Critical thickness criteria on stone arch bridges with low rise/span ratio and current traffic loads

J. A. Martinez Martinez, J. Moreno Revilla, A. Aragon Torre
Department of Civil Engineering. University of Burgos, Avda. de Cantabria, s.n. 09006 Burgos (Spain)
e-mail : ingciv@ubu.es

ABSTRACT: In this paper we try to find thickness criteria for stone arch bridges with low rise/span ratio subjected to the traffic loads envisaged in current Spanish road-bridge regulations. An analysis model is developed and criteria put forward based on geometric variables, such as thickness, and on mechanical variables, such as acceptable tension.

1 INTRODUCTION

The bridges in our road system have to carry increasingly intense traffic. The loads they must bear have also increased markedly in the last few centuries, despite various regulations which have tried to keep these loads within more or less stable limits. In spite of this, the sturdiness of old bridges built centuries ago is proven by the fact that many are still in use today. Their creators can scarcely have imagined the kind of loads they would have to support. One could at first think that these bridges, built with simple, natural materials such as rocks, and constructed basically by placing small rocks on top of each other, constitute a symbolic representation of the inventory of a country's bridges. In Spain this has been shown to be far from true. The Ministry of Public Works has published data showing that 20% of the bridges under its administration are arch bridges. Of these, some 80% are either of stone or concrete, with a similar number of both. The data also reveals that these bridges are not to be found only on secondary roads. A large number are to be found on important communication routes, with a significant percentage supporting traffic intensities of over 7,500 vehicles per day, while some even reach the figure of 20,000 vehicles/day.

2 AIM OF THE WORK

The aim of the work is to look at the general state of stone arch bridges in the light of current traffic loads. This will provide us with a document in the form of a set of easy-to-use abacuses which give us an idea of the safety conditions of each bridge, initially evaluated in terms of the geometry of the arch and of the resistance of the stone.

Moreover, in view of the variety of types of stone arch bridges in existence, regarding the curve or curves which define the geometric layout of the intrados, in this work we limit our attention exclusively to bridges whose intrados is comprised of a single circular curve, with rise/span ratios below 0.50.

3 LOADS ON ROAD BRIDGES

The regulation currently in force in Spain is the “Instrucción sobre las acciones a considerar en el proyecto de puentes de carretera (IAP)” (1998). This regulation establishes, with regard to different loads, that bridges will be calculated for the following vertical loads:

- a uniform load of 4,0 kN/m2 distributed over the whole surface or a part of it, whichever is the most unfavourable.
- a 600 kN vehicle divided into six loads of 100 kN as in the figure, where the loads are 1.5m apart lengthways and 2m apart transversally. Each load is placed on a rectangular surface 0.2m in length and 0.6m in width.

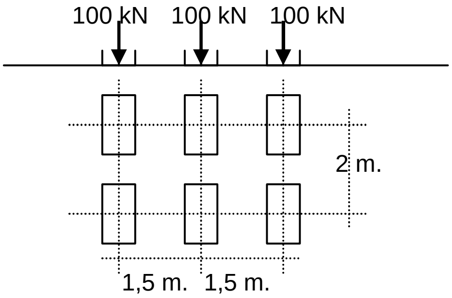

Figure 1: Vehicle regulation loads

4 BEHAVIOUR OF MATERIALS

Most stone bridges are made of either sandstone or limestone. Limestone is carbonated rock, sedimentary in origin, and with variable resistance to compression, normally oscillating between 20 and 80 MPa. Sandstone is sedimentary debris rock and its resistance to compression can also vary greatly, often from 25 to 100 MPa. Below we give values for the characteristics of the rocks in the geographical area we have worked in.

Table 1 : Limestone and sandstone

Limestone	Resistance to compression (MPa)	Density (gr/cm3)	Sandstone	Resistance to compression (MPa)	Density (gr/cm3)
Stone from Hontoria	22,57	2,14	Sandstone from Sotoscueva	25,63	2,02
Stone from de Bernuy	24,18	2,18	Sandstone from Salas	35,30	2,09
Pink Sepúlveda Rock	38,05	2,26	Sandstone from Aguilar	36,93	2,17
Limestone from Agreda	54,95	2,24	Partridge eye Sandstone	37,08	2,20
Stone from Campaspero	74,17	2,43	Maragata Stone	73,22	2,57
Stone from Boñar	82,41	2,43	Red stone from Brañosera	103,65	2,50
Stone from Caleruega	90,54	2,42	Yellow stone from Brañosera	104,07	2,59

The σ-ε ratio of the stone as a compound material really has a nonlinear character. Thus for example, Spanish regulations for FL-90 bricks consider a σ-ε diagram of the parabola-rectangular type.

