Thrust line analysis of complex masonry structures using spreadsheets

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ABSTRACT: Thrust line models of a number of more complex, three-dimensional structures have been built in recent years. Two examples are: a complicated system of flying buttresses supported on vaults at Wells cathedral and the vaulted supporting structure of Leeds railway station.
The paper describes the procedures adopted in building a three-dimensional thrust line model in a spreadsheet. It explains some of the more complex issues, which arise in three-dimensional studies, including aspects that have been ignored in simpler analyses. The implications of a wrenching action generated by the structure are also discussed
Problems of visualisation, interaction and control are described. The control tools provided through MS Visual Basic are discussed, and their application demonstrated.
A brief explanation is given of the procedures for defining the geometry of the structure and for computing its gravity response.

1 INTRODUCTION

1.1 The problems of assessment

It is easy to think of assessment as a simpler process than design. It does not, immediately, appear to demand the creative process. When it comes to analysis as part of assessment, however, the problems can be much greater than in primary design. This is because, in design, the structure being analysed does not exist. The designer's task is to ensure that it is constructed in such a way that the analysis he carried out is valid. In assessment, the structure is there, but much of its geometry and history are hidden.
Ted Happold said that design is all about achieving the confidence to build. In assessing an existing structure the confidence to approve may be more difficult to achieve.

1.1.1 Plastic Theorems

Modern structures, properly designed, naturally satisfy the demand for plasticity. That is they are capable of redistributing load before collapse. This means the analyst can be satisfied with a single positive result. If he finds a way for the structure to stand up, then it will also find a (possibly different) way. In dealing with masonry structures, engineers brought up on modern materials find it difficult to believe in plastic behaviour. Masonry is a brittle material but it is capable (after cracking) of developing very flat moment rotation characteristics. This is because the moment is caused by eccentricity of a large force. An analysis that shows the potential for redistribution in a graphic way provides a confidence-building route to peaceful sleep for the engineer.

1.1.2 The role of stiffness

To begin with another quotation from Happold, “all structures are a series of interconnected stiffnesses”. A problem with masonry is that, because it is used in great bulk, it is much stiffer than the materials most engineers are used to dealing with. Nevertheless, it is capable of bending without cracking if the loading regime is appropriate. The point here is that redistribution takes place with minimal deformation. It is therefore often possible to ignore deformation in analysis.

2 METHODS OF ANALYSIS

The toolbox of the analyst seems full to overflowing. With almost unlimited computing power available, it seems it should be possible to analyse any structure. In the course of the authors’ careers we have passed from slide rule and moment distribution through frame analysis and finite elements to discrete elements. Each depends on a sound knowledge of material behaviour, an effective material model and a measure of certainty about initial conditions. Even with all of these, the process of building and running a model can be very expensive. The engineer needs to have a simple tool available for a quick response.

2.1 Equilibrium

Equilibrium analysis is about the stability of a structure in whole or in part. Is this element, or this complete structure capable of sustaining the forces imposed upon it without rotating or sliding out of control. The primary tool of visualisation is the thrust line, effectively invented by Robert Hooke in 1695.

Engineers have used the thrust line in many forms. Through the nineteenth century, they tended to ignore the element of indeterminacy in an arch and think that the thrust would follow what they perceived as the best line through the structure. Gaudi, in the early 20thC went further and modelled his thrust lines first then disposed the material around them so that the thrust was in the centre of the structure. Unwittingly, everyone relied on the plastic theorems. The structures redistributed load and stood the test of time, though the calculated (or physically modelled) thrust lines bore no real relation to the actual path of forces through the structure.

Now, it is possible to build models very similar to those of Gaudi in the computer, and to use them to explore the possibilities of redistribution. If we let go of the desire to make the computer king, we can build interactive models that merely relieve the engineer of the chores, while ensuring he remains in control of the analysis. Ideally the analytical tool becomes a learning tool, allowing the engineer to progressively develop his understanding of the likely behaviour of the structure.

