An orthotropic damage model for non linear masonry walls analysis: Irreversible strain and friction effects

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ABSTRACT: In this work the damage model for masonry proposed by the authors in Saetta et al. (2000) is enhanced in order to better simulate the mechanical behaviour of masonry. Particularly a friction factor, which enables to account the friction effect across an open crack, and the possibility to take into account irreversible strain have been introduced. The simulations of some experimental tests carried out both on test shear walls and large scale walls show the capability of the proposed continuum model to well describe the failure strength, the hysteric response and the damage evolution of masonry structures.

1 INTRODUCTION

The necessity of preserving historical heritage and the need of a better understanding of the mechanical behaviour of structural masonry has led to great innovation in the development of specific numerical techniques able to analyse such constructions.

A reliable numerical method for analysing masonry structures must take into account the peculiar features of such a material. Indeed masonry is a composite, heterogeneous, non linear material that exhibits distinct directional properties, because the mortar joints act as planes of weakness.

A detailed numerical model of such a material can be developed using the so-called *micro modelling* (e.g. Lourenco 1996). For this approach each unit and joint has to be meshed separately to allow for only one material appears within each element. Obviously, such a technique is not suitable for analysing large, real structures, for which a different approach has to be adopted. In particular the so-called *macro-modelling,* where masonry is regarded as an orthotropic composite with different elastic and inelastic properties along the two main directions of the real material (i.e. the bed joints direction and the head joints direction), can be adopted. For such an approach, a relation is established between average masonry strain and average masonry stress so that the main aim is the knowledge of the global behaviour of masonry structures.

By adopting this approach the necessary computational effort of non-linear finite element analysis of large masonry structures is considerable reduced.

In this work the enhancement of a simple but effective macro-model (Saetta et al. 2000, Berto et al 2001) based on the continuum damage mechanics, suitable for orthotropic brittle materials, like masonry, is presented. In particular, since masonry walls are one of the most common construction typology used all over Europe, the proposed model, in this first draft, is specifically developed for analysing masonry structures which can be approximated as being in plane stress state. It is worth noting that masonry walls are often subjected to racking shear in addition to compressive loads, so it is crucial that a numerical model correctly reproduce the behaviour of the material to this type of loading.

Orthotropic elasticity is combined with orthotropic damaging in such a way that a completely different behaviour along the two principal axes is represented. One of the basic assumptions of the proposed model, is the acceptance of the natural axes of the masonry (i.e. parallel and orthogonal to the mortar joints) also as principal axes of the damage. In each direction two independent damage parameters are assumed, one for compression and one for traction, allowing the crack closure effect to be adequately described. The evolution laws of damages indexes are assumed similar to that proposed by Farja and Oliver (1998) for concrete, and used also by the authors (Saetta et al. 1998, 1999).

Moreover the model has been improved by introducing a shear factor (s. f.), which enables to account the friction effect across an open crack. By including a plastic strain tensor with a simple but efficient evolution law, the occurrence of irreversible deformations will also be allowed. Despite of such developments, the numerical model maintains a low computational effort and can be profitably used for the global analysis of real structures.

The simulation of some tests carried out both on a masonry specimen and shear walls experimentally tested at ETH Zurich (Ganz and Thurlimann, 1984) shows the capability of the present version of the damage model to well describe the failure strength, the damage evolution and the development of irreversible strain of masonry.

Finally the analysis of a large-scale wall, belonging to a masonry building prototype tested at the University of Pavia, CNR GNDT (1995), is carried out by using the damage model enhanced with the introduction of the shear factor, accounting for the friction effect across an open crack.

At now the improvement of the damage model is a work-in-progress, and next applications will be a better validation of the effect of irreversible strain by means of more specific experimental tests, so allowing the simulation of large-scale wall by using realistic parameters. Moreover the friction term has to be considered variable with the crack opening.

2 TWO-PARAMETER DAMAGE APPROACH

2.1 Basic assumptions

The model derives from the extension of a damage model for isotropic material, similar to that proposed for concrete structures by Faria and Oliver (1998) and used also by the authors (Saetta et al. 1998,1999)

To extend the isotropic damage model to an anisotropic one a vector or tensor damage representation must be used. In particular in the case of an orthotropic material it is reasonable to assume that the damage variables are scalars associated with each of the three principal directions of anisotropy of the material (Chaboche et al 1995). Hence for the masonry under plane stress conditions, we have defined two independent damage variables (one for compression and one for tension) for each natural axis (the so-called d_x^+, d_x^-, d_y^+, and d_y^-). So four distinct damage criteria have been introduced, which correspond to a damage bounding surface, in the space of the effective stress ($\overline{\sigma}_x$, $\overline{\sigma}_y$ and $\overline{\tau}_{xy}$), whose shape is a double pyramid with rectangular base.