In our study however, for simplicity’s sake, in the first phase we have worked with a linear compression diagram and with zero resistance to traction.

5 CONSIDERATION OF LOADS

For this study we have considered the current regulations as to road-bridge loads as well as the gravitational loads of the remaining elements, taking average paving densities and thicknesses. We have considered the following loads:

Variable loads:

- 4 kN/m2 distributed over the whole width and on the most unfavourable length for analysis of the arches.
- 6 loads of 100 kN regulation-size loads.

Gravitational loads:

- Paving: 2,4 kN/m2.
- Filler: we consider a filler density of 18 kN/m3, and an internal angle of friction of 300.
- Masonry: we consider a masonry density of 24 kN/m3.

6 ANALYTICAL PROCESS

We have constructed a computer model that analyses various rise/span relations in arch bridges with a circular section. For each rise/span relation we have studied several thicknesses of filler over the extrados of the arch at its keystone. The thicknesses were varied form zero to two metres in increases of 0.5 m. Each case was studied for the worst position of the variable loads. This was generally for the load of 4 kN/m2 distributed over only half the span, and the vehicle with three loads situated such that its central load is a quarter of the distance from the span.

Each 100 kN load was considered placed on its surface of 0.20 m by 0.60 m. We considered the reduction in stress with depth from the upper surface to open out at an angle of 45º. In order to properly simulate the regulation load vehicle, the model assigns each 100 kN load another load of the same size 2 m apart transversally. The effects of this load will be felt at the corresponding depth. The stress level calculated for each depth will have an effect on the extrados of the arch.

All vertical loads which effect the extrados of the arch via the filler of the vault will tend to exert horizontal thrust on the vault. This effect has been taken into account when calculating the stability of the arch.

With all these loads, plus the weight of the masonry itself, we have calculated the minimum thickness of arch capable of withstanding a thrust line in equilibrium with the system of loads. This value is considered in the analysis to be the minimum thickness of an arch with a specific intrados which would be able to balance the loads which gravitate over the arch

We analyse the thrust lines contained in the minimum thickness, selecting the planes of the joints where the eccentricity with relation to the intrados is minimum and maximum, and calculate the value of the normal and tangential forces on these points and their application points.

With the normal force value and the design tension of the stone (σ_{Sd}), we determine the necessary thickness of the voussoir supposing the tension distribution to be linear such that at no point on the joint is the design tension exceeded. With the value of the tangential force it is observed that there is no sliding between voussoirs, given that their quotient with normal forces (less than 0.05) is well below the stone-stone friction coefficient. This has been studied for an average thickness of filler of 0.50 m. on the extrados of the arch at its keystone.

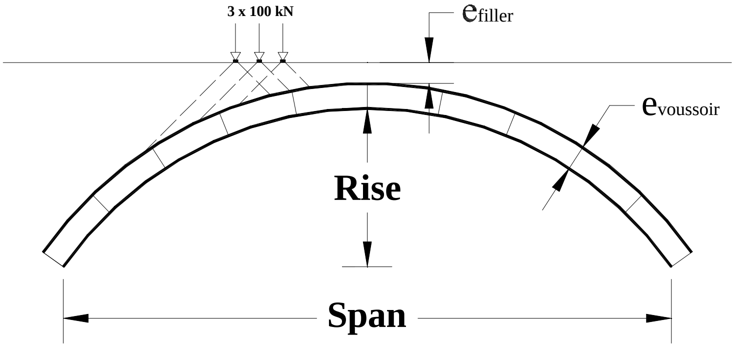

Figure 2: Geometry

7 THE SAFETY OF ARCHS

The safety of arches can be introduced either in terms of stability through the equilibrium of forces, it being essential that the line of thrusts remains within the stone with a certain margin for safety, or that the maximum tensional level is sufficiently below the acceptable tension of the material. In the first case, the safety will be based on geometric questions, while in the second the safety will be based on mechanical questions.