3 MODELLING

Effective equilibrium models depend on simple processes. We need a way to divide the structure, a way to compute the forces generated by and on each element and a way to trace the development and flow of force through the structure. Finally, a range of tools is needed, to provide effective visualisation of the effects and control of the process.

3.1 Combining forces and moments

Tracking the flow of force through a structure involves adding the small contributions of force from different elements and, occasionally, finding appropriate ways to divide them between different potential paths. Forces are best divided into components. Rather than fix them geometrically in space, it is better to locate them by computing the moment they have about a chosen origin. In this way, a force has six components (three forces and three moments) and they can be added simply by adding the components. This is a perfect application for tabular calculation, a wonderful application for spreadsheets. The remainder of this paper will be dedicated to an exposition of the processes used by the authors in dealing with two specific structures, and a discussion of how the process might be applied to further types.

4 SPECIFIC PROBLEMS

4.1 Flying Buttresses at Wells Cathedral

4.1.1 Introduction

Flying buttresses are usually seen as two-dimensional structures. In Wells, the buttresses that were built to take out the thrust from the window arches in the choir were modified, either during construction or shortly afterwards, in a way that has caused anxiety to a number of engineers.

When the Lady Chapel was built, east of the Altar, behind the Bishop's Cathedra, a linking circulation space was built, which became part of the ambulatory. The Original flying buttress, above the roof (figure 1) of this area looks perfectly normal, as do the columns and vaulting seen from beneath. Only in the roof void is the unusual construction obvious (figure 2). Here a block of fine ashlar masonry inclines far from the perpendicular to transmit load from the buttress above down to the slender column below. The buttress effectively steps sideways onto its support.

Challenged to prove that this structure, which has stood the test of 600 years, was structurally

Figure 1 The Wells buttress above the roof

sound, we built a spreadsheet model to explore potential thrust lines.

4.1.2 Modelling decisions

Surprisingly, the principle direction of the system was the vertical one. We therefore chose to cut the structure into horizontal slices. Ideally the cut lines should cross the thrust at right angles, but we thought, correctly, that such complexity would not be necessary here.

The buttress and column have complex and quite different shapes. The section in the roof void is essentially rectangular in plan, while the vaults must be modelled in some entirely different way. We modelled the slices as octagons in plan, allowing a different shape top and bottom. Because the slices were horizontal, it was best to use a local origin for calculating moments. This was chosen as the point of intersection of the relevant horizontal plane and the vertical axis through the centre of the buttress where it met the wall.

Figure 2 The buttress steps sideways in the roof space

We chose to trace the thrust downwards, treating the thrust from the wall as input. The forces sustainable by the surrounding masonry prescribed the limiting values.

4.1.3 The process

The principal forces involved are the self-weight of the structural elements, the thrust from the wall above, and the input of weight and stabilising forces from the vaults and roof. Each of the octagonal sections was modelled by dividing each face into two triangles, each subtending a Tetrahedron at the local origin. By ordering the vectors to the corner of the triangles, the computation of volume and centroids became systematic.

At each level through the structure, the forces from above were added to those from the current element. This also involved shifting the axis origin downwards for the input moments. The axis system was x&y horizontal and z vertically upwards. At each plane, the x and y moments were divided by the z force to find the location of the thrust in terms of co-ordinates. It will be immediately clear that our choice of horizontal sections greatly simplified this calculation since the force normal to the plane was always the vertical component.

It is important to note, though, that this is not the end of the process. The horizontal components of force do not act through the centroid of the section. Nor, indeed, do they act through the centre of vertical stress. We therefore had to find a way of representing the effect of all these forces. Because the forces are so small compared with the strength of the material, we chose to visualise the thrust as a curved cylindrical element that had sufficient cross section to resist the axial thrust. Around that we placed a ring of material large enough to transmit the horizontal shear. Outside that again, was a ring of material designed to resist the torsion resulting from the fact that the centre of shear did not correspond with the centre of thrust. At each level, we could therefore envisage a circular patch of material that carried all of the forces to the next layer. If we could show that a load path existed where this patch was within the actual section at every level, we would satisfy the equilibrium and yield conditions of the plastic theorems, and so demonstrate that the structure is safe.