Moreover at this stage of the research the evolution laws of damages variables are assumed similar to that proposed by Farja and Oliver (1998) for the concrete.

It should be noted that, since the adopted damage model is based on the strain equivalence principle (Lemaitre and Caboche 1978), it is crucial to define the transformation of the Cauchy stress tensor s into the effective stress tensor $\overline{s}$. In a general case of anisotropic damage this leads to the introduction of the fourth- rank damage effect tensor $\mathbf{M}$. Hence according to Chaboche's notation (Chaboche et al. 1995) the following 3D definition of the effective stress tensor holds:

$$\overline{s} = \mathbf{M}^{-1} : s \tag{1}$$

that in the case of isotropic damage becomes the well known relationship $\overline{s} = \left[(1 - d)\mathbf{I}\right]^{-1} : s$.

Detailed discussion of some different definitions of the damage effect operator **M** may be found in Skrzypek and Ganczarski (1999).

With reference to in-plane load conditions, and by using matrix notation, the equation (1) becomes:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\tau}_{xy} \end{Bmatrix} \tag{2}$$

where the expression of M_{ij} depends on the definition used for **M**.

Within the framework of the orthotropic damage model here developed for masonry we have adopted the following definition:

$$\mathbf{M} = \left[\mathbf{I} - \hat{\mathbf{D}}\right]^{-1} \tag{3}$$

where **I** is the third order identity matrix, and $\hat{\mathbf{D}}$ is a suitable 3×3 damage matrix, whose elements are functions of the damage variables (d_x^+, d_x^-, d_y^+, and d_y^-), and of the deformation tensor. The analytical expression of the adopted damage matrix together with a complete description of the proposed damage model can be found in Saetta and al. (2000) and in Berto and al. (2001).

3 FRICTION EFFECT

The damage model presented in the previous section underestimates the shear strength of the masonry. It can be seen (for the details see Saetta and al. 2000 and Berto and al. 2001) that when the damage parameters approach 1.0, the secant, as well as the tangential stiffness, matrices tend to be identically zero, in regard to the shear stresses. This is because it does not take into account the additional shear capacity due to the frictional shear capacity through an open crack that becomes really significant in presence of high normal stress.

This effect can be considered, within the framework of a simplified approach, by introducing the following expression for the residual shear, which is transmitted between the two surfaces of an open crack:

$$\tau_{att} = \frac{\left|\left\langle -\left(\sigma_x + \sigma_y\right)\right\rangle\right|}{2} f \tag{4}$$

where f is the frictional term; the symbol $\langle\,.\,\rangle$ indicates the MacAuley brackets that are defined as follows:

$$< x > = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases} \tag{5}$$

When in a point of the masonry the damage level is so high that

$$|\tau| < \tau_{att} \,. \tag{6}$$

the shear stress is assumed equal to (τ_{att} sign(τ)).

In such a way the introduction of the friction term introduces a reduction of the damage effect and specifically τ_{att} can be considered as a minimum shear strength assured even for a completely damaged material under tensile load.

In this first formulation the friction factor is assumed constant. As a consequence the numerical results demonstrated a significant improvement but still showing some shortcoming (see the paragraph of numerical results, Fig. 3)

A further improvement of the above formulation will be to take into account that the friction due to the roughness of the crack surface decreases as the opening of crack increases, so relating the friction coefficient to the value of shear deformation.

4 IRREVERSIBLE DEFORMATION

For the evolution of the plastic strain tensor we have adopted in the present work the following law:

$$\dot{e}^{p} = \beta\, \mathbf{H} : \dot{e} \tag{7}$$

where:

- β is a material parameter, which controls the rate intensity of plastic deformation (a zero value cancels plastic contributions, reducing so the material model to a elastic-damage one). In this first formulation (in order to avoid too many input parameters) we have considered β as a scalar value, but it is evident that it is possible to assign different values of β along each material axis.
- $\mathbf{H}(\dot{\hat{\mathbf{D}}})$ is a fourth order tensor whose elements are the Heaviside functions of the tensile and compressive damage rate. It has been introduced in order to avoid plastic evolution during unloading or partial reloading. Its matricial expression for in - plane load conditions is:

$$\mathbf{H} = \begin{pmatrix} H(\dot{d}_x^+ + \dot{d}_x^-) & 0 & 0 \\ 0 & H(\dot{d}_y^+ + \dot{d}_y^-) & 0 \\ 0 & 0 & \frac{1}{2}\left[H(\dot{d}_x^+ + \dot{d}_x^-) + H(\dot{d}_y^+ + \dot{d}_y^-)\right] \end{pmatrix} \tag{8}$$

