Virtual-3D analysis of Clifton suspension bridge

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ABSTRACT: The paper describes the fundamentals of a virtual-3D analysis of the historic Clifton Suspension Bridge to understand its structural behaviour under symmetric and eccentric traffic and footpath loadings and to evolve a repair strategy for its damaged parapet girders.

1 INTRODUCTION

One of Britain's foremost engineers, Isambard Kingdom Brunel (son of Marc Brunel) had won the competition for the design of Clifton Suspension Bridge for the picturesque Avon gorge in 1829. Brunel's efforts towards its construction during the 1840's were not entirely successful due to financial constraints and only the towers could be completed at that time. However, after his death in 1859, the engineering fraternity commissioned its completion to commemorate his aesthetic and technical genius and his contribution to the advancement of engineering on a wide front. Two-thirds of the links forming the chains of this Bridge are those reclaimed from Hungerford Bridge over river Thames in London, also of Brunel's design which was taken down due to railways' increased loading requirements. With some modifications to the original Brunel's design (Barlow 1867) the bridge was finally completed in 1864.

2 OBSERVED DAMAGE TO THE BRIDGE

The bridge had remained open for all local traffic until about 25 years ago, when concerns arising from significant movements in the riveted joints of the parapet lattice girders had led to the imposition of 4 Tonnes vehicle weights restriction on the bridge. During inspections of the bridge under live loads passing on the deck, it was noticeable that the stiffness of the cross-girders tended to cause significant bending of the parapet girders of the bridge that was essentially the cause of damage to the riveted joints of the parapet girders.

3 PAST LOADING HISTORY AND RECENT LOADING TRENDS

There are no records of any global analyses of the effects of loadings on the bridge having ever been carried out in the past. Judging from the levels of factors of safety used against material strengths and the types of loading prevalent in the nineteenth century, the bridge would have been quite competent for those earlier times. There are however indications that the bridge had been used for transporting unspecified and possibly heavy military hardware during the second world war period. Also during the fifties, Pugsley and Flint (1955) are known to have carried