The final stage of designing the model was to devise a scheme for visualisation and interaction. This must allow the engineer to explore the limits of top thrust that could be sustained by the system. Presenting a detailed geometric model of the structure was clearly unlikely to be either easy or effective. We decided to plot only the perimeter of each horizontal section and the line of thrust down through the structure. The model was set up in such a way that the view could be rotated. When viewed in elevation, the sections were simply horizontal lines as the view was tilted, the planes became obvious and the user could observe the flow of force more effectively.

The circular patches of stress described above were presented only for one section at a time. The section was identified on the main view by radial lines from the centre of thrust to the corners. Figure 3 shows the thrust pattern while figure 4 shows the detailed picture of the chosen section. Note the arrow indicating the direction of the shear force on this plane.

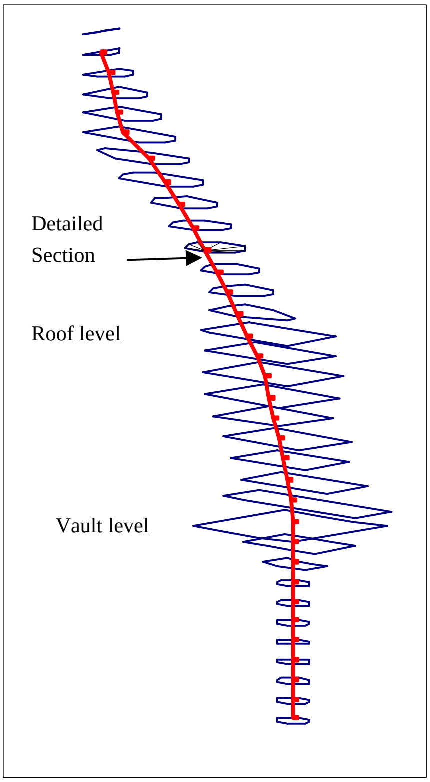

Figure 3 An oblique view of the Wells thrust line

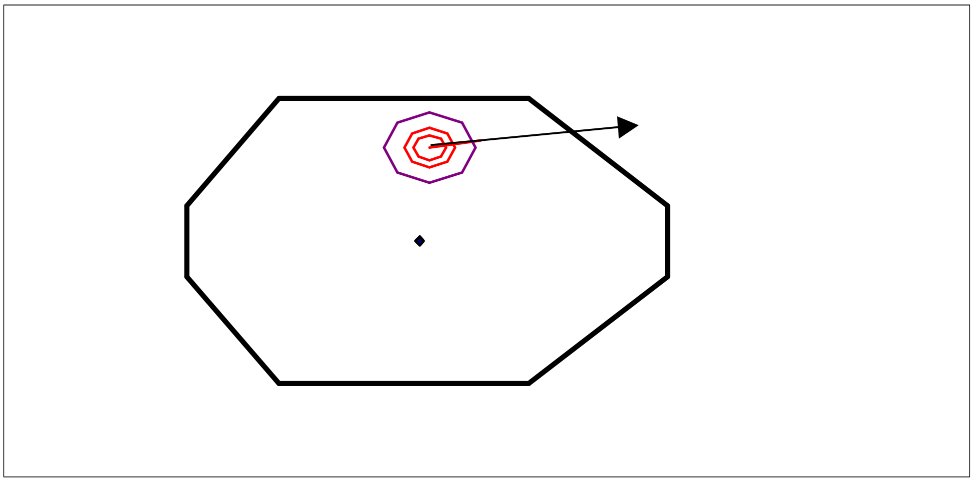

Figure 4 Plan view at chosen section exhibits the wrench action described

4.2 The Dark Arches, Leeds City Station

4.2.1 Introduction

Many of the railway stations in Britain stand above the surrounding land. Various types of structures were used to create warehouse space beneath the station. In Leeds there is a system of vaults with spans of about 8m and piers of 1.8m. The most complex area has vaults and piers skewed at 18 degrees.