5 NUMERICAL EXAMPLES

5.1 Tension-compression cyclic test

Let us consider a masonry specimen whose mechanical characteristics are summarized in Table1 and Table2. The β parameter in this example is set equal to 0.5. The specimen has been subjected to a cyclic test in order to check for the performance of the proposed plastic-damage constitutive law. Fig. 1 shows the response of the masonry specimen in terms of stress-strain curve. As it can be observed the model is able to reproduce the softening behaviour in tension, as well the hardening and softening behaviour in compression. Moreover the stiffness reduction due to the damage effect, the stiffness recovering (damage in tension is supposed to have no effect in material compressive behaviour) and the irreversible strain evolution are also well captured.

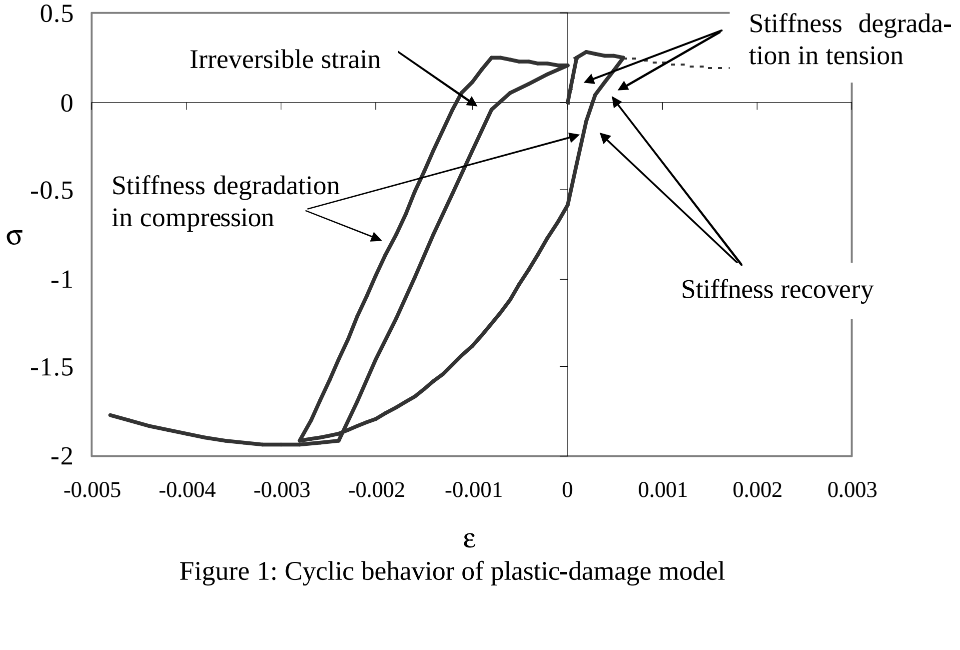

Figure 1: Cyclic behavior of plastic-damage model

5.2 *Shear wall: friction effect*

The numerical applications hereafter described are specifically focused on the testing of the friction term. In particular a shear wall experimentally tested is simulated by means of damage model including friction effect and the numerical results are compared with the experimental ones. The wall, formed from hallow clay bricks, was tested at ETH Zurich (Ganz and Thurlimann, 1984). As shown in Fig. 2, it consists of a masonry panel and two lateral flanges, at the top and bottom of them are placed two concrete slabs.

