The influence of the hoop's stiffness in the roofs strengths of bellesguard building

Javier López-Rey, profesor y Elisabet Vega, Estudiante
Dpto. de Estructuras en la Arquitectura

Francisco Muñoz, Doctor
Dpto. de Construcciones Arquitectónicas I, ETS de Arquitectura de Barcelona
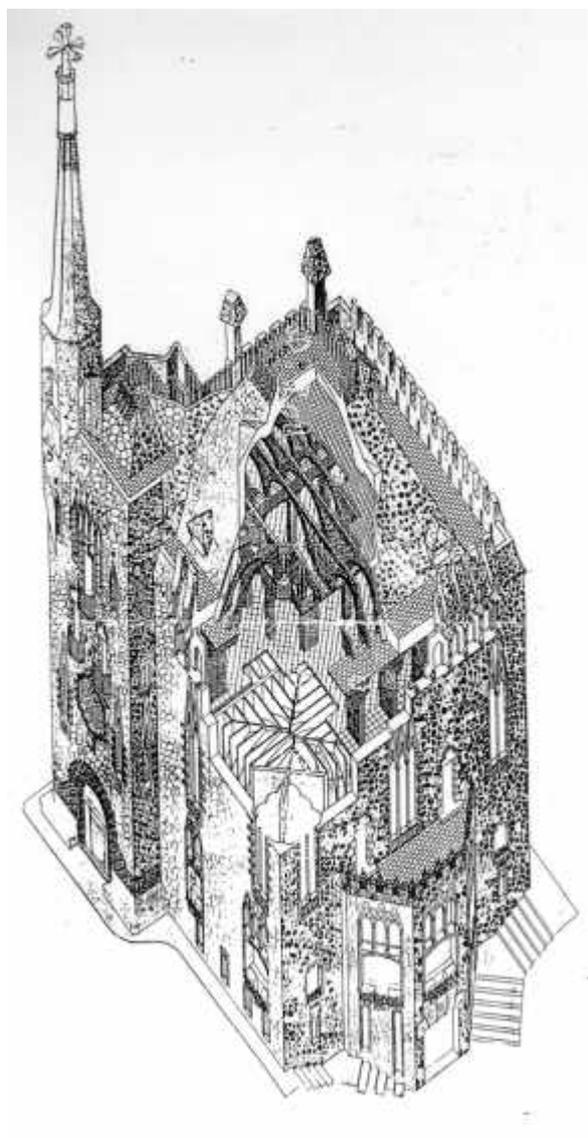Universidad Politécnica de Cataluña, España

Figure 1

1   INTRODUCTION

The signatory team of this project has been working on the structural model exposed in this seminary for over five years. What started as an excuse for introducing students into the use of analysis techniques by the finite element method, has ended up becoming, at the moment, an endless geometrical-structural "*testing field*". Throughout these years, different load transmission problems amongst the complex parts of the structure are being discovered. This report focuses on one of these transmissions. In particular, it refers to a series of load and bracing buttresses, transversely linked to the exterior walls of the building, on their connection to a base slab, apparently very weak. Punching pathologies are not shown, and we will try to explain the reason why. Communication leans on a way of working, which we have called "globally". It consists of approaching the complexity of a building as a whole, not dividing it in smaller and easier parts to study. The presented model has 12300 elements and 73000 degrees of freedom. Although the procedure may be more complicated, it avoids the inaccuracies that appear by the artificial decomposition of the structure in hierarchized portions.

The BELLESGUARD building is, according to the signatories of this document, one of the greatest creative works of Antonio Gaudí. Even though this project appears in all texts and writings about this famous architect, it is not as popular as his other buildings.

2   CONSTRUCTIVE DESCRIPTION OF THE BUILDING

The construction, made of brick and masonry, can be inscribed in a square, its sides measuring fourteen meters. It is a five-storeys building. The ability of the architect helps create a great amenity of different hollows, on each of the four façades. The apparent symmetry that could lead to the same presence for the four corners of the building, is distorted by a variation in one of them (the south-west) that becomes a semi-independent tower, on which stands a high-pitched pinnacle with a three-dimensional cross on its top. (fig.1).