The loading regime was changed as a result of remodelling the track layout in the station. The comfort of history was therefore no longer available so a calculation of capacity was required. A spreadsheet model was produced in which a set of four vaults sitting on a single pier could be analysed for various loadings.

4.2.2 Force flow

Before a model could be built, a mental picture of the flow of force must first be generated. The engineers responsible for assessment were familiar with the analysis of viaducts, and were concerned that stability of the piers could not be proved using normal analysis. The authors formed the view that the vault system behaved more like a membrane in plan. The slight hollows at the piers detracting nothing from this membrane behaviour. We were, however interested to explore the extent to which shear in the membrane would be used to stabilise the structure. It was also important to ensure that the abutments at the boundaries of the vault field were sufficient to sustain any horizontal forces carried there.

4.2.3 Geometry

The geometry of the system was complex. For each vault, the local origin was set at the centre of the skew span, but with the axes square. X is in the span direction, Y along the centre of the curve of the vault and Z vertically upwards. The vaults were circular segments, and each was divided into 20 blocks on planes radiating from the Y-axis. Each segment of arch was thus a prism, but with inclined ends.

Each face of the prism was divided into two triangles, each subtending a tetrahedron at the origin. Computing the volume and centroid of the tetrahedron is simple. The block is then assembled from its component parts. Above the vault are masonry infill, soil fill and ballast. Each was similarly divided and reassembled to provide forces and moments at the origin.

4.2.4 Loads

The self-weight loads were dealt with as above. The soil fill exerted horizontal, as well as vertical, force on the vault blocks. Live loads were railway loading through track systems. These were divided into a system of small point loads. Eight such loads represented the force from each sleeper. By using these small divisions, it was possible to allocate load simply to the vault on which it appeared.

4.2.5 Thrust

The sum of all the forces acting on a block delivers a resultant, which is a component of the forces acting on the next block. The model was designed to allow the user to input an estimate of the thrust at one vault springing and so see where the thrust travelled through the structure.

4.2.6 Manipulation

With the more recent versions of Excel, there are many tools for effective manipulation of numbers. The use of scroll bars to control thrust input is a typical example. In this three-dimensional problem, the end thrust might be skewed, and the thrust might not follow a straight line in plan. Two views were presented, plan and elevation, and the effect of any adjustment to the thrust could be observed in any view (figure 5).

Once a suitable trust line has been found for one quarter of the system, it is possible to move on to the next. The computation automatically deals with the reversed skew in the orthogonal direction.

When all four quarters have been dealt with, the resultant thrust on the pier can be assembled and the thrust carried down to the ground.

4.2.7 Output

A simple thrust line is not sufficient for dealing with such a complex structure. We built tools for exploring the stress regime in cracked sections of vault. Both graphical and numerical results could be obtained.