As regards safety criteria based on geometric considerations, the result obtained must be considered a minimum thickness for each of the cases studied. If the adequate safety coefficient is applied, the abacus obtained allows to make a direct calculation of the thickness of the voussoir of the arch. Usual values of this geometric safety coefficient are between 3 and 4.

In the ideal case where materials have infinite resistance the thickness of the arch obtained with the above criteria would be the minimum. That is, that in which the line of pressure is at a tangent to the intrados and the extrados of the arch. If one wants to obtain a sufficient safety margin based on tensions, then the acceptable tension of the material must be reduced and the minimum thickness increased in order to not exceed the design tension. A usual value for this is also between 3 and 4, although some authors have even advised values as high as 10.

8 RESULTS

A series of graphs have been drawn up, based on our studies, which make it easier to interprete the results.

In the five graphs below, the most significant variables are summarise form the point of view of the use of the results, considering a minimum geometric thickness. That is, that the line of pressure is a tangent to the upper and lower surfaces of the voussoirs. The variables of the problem in hand are the rise of the arch, the span of the arch, thickness of filler over the keystone and minimum voussoir thickness.

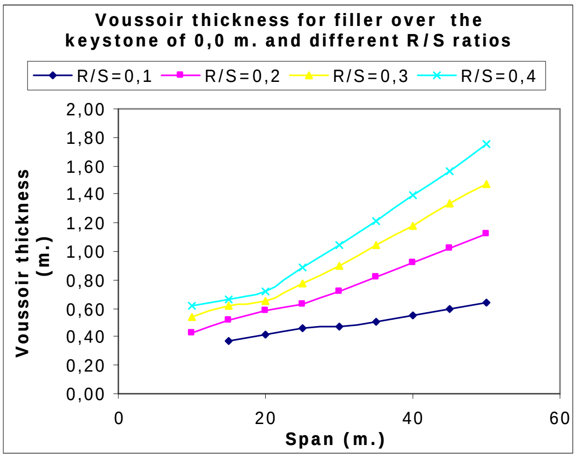

Figure 3

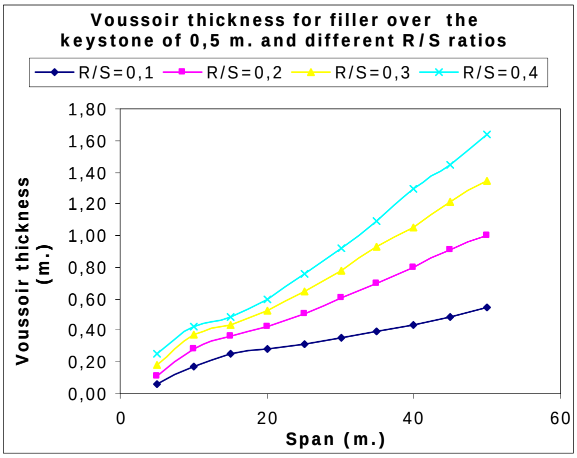

Figure 4

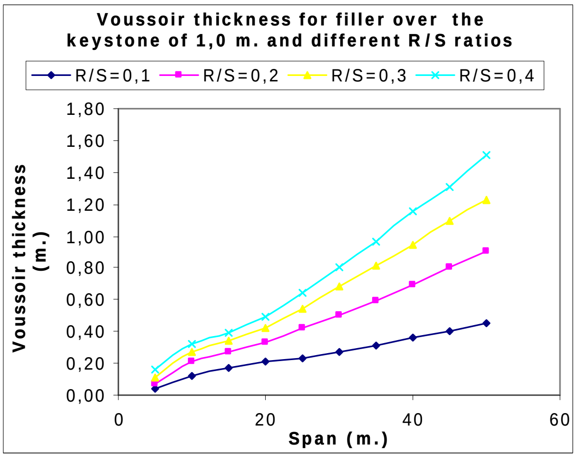

Figure 5

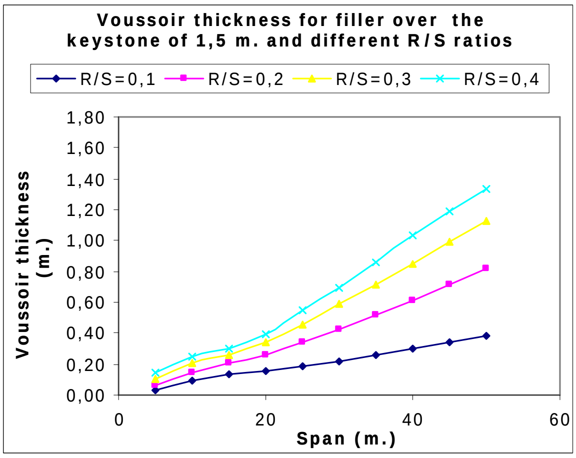

Figure 6

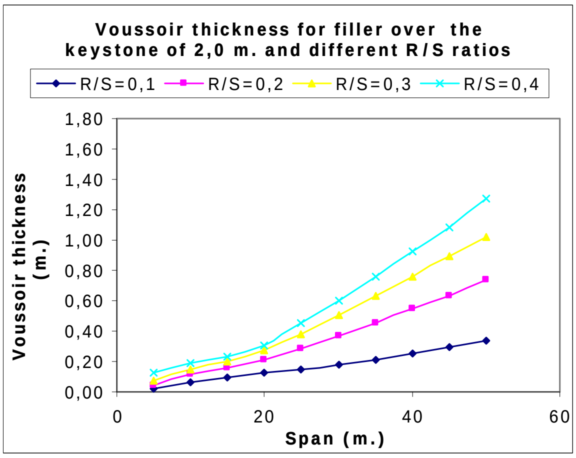

Figure 7

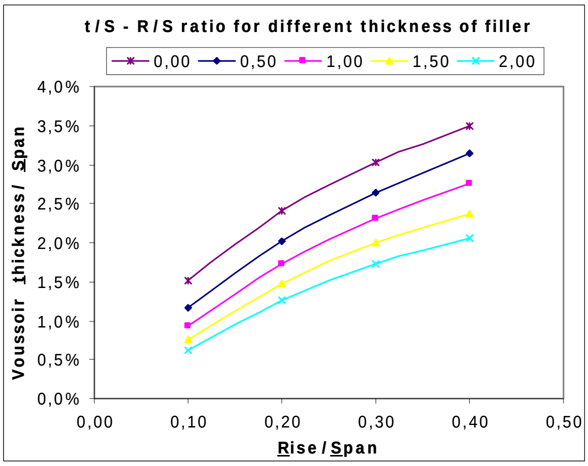

Figure 8

In the following four graphs, the parameter of the problem is the resistance of calculation of compression of the stone, in addition to the rise, span and thickness of the voussoir. For these four tables, we have taken a thickness of filler over the keystone of 0,50 m, which is close to the normal value for stone arch bridges. As in the previous case, the use is direct, and it suffices to enter in the tables the known geometric values of the bridge and obtain the minimum thickness of the voussoir in order not to reach calculation tensions greater than those set. For example, in a bridge with 30 m. of span, and 3 m. of rise (R/S= 0,1), we are going to use a geometric safety factor, and a tensional safety factor both with valor 3. In figure nº 9, we get a thickness of valor 0,35 m. If then we apply the geometric safety factor we get a thickness of 1,05 m. Nevertheless, if we have a hard stone (i.e. 60 MPa), we must use the curve of 20 MPa and then we get a thickness of 0,44 m. Now, if we have a soft stone (i.e. 6 MPa), we will use the curve of 2 MPa, and we will obtain a thickness of 1,29 m.