Out of the five storeys, the most interesting ones are the third and the fourth. Above the latter, still another exists. It could be described as a four-sided attic. The gravity loads of this last space (with no columns or partitions inside), is distributed along the perimeter.

This seminar presents the analysis of the fourth storey, which consists of six loading brick planes 10cm thick. Its lower limit is a sequence of three very attractive asymmetry arches. These planes are each separated 90 cm and follow the general east-west direction. The area covered by this storey is approximately a third of the total. The rest is covered with two complementary structural types:
a) a series of planes that follow the same constructive form as the previous six, but which develop in oblique or perpendicular directions. Their structural presence is less powerful than the first explained above, but they give the whole of the scene a stimulating organical disorder. This second system covers another third of the storey. All the planes mentioned up to now will be referred to as *cuadernas.*

Figure 2a

Figure 2b

Figure 2c

Figure 3
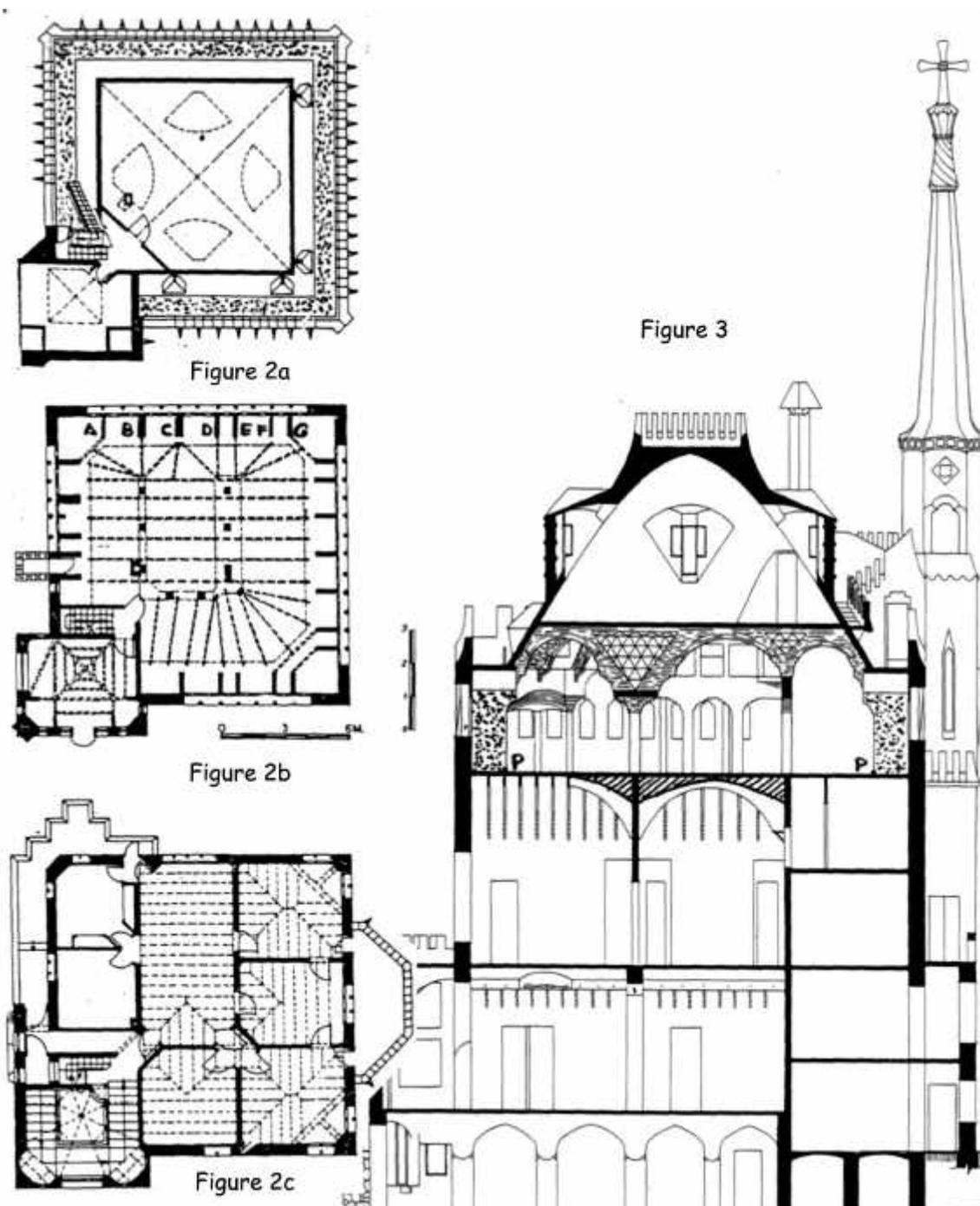
b) Finally, the remaining portion is covered by a series of buttresses linked to the façades. The separation between these elements follows the module of the exterior hollow, which is taken single or double, with great freedom.