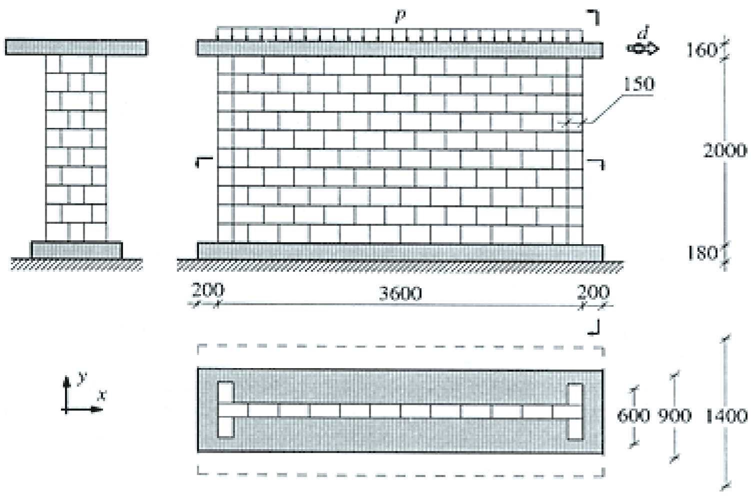

Figure 2: ETH Zurich clay brick masonry test

The wall has been firstly subjected to a vertical uniformly distributed load *p*=1.91 MPa, then a horizontal monotonically increased force was applied on the top slab under displacement control. Table1 and Table2 show the mechanical parameters used in the numerical analysis, they are derived from the biaxial tests carried out from Ganz and Thurlimann (1982) and simulated by Lourenço (1996). As made from Lourenço in his analysis, we have assumed for the flanges in the x direction the tensile and compressive strength of the clay brick (f_{tx}=0.68 MPa f_{cx}=9.5 MPa). A regular mesh, made up of linear triangular elements, has been used.

Table 1: Elastic material properties of ETH Zurich shear wall

undamaged elastic modulus in x direction	Ex = 2460 MPa
undamaged elastic modulus in y direction	Ey = 5460 MPa
Shear modulus	Gxy = 1130 MPa
Poisson's ratio	νxy = 0.18

Table 2: Material properties of ETH Zurich shear wall used in the damage model

	x direction	y direction
uniaxial elastic limit in compression	fcx0 = 0.56 MPa	fcy0 = 1.37 MPa
uniaxial initial compressive strength	fcx = 1.87 MPa	fcy = 7.61 MPa
uniaxial initial tensile strength	ftx = 0.28 MPa	fty = 0.05 MPa
shear strength	fτ = 0.3 MPa	
Fracture Energy	Gfx = 0.02 N/mm	Gfy = 0.02 N/mm
A parameter	Acx = 0.12	Acy = 0.24
B parameter	Bcx = 1.0	Bcy = 2.5

It is worth noting that the effect of self weight is also considered during the analysis.

The obtained results in terms of load-displacement diagrams are plotted in Fig. 3 for different values of the friction factor f and compared with the experimental ones, showing the benefit due to the introduction of the friction factor.

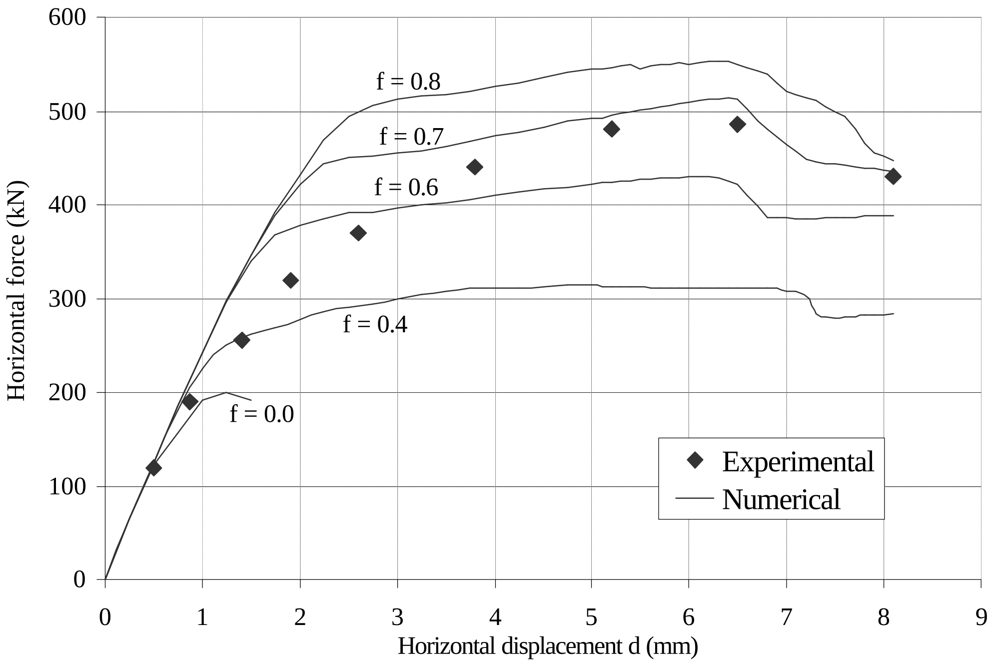

Figure 3: Load-displacement diagrams with various friction terms

It can be evidenced that, in the absence of this factor, unrealistic result has been found (see the curve corresponding to f = 0), in particular the calculated collapse load is much smaller than the experimental one. Instead by introducing friction factor it is possible to numerically reproduce the real behaviour of the masonry wall.