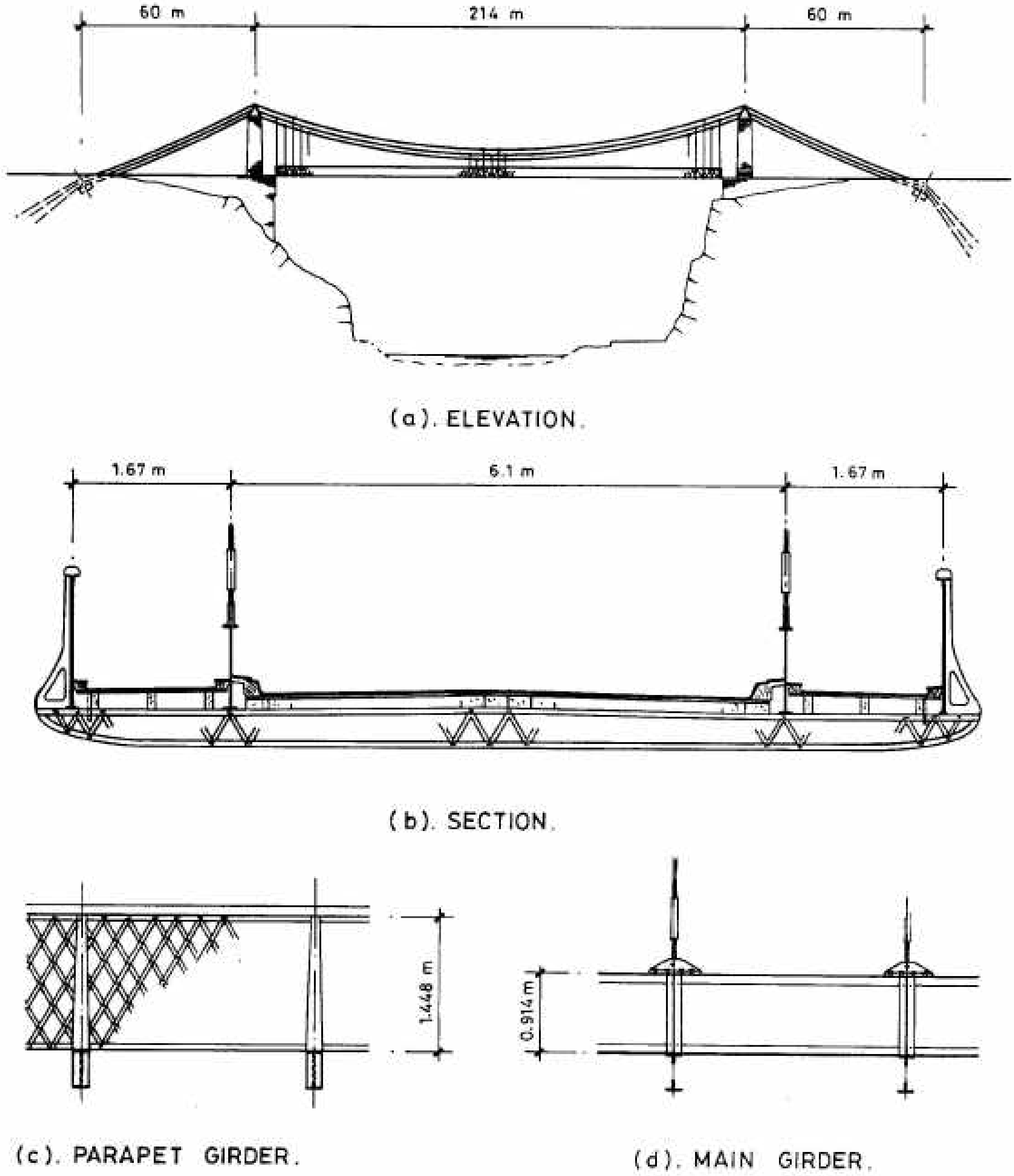

Figure 1 : Clifton suspension bridge.

out tests on the bridge to determine its influence lines for certain load effects by incrementally moving 8 and 16 Tonnes loading trains along the centreline of the bridge.

Due to the dramatic increase of traffic during the recent decades and the random tendency of vehicles to 'bunch' behind slow moving vehicles such as bicycles, the effects of random convoys of the specified 4 Tonnes vehicles in the lanes of the bridge were also needed to be determined, to justify such restriction.

In view of the movements in the riveted joints of the parapet lattice girders, global analyses of the bridge for a variety of symmetric and eccentric loadings on the deck were undertaken to understand the global behaviour of the bridge, to determine the force distributions in the various parts of the bridge including the parapet girders and to arrive at a rational repair strategy for the failed joints in the parapet girders of the bridge. The paper is based on the work described in detail in Reference 3.

4 BRIDGE CONFIGURATION

As shown in Figure 1, the bridge consists of 2 planes of 3 chains each with 7.2m links with hangers at 2.4m spacing from the 3 chains, supporting the two 0.9m deep plate (main) girders in sequential order. Shallow but fairly stiff Warren truss type of cross girders is attached to the bottom flanges of the main girders below the hanger positions. These cross girders support the longitudinal timber decking with asphalt surfacing between the main girders for vehicular traffic. Outside the main girders, wide footpaths are supported on the overhanging parts of the cross girders with 1.45m deep lattice (parapet) girders acting as the handrails. The entire deck is freely suspended from the chains between the abutment towers with only the lateral restraints at the ends of the deck. Two short hinged spans connect the suspended bridge deck to the roadways at the abutments. Saddles mounted on large slider bearings located at the top of the towers are connected to the side chains anchored deep down in the underlying rocks well behind the towers.

5 CONCEPTUAL BASIS FOR REPAIR

The parapet girders, judging from the age and the light form of construction, would almost certainly have been designed for the pedestrian hand loadings and for decorative purposes only, without any proper understanding about their participation in the global behaviour of the bridge. Since these are deep (lattice) girders, if the analyses for live loads on the bridge indicated their contribution towards stiffening the deck (i.e. sharing the load effects with the main girders) to be significant, then clearly the joints must be repaired for the ensuing forces so as not to overload the main girders of the bridge. Otherwise, the movement in the riveted joints in the parapet girders could be acceptable and be largely inconsequential.