As a whole, the ceiling of this storey is one of Antonio Gaudí's finest and most fascinating productions.

Continuing with the description of the rest of the building floors in descendent direction, the following one (the third) consists of a series of loading planes, that have been widely studied. In this case, they are not 10 cm walls, but doubled partitions, separated 40 cm.

The two remaining floors have not been as studied as the rest; they seem to consist of a series of domes. This is not the first building where Gaudí experienced with different

structural types and just one material: ceramics. The masonry is only used in the façade walls. What makes this building specially interesting is the lack of a single structural type.

3   DEFINITION OF THE CASE STUDY

This project presents, as explained at the beginning, an analysis of the atypical organization of a series of buttresses. In particular, we concentrate on their transmission on the slab that separates the third from the fourth floor. This slab seems to be made of two thin layers of flat ceramic bricks which rest on some "*microcuadernas*" thinner and closer than those on the upper floor, already described.

The resulting mesh of planes shows a very acceptable general stiffness. In addition, as figure 3 shows, the lower side of the slab is vaulted at the support, in the perimeter. The area between its two sides, convenietly filled, becomes a spledid continuous capital. The strength transmission problem would not exist if the buttresses rested on this perimeter capital. But one can apprecciate in figure 3 that the two buttresses illustrated (shown as dotted areas) rest either on a void, the right one, or transversely on to the *microcuadernas* and without a capital, the left one. In fact, looking at the buttresses plan (figure 2b) and superimposing it on the *microcuadernas* one, we will deduce that only the F and G buttresses on the north façade rest on the perimetrer capital of the inferior slab.

The case is specially critical on buttress C. Three diagonal *cuadernas* reach it, triplicating the loads on the other buttresses (see table 1). Besides, buttress C does not have a capital for transmitting its loads to the *microcuadernas,* which take a perpendicular direction.

We tend to think that this unconcern towards the way of supporting the buttresses of the fourth storey onto the third floor slab follows an intrinsic reason of the buttress itself (seen from the structural point of view). The reason could be that the compression stresses near the support area are very low. It could also be that these were tension stresses near the P points. This work tries to give an answer to such a question.

The possibility of having tensions in the mentioned points would imply a strong deflecting behaviour in the butresses rather than an axial one. We should not forget that the role of these structural elements is to transmit the vertical and horizontal strengths that the dome situated on top creates.

In the strength transmission chain, from the dome to the basis of the buttresses, the perimetrer hoop plays an important part. It is a square in plan, and is situated between the *cuadernas* and the buttresses. This hoop appears in the façade just like a sentry walk in an imaginary medieval castle wall.

Consequently, we can think that the global stiffness of this hoop may have a direct influence in the arrival of the strengths at the point P. Such a structural element will have some parts in tension. Ceramics, just like all non metallic materials, is not good for supporting such tensions. Thus, the material will crack. Then the resulting hoop does not have the continuos original geometry. Introducing this fact in this study implies entering into the non-linear geometry concept.