The numerical results obtained using f = 0.7 are reported in terms of damage contours (Fig. 4) and deformed configuration (Fig. 5), at the displacement of 6.0 mm.

It is worth noting that at this level of displacements (around the experimental peak load), the wall shows a diffuse tensile damage distribution d_x^+, in quite good agreement with the experimental crack pattern reported in Fig. 6. The corresponding compressive damage, although just present, maintains sufficiently low with respect to the limit value.

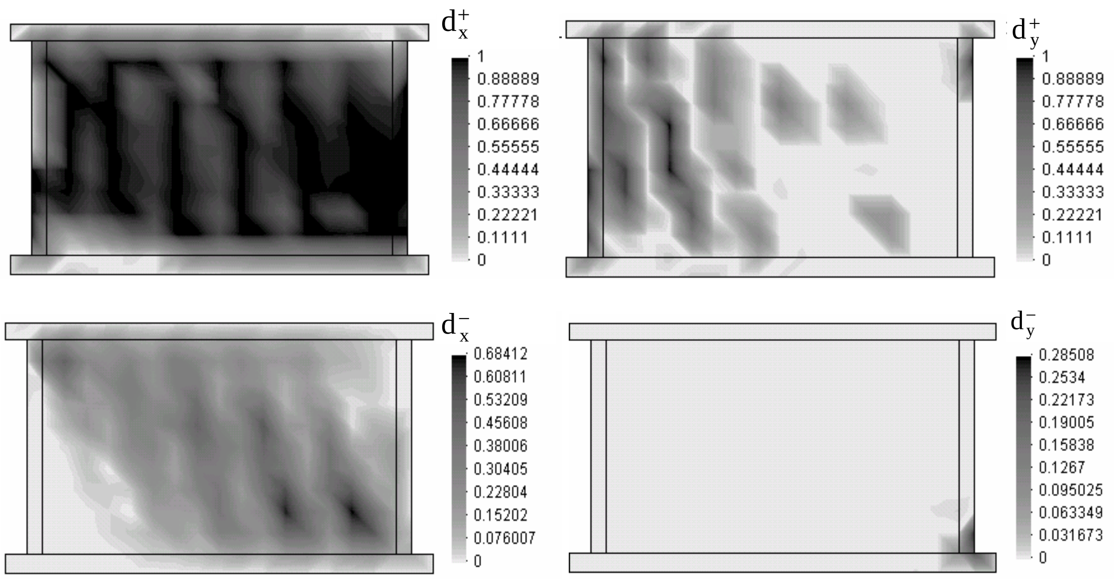

Figure 4: Damage contours at the displacement of 6.0mm

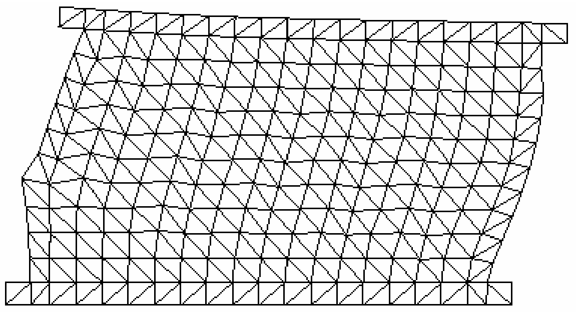

Figure 5: Deformed configuration at the displacement of 6.0mm

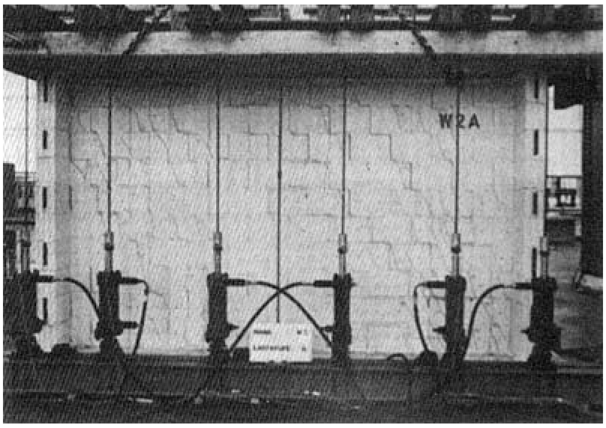

Figure 6: ETH Zurich: Experimental crack patterns at the displacement of 6.0 mm

At collapse the damage contours and the deformed configuration are shown respectively in Fig. 7 and Fig. 8. The mode of failure is for masonry crushing, as demonstrated by the development of compressive damage, Fig. 7. This result is in accordance with the experimental evidence (Fig. 9).