6 BASIS OF COMPUTER MODELLING AND ANALYSES

Suspension bridges, although stiffened by the suspended deck, due to the deformability of their profiles of the cables (or chains) should ideally be analysed as geometrically non-linear structures, not least to ascertain the degree of non-linearity likely to be occurring in the applied loading range. Due to the potential non-linearity of behaviour combined with the extremely large number of elements forming this bridge, it was judged that the full three dimensional non-linear analyses of the bridge would be very time consuming, cumbersome and expensive even on main-frame computers of the late eighties.

It was therefore decided to carryout geometrically non-linear analyses in the plane of one of the suspension chains of the bridge only, with an 'equivalent' in-plane model of the deck suspended from it. The interaction of the main, parapet and cross girders and any eccentricity of traffic loadings about the centreline of the bridge should therefore have to be reflected in the derivation of the 'equivalent' in-plane model of the deck.

The linear elastic deck structure consists of the main and parapet girders with cross-girders effectively hinged to the bottoms of these girders. Live loadings symmetric to the centreline of the bridge would cause symmetric deflections but no net rotations of the cross-sections of the deck. Eccentric loadings would cause asymmetric deflections that can be resolved into symmetric deflections plus net rotations (i.e. tilting) of the sections of the deck. Since the bridge has two traffic lanes and two footpath lanes which can each be loaded independently of the others, loadings on the bridge are, in general, eccentric. Symmetrically loaded lanes would represent heavier loads on the bridge, however, since the parapet girders are located well outside the main girders, the 'tilting' of the cross-sections of the bridge under single traffic and/or footpath lane loadings could also possibly result in critical load effects in the parapet girders of the bridge.

It became clear that the effective stiffness of the parapet girder and its modes of interaction with the main girders would be different for the symmetric and eccentric live loads on the deck and therefore separate 'equivalent' models would be required for the global virtual-3D analyses of the bridge for such load cases. In order to facilitate the transformation of the actual 3D structure into its 'equivalent' in-plane representations in the plane of one of the main girders and its suspension chains, it was necessary to determine the stiffness characteristics of the cross girders for the symmetric, anti-symmetric and eccentric live loadings on the deck.

6.1 *Deck in-plane 'equivalent' model for symmetric loadings*

The cross girder model of Figure 2(a) was analysed on a micro-computer to determine its deflection stiffness for unit loads applied at the parapet girder positions. The stiffening effect of the timber deck above the cross girders was incorporated by representing the decking by a 'virendeel' frame reflecting the lateral shear properties of the timber decking elements. Some old data relating to the measurement of strains in the top and bottom chords of the cross girders under passing live loads had indicated the stiffening effect of the deck to be significant and the degree of interaction with the timbers used in this model was correlated against this data.

From the analysis of the cross girder, it was possible to calculate a cross-sectional area 'As' of a 'bar' element of a selected 2m length to have the same deformability i.e. axial stiffness as the cross-girders. The equivalent in-plane model of the deck would therefore be the upper beam (main girder inertia 'Im') and a suspended lower beam (parapet girder inertia 'Ip') interconnected by bar elements of the calculated cross-sectional area As in the plane of one of the main girders. The structural equivalence was tested by comparative separate runs of the deck structure verses this in-plane model on a micro computer for a variety of loadings, confirming the following parametric equivalences:

	Actual deck structure	In-plane model
i)	Bending moment in main girder	= Bending moment in upper beam
ii)	Shear force in main girder	= Shear force in upper beam
iii)	Deflection of main girder	= Deflection of upper beam
iv)	Bending moment in parapet girder	= Bending moment in lower beam
v)	Shear force in parapet girder	= Shear force in lower beam
vi)	Deflection of parapet girder	= Deflection of lower beam
vii)	Shear force in the cross girder	= 'Bar' force

6.2 *Deck in-plane 'equivalent' model for anti-symmetric loadings*

All eccentric loadings on the bridge can ultimately be resolved into their symmetric and anti-symmetric components, although the latter can not occur by itself for practical deck live loadings. This anti-symmetric component generally causing rotational deformations of the deck about its centreline, was considered next.

The cross girder model of Figure 2(a) was analysed on a micro-computer to determine its deflection stiffness for equal and opposite unit loads applied at the parapet girder positions. From this it was possible to calculate the cross-sectional area 'Aa' of a bar element of a selected 1m length, to have the same axial stiffness as the cross-girders.

