behaved in the same manner as in the experiments. Warping of the elements was observed, compare fig. 11 and fig. 13. Further analysis is part of subsequent research.

It showed in the test that the shortest tensile element, the one that spaned over the shortest diagonal, took the largest force, indicating that the arch took the largest load in the shortest direction. As measuring of reaction forces is difficult, a numerical simulation could give some answers. Also the effects of bed joint direction could be simulated.

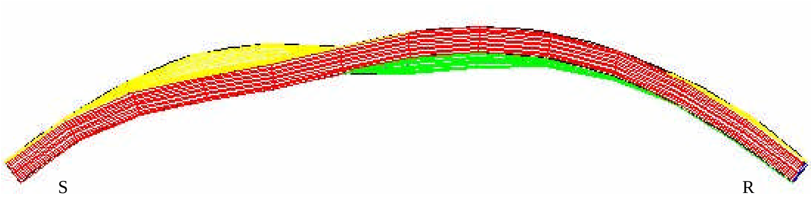

Figure 13 Deformed shape of skew arch calculated with ANSYS

5. DISCUSSION

5.1. The experiments

Loading and unloading with hydraulic jacks gave difficulties while the arch was relative flexible and loading one point caused displacements of the other points and consequently a change in loads occurred. At the end of testing deformations were so large that the jacks got out of plumb. In this way, simulation of dead load was not realistic. A real dead load, e.g. steel-chains can be used for scale models, Bertrand (1999). However, for larger (semi-full scale) tests an unsafe situation would occur. Other ways of loading e.g. by means of air mattresses should be considered. Then, the load would be more perpendicular to the surface giving a better representation of wind load too.

The effect of load distribution by a back fill by sand on the arch nor the effect of additional masonry to flatten the top of the arch were not studied yet.

Test and numerical results were similar. The arch cracked as expected and hinges developed at positions were they were expected. An acceptable prediction of force transfer and displacements under service load (small deformations) can be given. In the failure state the magnitude of the displacements is unpredictable while the arches failed suddenly. Perhaps failure should be defined in terms of maximum displacement, e.g. 30 mm = 0.01 of span

The arch was relatively thick; subsequently the behaviour of thinner arches may be studied. Tensile strength was relatively high for this kind of masonry.

FEM analysis for the arch modelled as a frame was discussed. Some linear analysis with with plate elements using ANSYS on the skewed arch has been done. In a subsequent project the numerical analysis, in stead of the experiments, could be the main item.

5.2. Education

More time than the required 240 hours was spend by the students in this case. Laboratory projects often take longer because students first have to develop ideas about tests. Then, specimens have to be made and harden and laboratory work has to be scheduled. After that, tests can be performed and finally reported. Construction and hardening of specimens causes a break of a few weeks in this type of projects.

Students work on individual projects and decisions are made independently. Discussion about the choices made is part of the learning process.

6. CONCLUSIONS

- Arches offer a challenging educational environment; students spent more time than required.
- Spreadsheet programs were useful for thrust line modelling and analysis.
- The set up of the test was satisfactory. Few points need improvement, such as load introduction. The deformation measurements should include horizontal displacements.
- An acceptable prediction for deformation under service load is possible.

ACKNOWLEDGEMENTS

The financial support of the Dutch Masonry Foundation is gratefully acknowledged. Participating in the foundation are the clay brick, calcium silicate, AAC and concrete units industries and mortar manufacturers.
The contributions of Tom Bertrand and Helen Kok are acknowledged. Thanks also goes to the staff of the Pieter van Musschenbroek laboratory for their support.

REFERENCES

Reports made by students under supervision of the author:
Bertrand, T., 2000, *Het draagvermogen en de bezwijkvorm van gemetselde bogen,*
Kok, H., 2001, *Onderzoek naar het bezwijkgedrag van twee gemetselde bogen,*
Reports made by students under supervision of Harvey, B. and the author :
Bertrand, T., 1999, *Stageverslag Exeter, the behaviour of masonry bridges*
Fiege, M. 1998, *Stageverslag Exeter, the behaviour of masonry bridges*
Boekel, R. van, and Ven P. van de, 2001, *Stageverslag Exeter 2001, Bill Harvey Associates, Structural Engineering, Conservation and Repair,*
Drysdale, R.G., Hamid, A.A., Baker, L.R., 1994, *Masonry Structures, Behaviour and design,* Prentice Hall.
Harvey, B., 2001, *Thrust line analysis of complex masonry structures using spreadsheets,* 3rd Int. Sem. structural analysis of historical constructions.
Hendry, A.W., Sinha, B.P., Davies, S.R., 1987, *Load bearing brickwork design*, Ellis Horwood series in Civil Engineering, 2nd edition, pp 11.
Heyman, J., 1996, *Arches, vaults and buttresses,* Variorum, pp. 279..295.
Huisman, W.J.J., 1994, *Vormbepaling van de ALCO dome,*Syllabus KO Research dag EUT pp. 135-147, In Dutch.
Martens, D.R.W., Vermeltfoort, A.T., Bertram, G., 1999, *Stapelbouw, Lecture notes* (In Dutch)
O'connor, C., 1993, *Roman bridges,* Cambridge University Press.
Roberts, J. and Simm, D., 2001, *What is learnt About Masonry by students of Civil and Structural engineering ?,* Masonry international, vol. 14, no 1, pp 5-8.
Shrive, N.G. and Sturgeon, G., 2001, *Masonry education in Canadian Post-secundary intitutions,* proc. 9th Can.Mas. Symp, Fredericton.
Shrive, N.G., Reda, M.M. and Huizer, A., 2000, *Simle design procedures for masonry arches,* proc. 12th IBMaC, Madrid, pp. 1687..1696.
Vermeltfoort, A.T., 2000, *Experiences with masonry education projects,* Annex of proc. 12th IBMaC Madrid, pp. 2215..2227