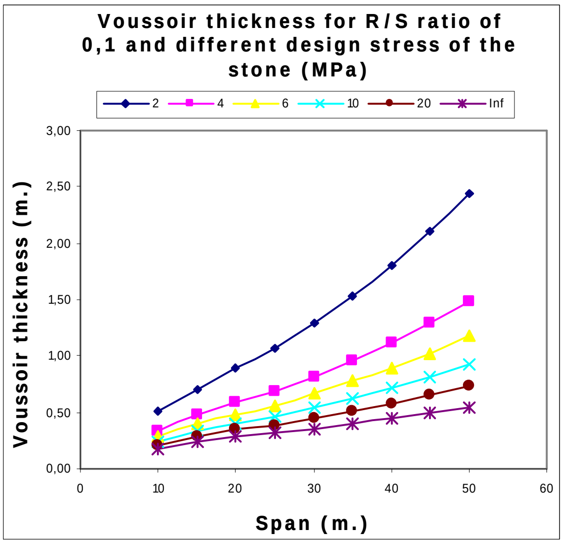

Figure 9

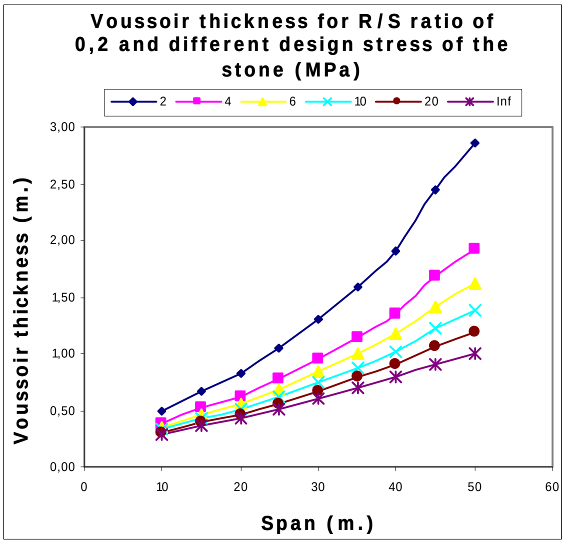

Figure 10

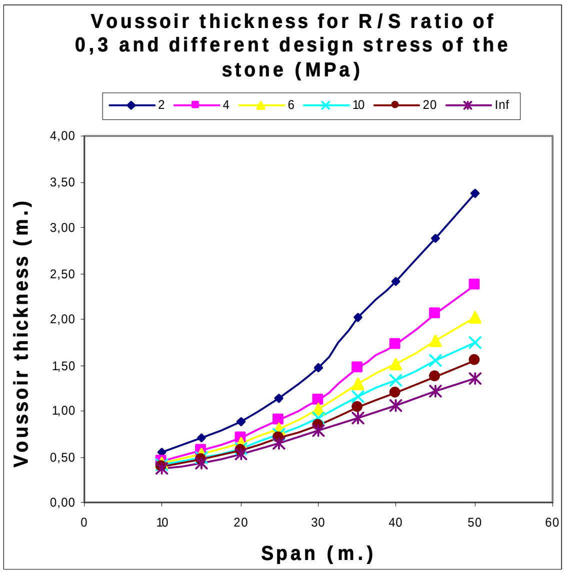

Figure 11

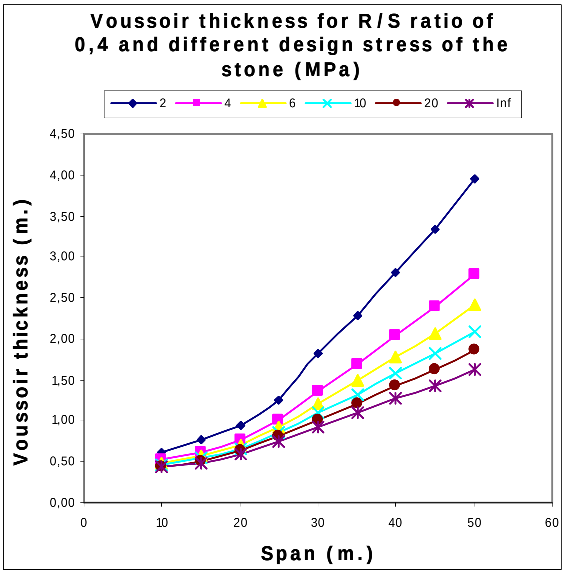

Figure 12

9 TESTING OF RESULTS

In order to evaluate the practical application of the study we have analysed several real cases of Spanish stone arch bridges with low rise/span ratio. Below are the geometric parameters of three of the bridges studied::

Table 2 : Real cases

Bridge		Span (m)	Rise (m)	Voussoir thickness (m)	Filler thickness (m)
“Larrate” bridge	River Linares (La Rioja)	16	2,0	0,9	0,5
“Castilla” bridge	River Arlanzón (Burgos)	15	1,5	1,0	0,0
“San Marcos” bridge	River Bernesga (León)	14	4,0	0,6	0,5