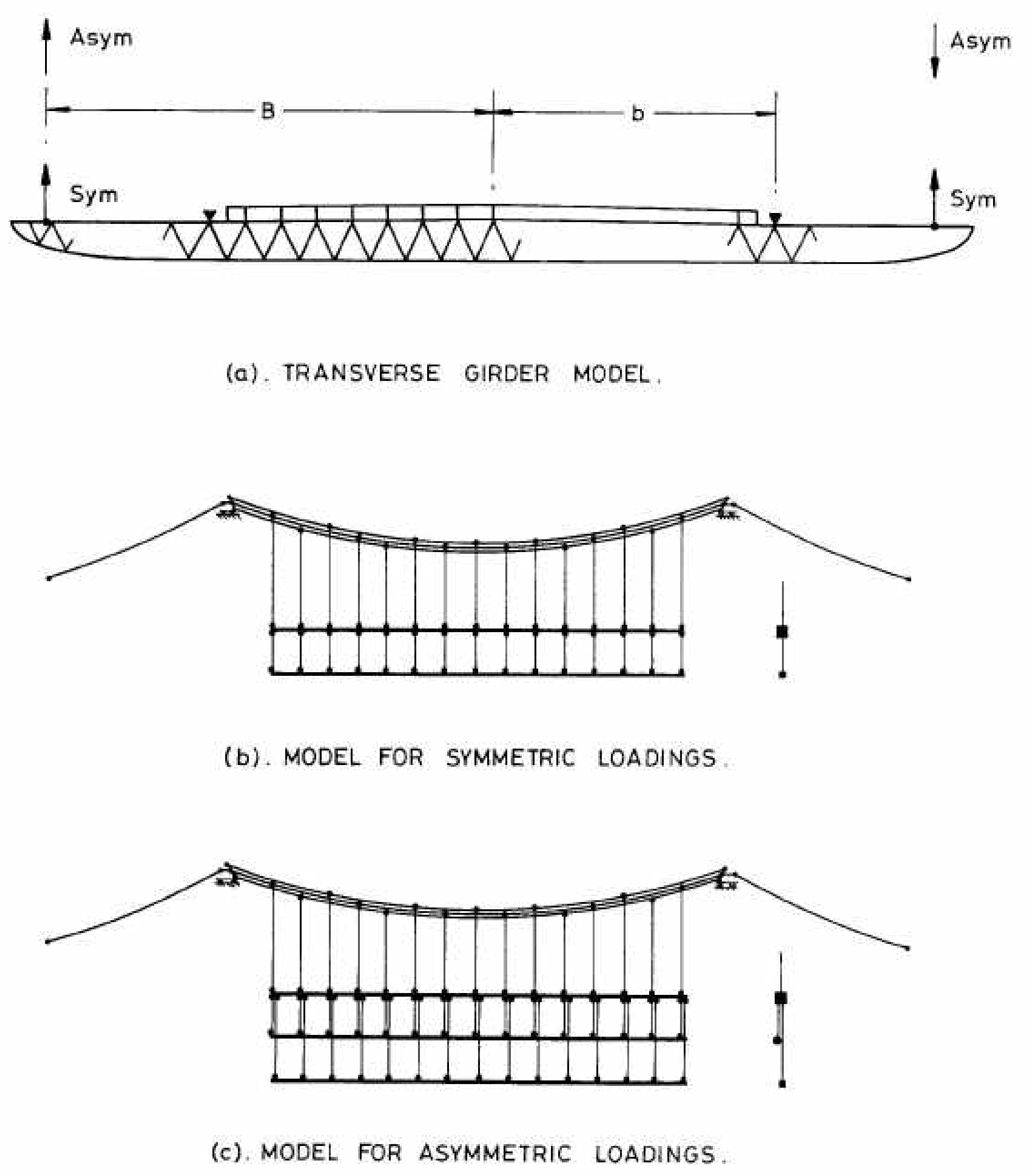

Figure 2 : Analytical models.