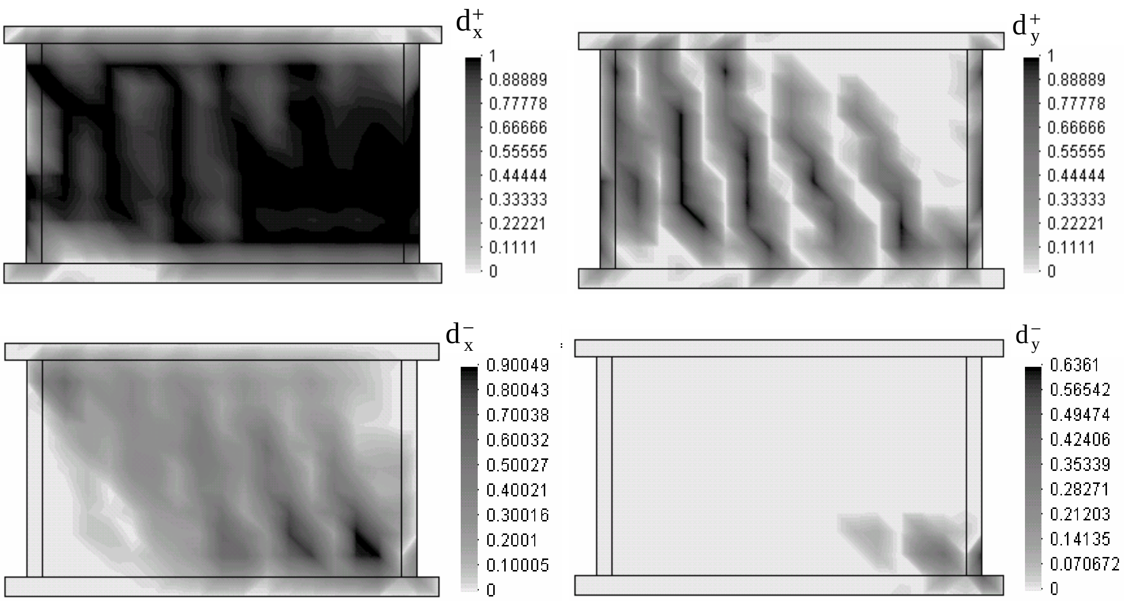

Figure 7: Damage contour at the displacement of 8.0 mm (end stage)

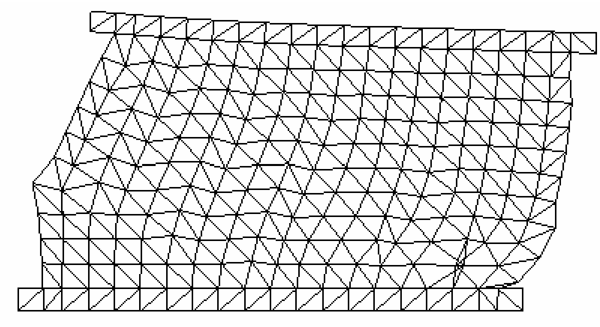

Figure 8: Deformed configuration at the displacement of 8.0 mm (end stage)

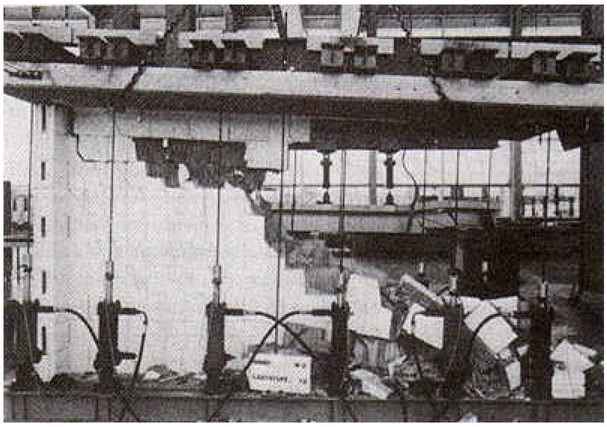

Figure 9: ETH Zurich: Experimental crack patterns at the displacement of 8.0 mm (end stage)

6 BRICK MASONRY BUILDING PROTOTYPE: NUMERICAL SIMULATION OF A LARGE SCALE WALL

With reference to the testing of the full-scale two-storey building carried out at the University of Pavia (CNR GNDT 1995), the numerical investigation of the wall B is developed by using the damage model, taking into account the friction effect.