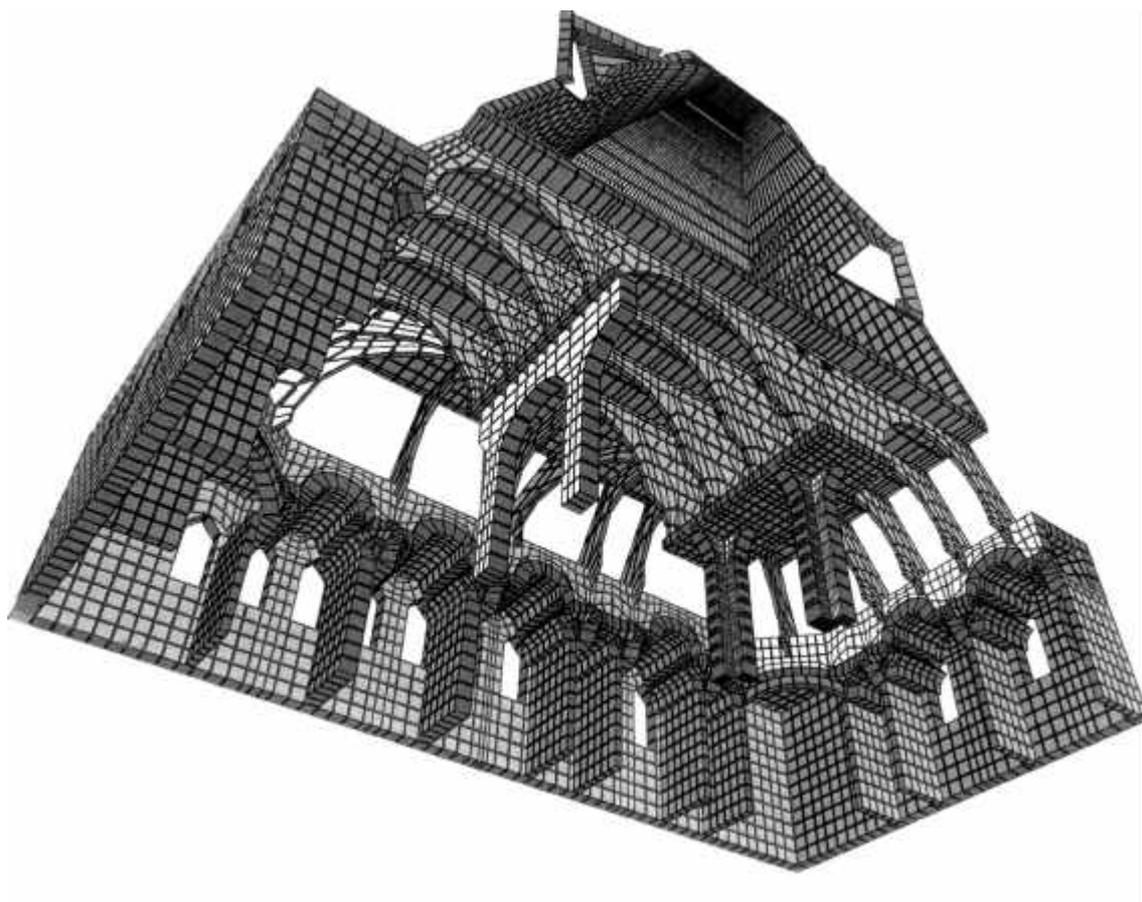At this time, the title of this work becomes justified.

Figure 4

4 THE STRUCTURAL MODEL BUILT

We propose a "global" analysis of the last two floors of the BELLESGUARD building, by a modelization by bidimensional finite elements, of the Shell type.

Their behaviour will be mechanically linear and will have four corners. Consequently, there will be 24 degrees of freedom. The complete model implies half the building, divided in two pseudo symmetric sides by an east-west line.

The half of the building that we are not studying implies the tower and the antiparallel transverse *cuadernas*.

The half we are modeling takes up four of the parallel *cuadernas*, which follow the east-west direction, and the 9 oblique ones situated on the north side.

Some massive parts of the structure will be modelled by flat finite elements, assimilating them to cross sections with their adequate thicknesses and inertia.

Such thing happens in the hoop on the fourth floor, and in the crowning of the dome.

The part of the dome situated under the last slab, which covers the walls at the end of the *cuadernas*, has been eliminated in the model to leave the structural transmission between the *cuadernas* and the hoop much clearer. The complete model has 12331 elements and 73674 degrees of freedom in the continuous geometry version.

The cracked version has exactly the same number of elements, but 73851 degrees of freedom in the nodes. (Fig. 4).