Since the parapet girders are located at a larger distance from the centreline of the bridge, their effectiveness towards resisting rotations of the deck is magnified in the plane of the main girders which is the plane of the analytical model. Conceptual reasoning described in Howard Humphreys & Partners (1989) report, points to similarly increased effectiveness of the cross girders in the plane of the main girder. If 'b' and 'B' are the distances to the main and parapet girders from the centreline of the bridge respectively then the parapet girder and the cross girder stiffness magnification factor in the plane of the main girders would both be $(B/b)^2$.

The equivalent in-plane model of the deck would therefore be the upper beam (of inertia Im) and a suspended lower beam (of inertia $Ip(B/b)^2$) interconnected by bar elements of the calculated cross-sectional area $Aa(B/b)^2$. The structural equivalence was tested by comparative separate runs of the deck structure verses this in-plane model for a variety of anti-symmetric loadings which confirmed the following parametric equivalences:

	Actual deck structure	In-plane model
i)	Bending moment in main girder	= Bending moment in upper beam
ii)	Shear force in main girder	= Shear force in upper beam
iii)	Deflection of main girder	= Deflection of upper beam
iv)	Bending moment in parapet girder	= Bending moment in lower beam x (b/B)
v)	Shear force in parapet girder	= Shear force in lower beam x (b/B)
vi)	Deflection of parapet girder	= Deflection of lower beam x (B/b)
vii)	Shear force in the cross girder	= 'Bar' force x (b/B)

6.3 *Deck in-plane 'equivalent' model for eccentric loadings*

This is the general case combining both the symmetric and anti-symmetric components of eccentric loadings on the bridge. Because of the differing modification factors operating for the two modes of behaviour (in the derivation of the in-plane equivalent deck models and subsequently to the corresponding results to derive the final load effects), it was finally concluded that parapet and cross girders would have to be represented by retaining both the symmetric and anti-symmetric modes of their behaviour in the equivalent in-plane model of the deck throughout the eccentric load cases analyses. As derived in Howard Humphreys & Partners (1989) report, from the eccentricity of the loading on the bridge such as due to single lane or footpath loadings, the proportions of symmetric mode (Fs) and anti-symmetric mode (Fa = 1 - Fs) operating for such loadings can be analytically derived.

The in-plane model of the deck would therefore be the upper beam (main girder inertia Im) and two suspended parapet beams: one lower beam allocated the symmetric mode stiffness Fs x Ip with its connecting bar elements of sectional area Fs x As and the other middle beam allocated the anti-symmetric mode stiffness Fa x $(B/b)^2$ x Ip with its connecting bar elements of sectional area Fa x $(B/b)^2$ x Aa. The structural equivalence was tested by comparative separate runs of the deck structure verses this in-plane model for a variety of eccentric loadings which confirmed the following parametric equivalences:

	Actual deck structure	In-plane model
i)	Bending moment in main girder	= Bending moment in upper beam
ii)	Shear force in main girder	= Shear force in upper beam
iii)	Deflection of main girder	= Deflection of upper beam
iv)	Bending moment in parapet girder	= Bending moment in lower beam + bending moment in middle beam x (b/B)
v)	Shear force in parapet girder	= Shear force in lower beam + shear force in middle beam x (b/B)
vi)	Deflection of parapet girder	= Fs x Deflection of lower beam + Fa x deflection of middle beam x (B/b)
vii)	Shear force in the cross girder	= Lower 'bar' force + Middle 'bar' force x (b/B)

By transforming the deck into its in-plane equivalents for the symmetric and eccentric load cases, the three dimensional suspension bridge is effectively transformed into its two dimensional analytical models for its virtual-3D analyses purposes.

7 ESTABLISHMENT OF THE DEAD LOAD GEOMETRY OF THE CHAINS

The old survey data about the geometry of the chains profile was very approximate and unreliable. This could have resulted in large locked-in forces in the girders due to any sharp 'kinks' being present in the profiles of chains suggested by this data. There are however known to have been attempts at smoothening the profile of the chains. Therefore, initially a computer model of the chains only (i.e. without any suspended deck) was repeatedly analysed using a geometrically non-linear finite element analysis program in which the dead loads of the chains and the deck with a 22.2^{o}C drop in chain temperature were applied in small increments. This drop in temperature was to ensure that there was no net lengthening of the chains during this loading process. After 4 or 5 iterations, each using the latest calculated chains profile, the process converged during which some of the nodes had moved by upto 500mm from their initial positions, resulting in a satisfactory (i.e. smoothened) profile of the chains. From this chains profile, the in-plane 'equivalents' of the deck previously derived were suspended, forming the complete models for the symmetric and eccentric live loads analyses of the bridge as shown in Figures 2 (b and c).