The result of the analysis is the minimum necessary voussoir thickness, which is compared with the actual thickness, to obtain the geometric safety coefficient. In addition, we obtain a reference of the maximum tensional work state that should be compared with laboratory tests on the material the bridge is made of:

Table 3 : Results

Bridge		Minimum Thickness (m)	Geometric Safety Coefficient	$\sigma_{\text{máxima}}$ of Work (MPa)
“Larrate” bridge	River Linares (La Rioja)	0,25	3,60	1,60
“Castilla” bridge	River Arlanzón (Burgos)	0,30	3,33	1,40
“San Marcos” bridge	River Bernesga (León)	0,35	1,70	2,70

The photograph in figure 13 is of the “San Marcos” bridge. Figure 14 shows the graphic output of the programme with data corresponding to this bridge, where the different lines represent the geometry of the bridge, the loads applied and the thrust lines.

Figure 13: “San Marcos” bridge

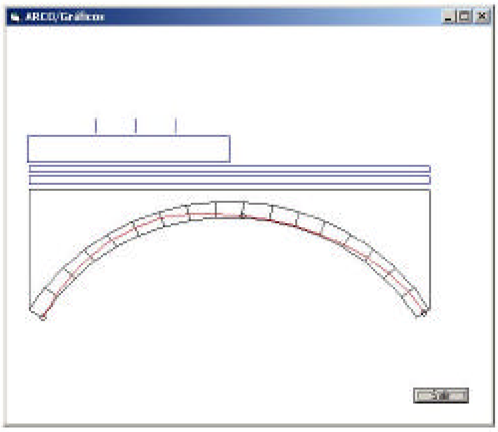

Figure 14: Output graphic

10 CONCLUSIONS

Observing the figures 9 to 12, and with the example we have made, it can be seen that with highly resistant stones is bigger the minimum thickness necessary is greater based on geometric safety criteria. Both criteria, geometric and tensional, become closer as the quality of the rock diminishes. With very low work tensions, the minimum thickness necessary is even greater using tensional safety criteria.

If we analyse the two criteria as a function of the variation in the rise/span (R/S) ratio, it can be seen that the difference between them falls as the rise/span ratio diminishes. By other way, thinking about the validity of use geometric safety factor, it is necessary to say that we can use them only like a first approximation or a preliminary comprobation. Its use is similar as the use of the knowledge of usuals ratios h/L in beams. This knowledge is employed in order to get the first dimensional value of the beam, but afterwards it is always necessary to make a tensional comprobation.

If we now turn our attention to the curves based on geometric criteria, we see that a linear relation is observed between thickness and span for lengths from around 20 metres. For smaller spans this linear relation is slightly distorted, probably by the effect of the dimensions of the loads with respect to the span of the arch. However, this linear relation varies in relation to the thickness of filler over the keystone; the thicker the filler, the less thick the voussoir needs to be to contain the thrust lines inside the vault. This would imply an increase in the stress of the bridge, a matter that requires a separate study.

Another interesting graph (Fig. 8) is that which relates the percentage of voussoir thickness necessary for each type of low rise/span ratio ridge. In this graph we can observe a global variation of the cases studied between 0.5 % and 3.5 % of the voussoir thickness in relation to the span of the bridge and different fillers. For each specific rise/span ratio it is seen that on average the voussoir thickness necessary can double as the filler thickness is reduced from two metres to zero.

Finally, it was observed that the lower arch, for the same span, the less thick the voussoir needs to be. This is because the filler distributes the loads more uniformly. However, as we have already mentioned, there may be additional phenomena that have the opposite effect, such as the increase in stress and the increase in horizontal thrust on the supports.

REFERENCES

Corradi, M. (1998). International Arch Bridge Conference (Venice, 1998)

Heyman J. (1995). Teoria, historia y restauración de Estructuras de fábrica. CEHOPU.

Heyman, J. El esqueleto de piedra. Mecánica de la arquitectura de fábrica. CEHOPU

Hendry Arnold W.. Structural Masonry (Second edition, 1998)

Ministerio de Fomento (1998). Instrucción sobre las acciones a considerar en el proyecto de puentes de carretera. (IAP’98)

Yáñez Hernández, M.A., Ortega Basagoiti, L.M (2000). Puentes de fábrica: Situación actual. Asociación Española de la Carretera.