The way of introducing the cracks in the hoop is very simple. Once detected the areas with the highest tensions (and their directions), we cut them perpendicularly with a *"computer scalpel"*. The cut is made in the following way: the nodes of the elements that will now be separated as a result of the crack, will be numbered and counted twice. The increase in the number of nodes implies the same increase for the degrees of freedom in the second version. In this first project only 16 cracks have been modelled in the hoop, but the process can be of a growing complexity. Taking a correct criterion in the orientation of the element's local axes, in reference to the global axes, will ease the understanding of the results.

Since the universal criterion establishes local axis 3 as perpendicular to the plane of the element, it only remains to decide the direction of the other two.

Taken as obvious that when planes have different orientation in space, their local triedrums cannot all have the same orientation, the following criteria have been adopted for the three planes on which we base this study:

a) In the horizontal plane of the sentry walk, and in the vertical one that follows the interior perimeter with the previous plane and with which it shares a horizontal edge: axis 1 is parallel to the edge and axis 2, perpendicular and inscribed on the plane of the element.

b) In the vertical planes of the buttresses, local axis 1 is horizontal and axis 2 is vertical.

The computer program used is SAP-2000, one out of the many commercial offers. It has an easy pre-process, it can exchange data with AUTOCAD and its post-process offers good visualization of the results.

5 ANALYSIS STRATEGY

Since this investigation project aims to determine the influence of cracks in the hoop, on a non-linear state, the analysis will be carried out in two different versions:
a) a study of the geometry when continuous
b) a study of the geometry when cracked. This is done by introducing controlled cracks where tensions are maximum. The process is purely geometrical. The originality of this project stands on such an immediate technique. At the moment, the number of cracks introduced is 16. Eight of them have been applied on the sentry's walk horizontal plane which covers the hoop, the remainig eight being applied on the vertical plane that shares an edge with the other surface. (Fig. 5).

For the comparison of models A and B, we will use the following parameter: the addition of the reactions in the four interior nodes, on the base of each of the buttresses. We think that this criterion, in such a case, is better than the analysis of the strengths in the same base, or than the movements of the upper nodes in the same buttresses.

Such a comparison tries to find out if the appearance of cracks increases the reactions in the P points.
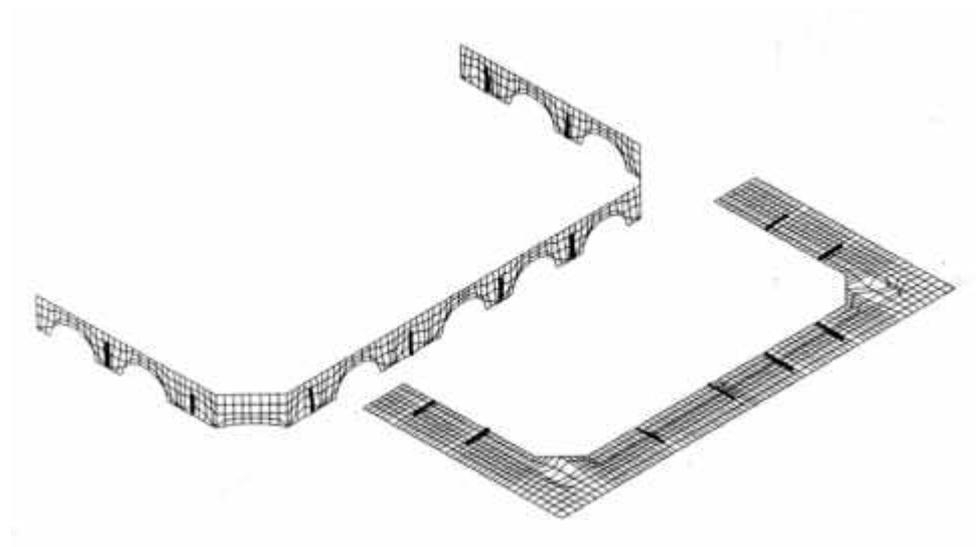
Figure 5

6 STUDY OF THE RESULTS

The following table shows the reactions in the four nodes in each of the seven buttresses on the north side of the model. They are called A, B, C... G. For each node, the following data is given:

Table 1

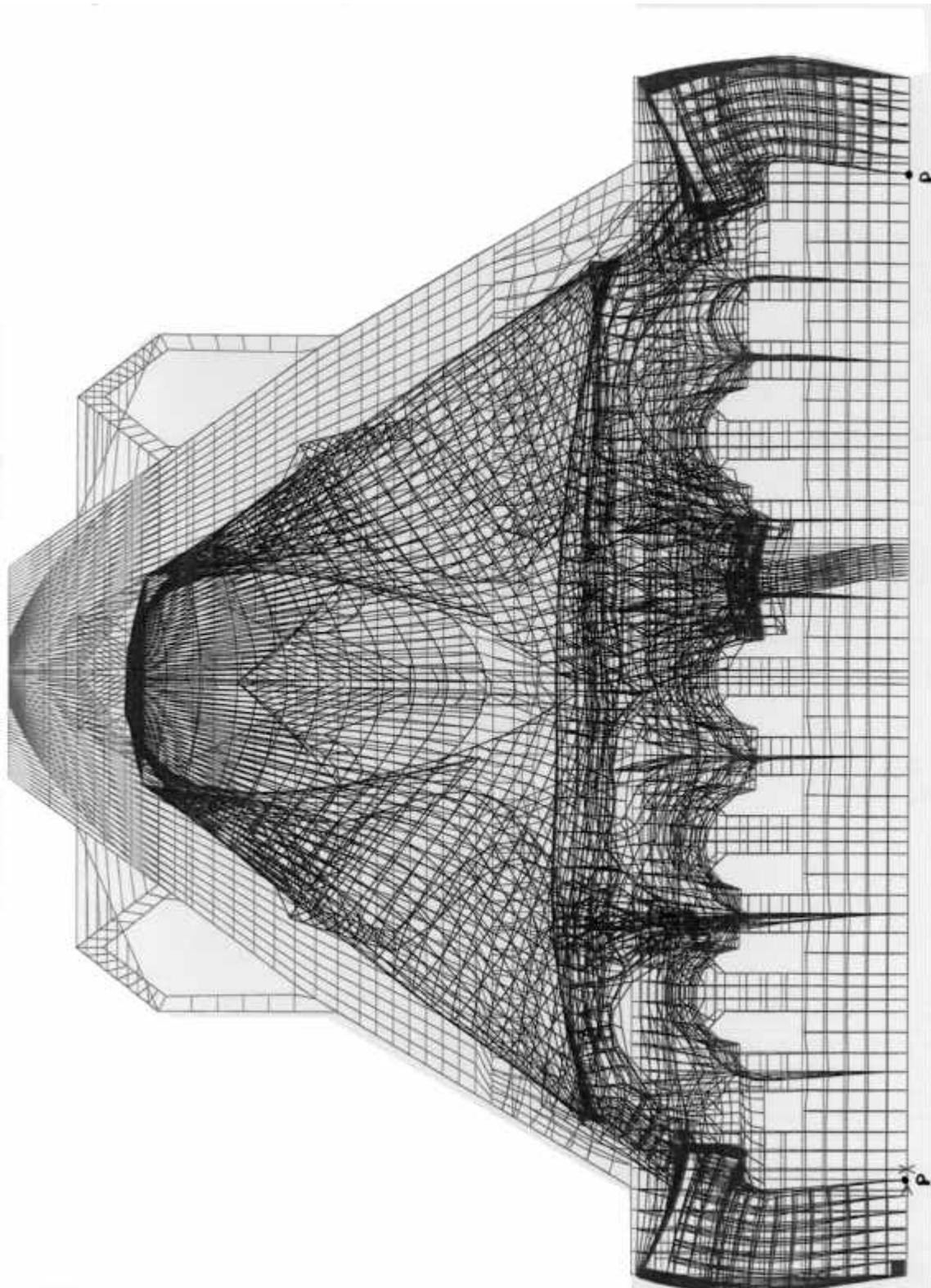| Buttness appelation's	Nodes numeration's Reaction value in the continous version Reaction value in the cracked version				
	2351	2350	2349	2351	Adition
A	0,042	0,244	0,359	0,403	**1,408 T**
	0,041	0,243	0,359	0,402	**1,045 T**
	2417	2416	2415	2414	Adition
B	0,306	0,766	0,908	0,949	**2,929 T**
	0,308	0,768	0,908	0,949	**2,933 T**
	2423	2422	2421	2420	Adition
C	0,327	0,895	1,094	1,154	**3,470 T**
	0,331	0,900	1,097	1,156	**3,484 T**
	2527	2526	2523	2522	Adition
D	0,181	0,485	0,579	0,595	**1,840 T**
	0,183	0,488	0,581	0,597	**1,849 T**
	2529	2528	2525	2524	Adition
E	0,160	0,451	0,550	0,571	**1,732 T**
	0,162	0,454	0,552	0,573	**1,741 T**
	2797	2796	2793	2792	Adition
F	0,109	0,372	0,484	0,517	**1,482 T**
	0,111	0,374	0,485	0,517	**1,487 T**
	2799	2798	2795	2794	Adition
G	0,015	0,222	0,3555	0,407	**0,999 T**
	0,017	0,224	0,356	0,407	**1,004 T**