8 GLOBAL ANALYSES FOR SYMMETRIC AND ECCENTRIC LIVE LOADS

The analyses of such global bridge models took some 2.5 hours and 3 hours of CPU time for the symmetric and eccentric load cases respectively even on a main-frame computer of the time. As is necessary for non-linear structures, the entire dead, live and temperature drop loads were applied at the appropriate nodes of the deck in-plane 'equivalents' and of the chains in small increments. The computer analyses resulted in the final forces distribution in all parts of the model for the particular load cases. Using the equivalences established earlier it was possible to determine the forces in any part of the bridge including the parapet girders which were of special interest in the context of the damage which had occurred to these members.

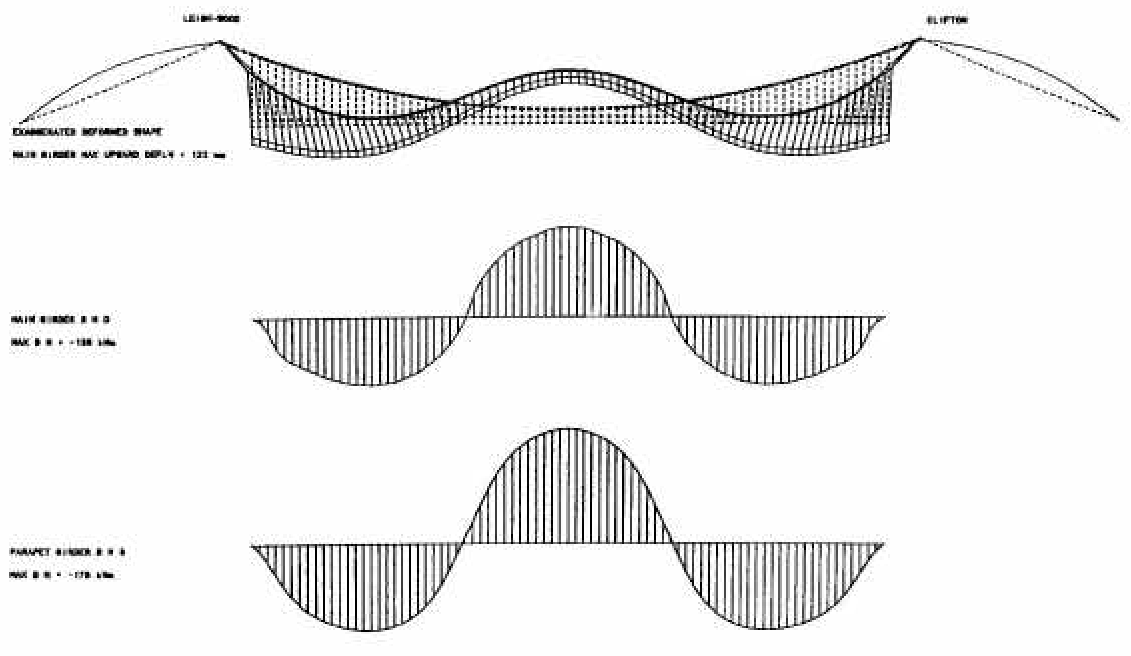

Figure 3 : Both lanes at the ends of the span loaded with UDL.