All the details of experimental set-up and material characteristics can be found in the CNR-GNDT report, while the numerical parameters are reported in Berto et al. (2001). Fig. 10 shows the geometry of the wall B of the prototype and the finite element mesh used in the analysis. According to the experimental test set-up, the load condition has been realized by a displacement history imposed to the first and second floors in such a way as to maintain the forces applied at the first floor equal to the ones at the top floor level. The main controlling parameter for the loading was the drift, i.e. the ratio between the top floor displacement and the height of the actuator (5.77m). The maximum value of the drift is 0.40%.

In addition to the masonry gravity loads (18.8 kN/m^3), the concentrated vertical forces P_1 = 11.3 kN and P_2 = 10.8 kN have been applied on each nodes of the steel beams of the first and second floor respectively to reproduce the experimental conditions.

At the maximum drift, corresponding to the end of the displacement history, Fig. 11 shows the damage contours. At this stage of the displacement history, the wall is nearly totally damaged. Although not negligible compressive damage appears (see Fig. 11a-b), it is evident that the failure of the wall is governed by shear deformations localized in the piers.

By comparing the damage contours with the experimental crack pattern reported in Fig. 12, a global good agreement can be ascribed.

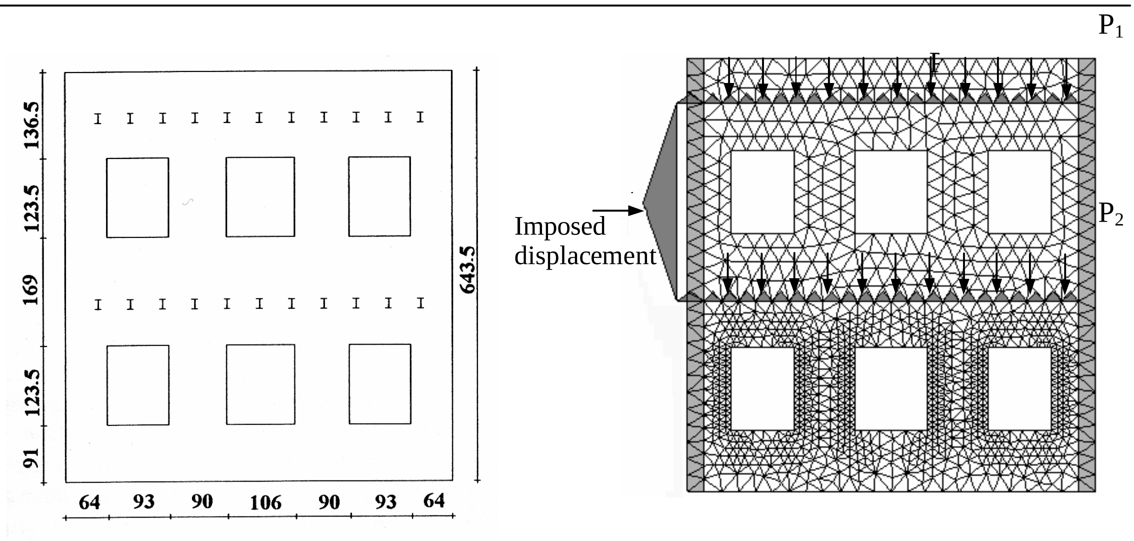

Figure 10: geometry of the wall B (dimension in cm) and finite element mesh of the large scale wall