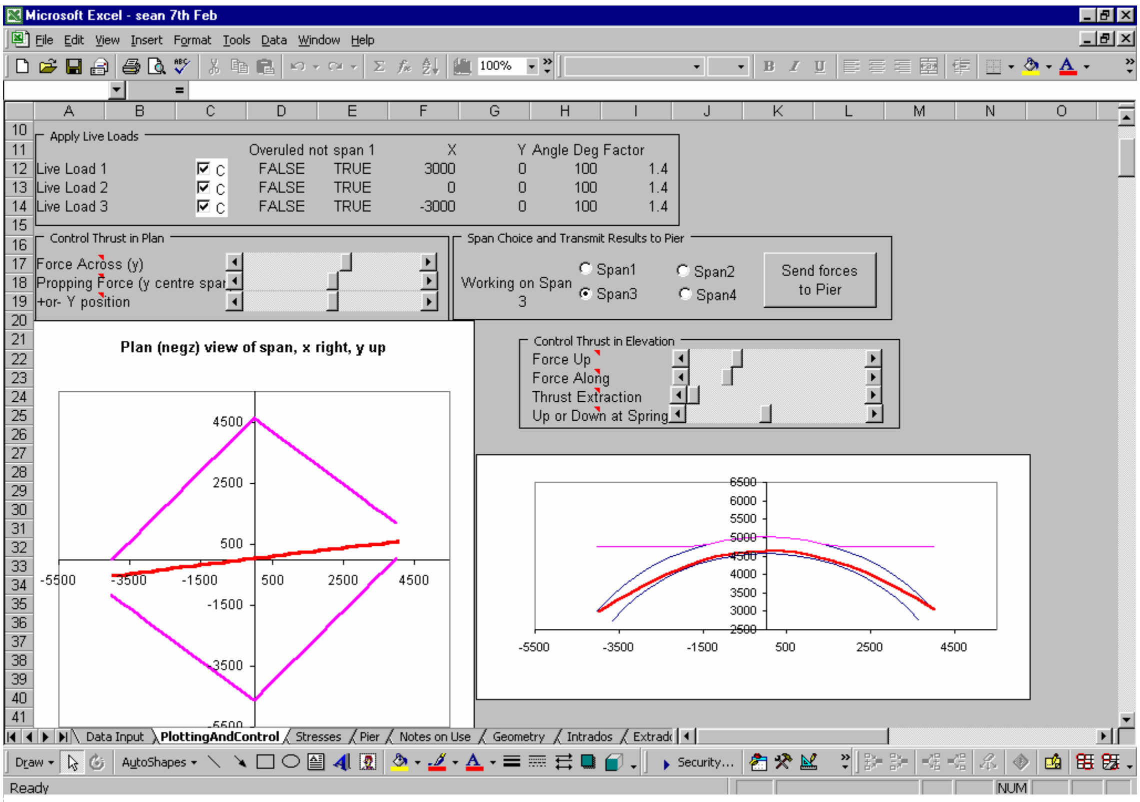

Figure 5 The working screen showing controls and visualisation

5 PROCEDURES

Successful implementation of models of the type described depends on the observation of a number of simple rules. Most important, as in all programming, is the rigid application of sign conventions. Perhaps less obvious, are the techniques for keeping the process clear and secure within the spreadsheet.

Spreadsheets are so simple to work with, that there is a great temptation to begin work before a design is complete. As with writing, it pays to begin with an outline of the spreadsheet, using one page for one concept. The speed of operation of spreadsheets on modern computers is such that there are few constraints on the use of cells, pages and even links between files. We prefer to take one step per cell. **If** statements are dealt with independently, first developing a truth table by considering one condition at a time, then combing conditions.

Addressing can be a complex issue. Careful design can deliver massive benefits. It is important that the minimum of individually typed cells is used. Design everything to be expanded by simple copying. This may mean leaving blanks in blocks of cells to ensure that relative addressing works consistently for many blocks. In this way, once a referencing system is correctly established, it will be repeated without error.

We have found, through bitter experience, that even the author of a program may find it difficult to carry the overall plan and the detail in his head. It pays to make maximum use of comments, both in text boxes on the surface of the sheets and in hidden comment attached to individual cells. A well-commented spreadsheet, like a well-commented program, is a robust tool that can be serviced and upgraded as necessary.

6 CONCLUSIONS

- Modelling arching structures remains difficult and expensive using modern FE tools.
- Simple models built in a spreadsheet can provide adequate results.
- Care must be taken in addressing to ensure consistent application.
- Prolific use of comment will secure the long-term value of a model.
- A model can be built for a specific problem in an acceptable timescale.
- The models described here showed that the structures concerned were satisfactory.