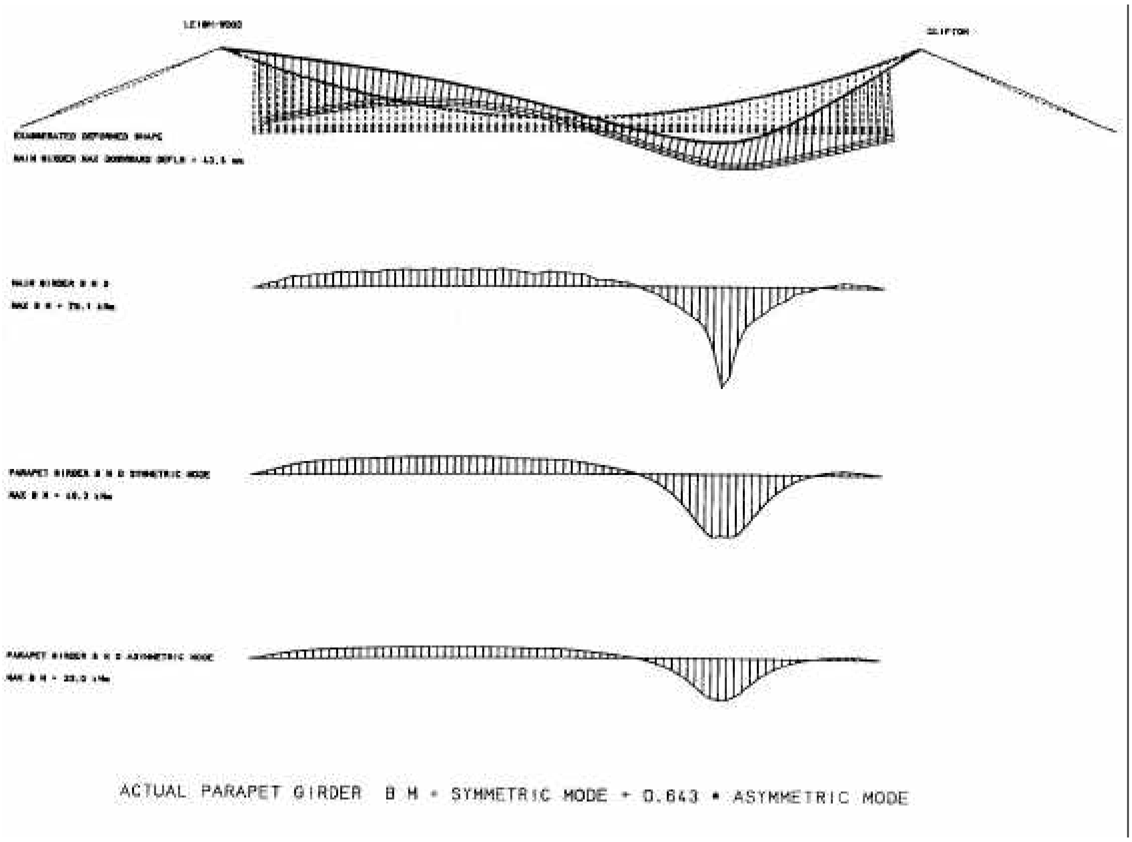

Figure 4: A4 Tonnes vehicle in one lane at quarter span.

For the eccentric load cases, in theory the two chains being differentially loaded could assume slightly different profiles causing a degree of non-linearity which can not be included in any in-plane analyses. Several analyses however indicated that the analytical results were remarkably linear with the applied loads in the range of the serviceability loadings likely to occur on the bridge. This is essentially due to the elastic stiffening (i.e. restraining) effect of the deck on the deformability of the chains profile, the only restrained lengths of chains being between the towers and the first suspension hangers. Numerous live load cases in various configurations on the deck were analysed using this approach. In general very close agreement with the influence lines of Reference 2 was obtained. Typical analyses results printed out by the computer's graph-plotter are indicated in Figures 3 and 4.

9 RESULTS

The worst bending moments resisted by the parapet girders were of similar magnitude as the main girders. Also some of the eccentric load cases such as a traffic and a footpath lanes loaded produced some of the largest bending moments in the parapet girders. This confirmed the need for ensuring the continued participation of the parapet girder stiffnesses in resisting the live loads using the bridge, so as not to overload the main girders of the bridge. The failed riveted joints in the parapet girders were replaced by their modern friction grip equivalents. Subsequent measurement of the forces in the parapet girders under passing live loads on the deck further confirmed the validity of the analytical work.

10 CONCLUSIONS

It is hoped that the concepts of transforming a horizontal deck into its vertical in-plane equivalents incorporated in the computer analyses of Clifton Suspension Bridge, would lead to a better understanding of the behaviour of such structures and could possibly have application in the derivation of transforms for other structures.

REFERENCES

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