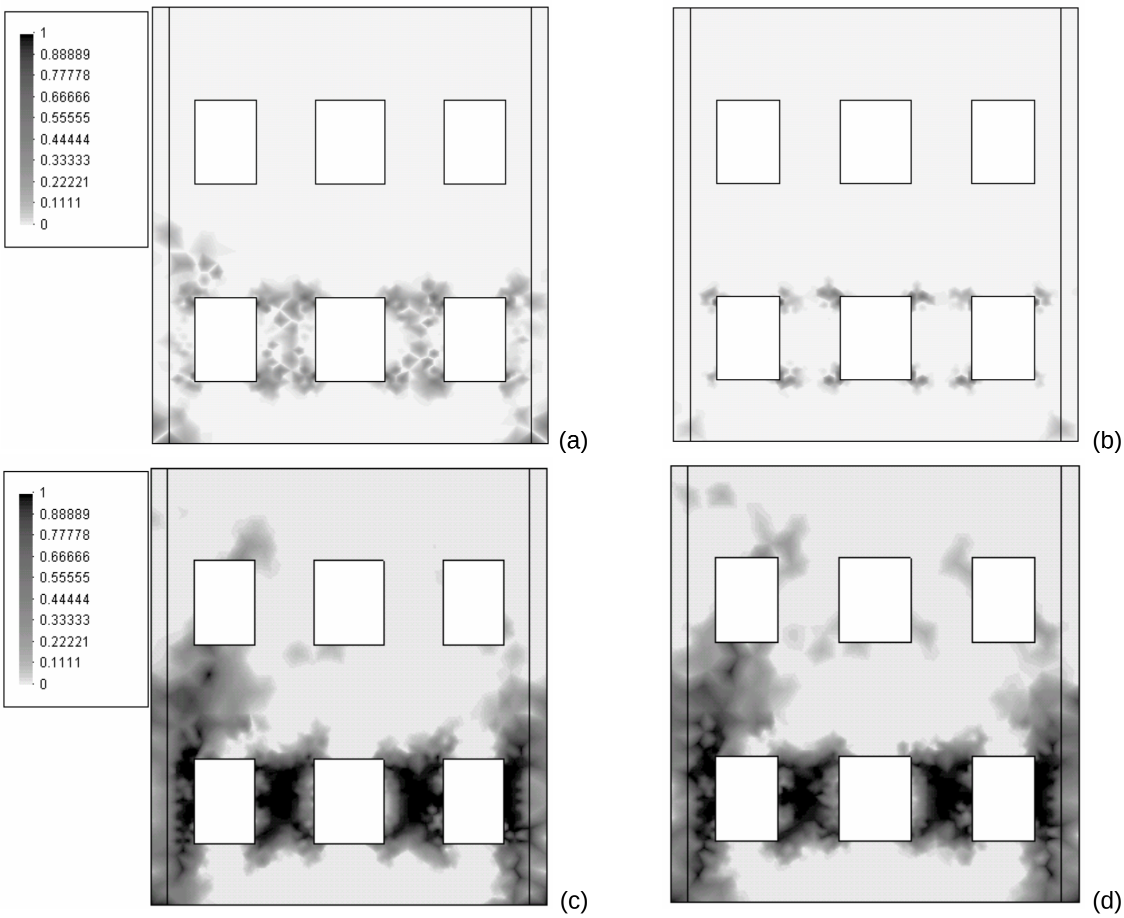

Figure 11: Max. nominal drift 0.4%: (a) d_x^- (b) d_y^- (c) d_x^+ and (d) d_y^+ damage contours

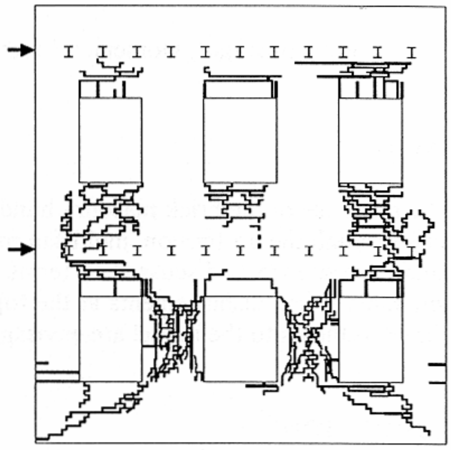

Figure 12: experimental crack pattern at the maximum nominal drift 0.4%

7 CONCLUSIONS

The in-plane loaded masonry constructions has been simulated by means of an orthotropic damage model, based on the definition of four independent internal damage parameters, two in compression and two in tension. The four parameters are referred to the natural axes of the masonry (i.e. respectively the bed joints and the head joints directions), so allowing to reproduce different inelastic behaviour along each natural axis. Friction effect has been accounted for as constant parameter, and the development of irreversible strain has been introduced.

The presented results demonstrated that the proposed damage model is able to capture the global behaviour of masonry structures. However in this form it needs some further improvements: the friction term has to be considered variable with the crack opening and the effect of irreversible strain has to be better substantiate with more specific experimental tests.

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