Figure 6

We can note that, except for buttress A, the value of the reaction is always higher in the cracked version, than in the continuous. The greater reaction in buttress C appears because three converging *cuadernas* rest near it. It is obvious that the differences can only be detected on the last two decimals, but the tendency seems clear.

The reason for the last comment is the fact that only a few cracks have been introduced. We verify that they have been scarce in number and length, and their influence in annuling the tensions is basically local.

From the deformational point of view, the following fact becomes obvious: the centripetal effect of the dome's weight vertical component is higher than the centrifugal effect of the horizontal component. This is close to the fact that the dome's design is very vertical. The small slab situated on the *cuadernas* may also contribute, introducing a hooping effect. (Fig. 6).

However, both effects are in contrast. This had already been verified in previous conclusions for this analysis. The extremely low compression stresses that appear in the inner sides of the buttresses give prove of it. This fact would explain the lack of pathologies in the contact with the thin slab, which is supported by a dense net of structural partitions or *microcuadernas*.

7   CONCLUSIONS

The dome's weight acting upon the buttresses through the perimetral hoop is the result of two contrary effects: the centripetal due to the vertical component, and the centrifugal due to the horizontal one. The first one is the winner. Therefore, compression stresses appear on the inner part of the buttresses' base, though low in value. We reached this same conclusion on the IV CONGRESO MUNDIAL DE MÉTODOS NUMÉRICOS which took place in Buenos Aires in June 1998. In this case, we corroborate this conclusion with a more refined and complete model.

The artificial cracking introduced by using a "*computer scalpel*", although a modest resource, detects quite clearly the appearance of a tension concentration in the buttresses area of conflict.

This means that the non-linear cracked model is more dangerous than the elastic one. The supposed lack of reinforcing bars inside the hoop makes us think that the real state of construction is very close to the craked model. The fact that possible cracks are not detectable (because of their micro-metrical size) does not imply that they do not exist.
We have not studied, however, the danger in the increase of compression stresses in the inner parts of the buttresses (P points), when we introduce more cracks in the model's hoop.

Introducing the cracks the way we have previously explained has turned out to being very effective for analysing the tension problems in stony materials. However, this method should be used together with an increasing process; the latter would be adequate for modelling the compressing behaviour in plasticity, but still using the commercial programs that do not offer such an option. In a new study of this case, this problem will probably be solved.

The global analysis of complex geometrical models of historic buildings by finite elements, is visually and conceptually complicated. However, it is more often applied than other analysis methods, more artificial and schematic, AS FOR EXAMPLE THE STUDIES BASED ON VECTORIAL MECHANICS. This method allows simultaneous complex structural interactions, impossible to take into account with other methods.

REFERENCES

Oc Zienkiewicz El de Los Elementos Finitos, 4ª Edición
Eugenio Oñate Cálculo de Estructuras por el método de Elementos finitos
Y.C. Fung , E.E. Secheler Thin Shell Structures, Theory, Experiment and Design
J. López-Rey Apuntes de Elementos Finitos