Stochastic analysis of historical masonry structures

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ABSTRACT: The aim of this paper is to show the possibilities of FEM modelling and probabilistic assessment procedures applied for problems of historical structures. It will be demonstrated that above mentioned tools supported by detailed material testing and investigation should give us information on the accurate prediction of the actual safety level of the structure and enable us to choose the optimal way of reconstruction and preservation. The differences between deterministic and stochastic procedures are demonstrated on parametric studies. The suggested approach technique is also applied to the analysis of the one of the most famous historical structure in the Czech Republic – Náměšt nad Oslavou Bridge.

1 INTRODUCTION

The preservation of historical buildings is of growing interest in many countries worldwide. Scientists and engineers perform historical, archeological and in situ investigation to repair, maintain and extend its lifetime. The repair and maintenance of historical masonry structures, generally a structural intervention of the existing structure, are reasons for the need of better understanding of these structures. The basic question arises: What is the carrying capacity, serviceability and the safety degree of the structure before and after intervention? To answer this question is extremely difficult in case of masonry structures, because the determination of material and geometrical parameters is generally uncertain.

Experimental testing of used materials is usually a necessary and important step before numerical structural analysis. The results are quite often spread in such wide range that using mean or “safe” values for a deterministic analysis would not be acceptable. Insufficient knowledge of input parameters of a computational model calls for the rational alternative of analysis - to work directly with uncertain data considering them to be random variables described by appropriate probability distribution function (PDF). Then there is a possibility of numerical quantification of statistical scatter, sensitivity and reliability (unreliability) using a reliability approach and relevant mathematical methods. Monte Carlo type simulation is straightforward tool for estimation of statistical parameters of structural response (stresses, deflections). A special stratified sampling technique Latin Hypercube Sampling is very suitable for computationally intensive finite element complex models as it requires relatively small number of simulations (repetitive analysis of relevant computational model).

Masonry is quite complicated material with non-linear behaviour. The starting point for understanding the behaviour of masonry structures can be a linear elastic analysis under the assumption of masonry as a homogenous material. Carrying capacity expressed e.g. by limit loading cannot be estimated and for serviceability limit state deformation stage is generally underestimated. Then consequent step is the non-linear modelling utilising achievements of computational mechanics. The difficulties arising from the need of appropriate data and material model for masonry are obvious when using those sophisticated approaches. The necessity to adopt a proper material model and special contact and link elements is essential. Simplified models using beam or plate elements can give good results but in some cases may not bring an adequate response of real structure. The speed of nowadays computers and the efficiency of modern stochastic procedures encouraged us to use 3D solid elements for the size of problems like never before even in a combination with stochastic approach.

Similar approach using 2D ANSYS elements was used for the analysis of the Charles Bridge in Praque (Novák and Žák 1995). Current technique using 3D non-linear elements and advanced stochastic procedures was applied to the analysis of Náměšt nad Oslavou Bridge.

2 THE HISTORY OF THE BRIDGE

Náměšt nad Oslavou Bridge, Fig. 1, belongs to the most valuable historical monuments in the Czech Republic and is one of the main tourist attraction in Moravia.

The bridge was completed in 1737, never suffered by heavy floods and is 75.65m long. It has 7 arches with the spans from 6.40 to 9.75 supported by 8 piers.

The last reconstruction (1988-1993) was done in a similar way like the reconstruction of the Charles Bridge in 1966-1975. The filling materials were removed and replaced by expanded clay concrete and a reinforced concrete slab without expansion joints that was anchored to the parapet walls. These changes caused the bridge to become too rigid and resulted in excessive cracks and other problems. The bridge is now treated as a historical monument and any traffic is excluded except pedestrians.

The bridge was built of sandstone, arenaceous marl, unknown materials and granite paving in sand. The current materials are sandstone, arenaceous marl, concrete slab and granite paving in mortar, Fig. 2.

Figure 1 : The view of Náměšt nad Oslavou Bridge

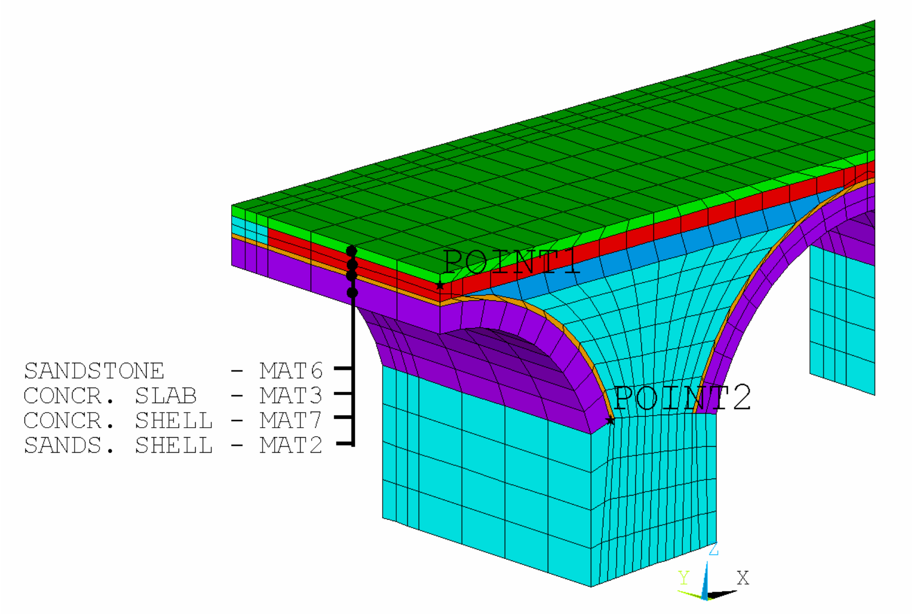

Figure 2 : Materials of the bridge

3 DETERMINISTIC COMPUTATIONAL MODEL

Finite element package ANSYS was found to be a very suitable tool for the analysis. FE model covers the quarter of the whole bridge using non-linear 3D elements together with a special link elements, Fig. 3.

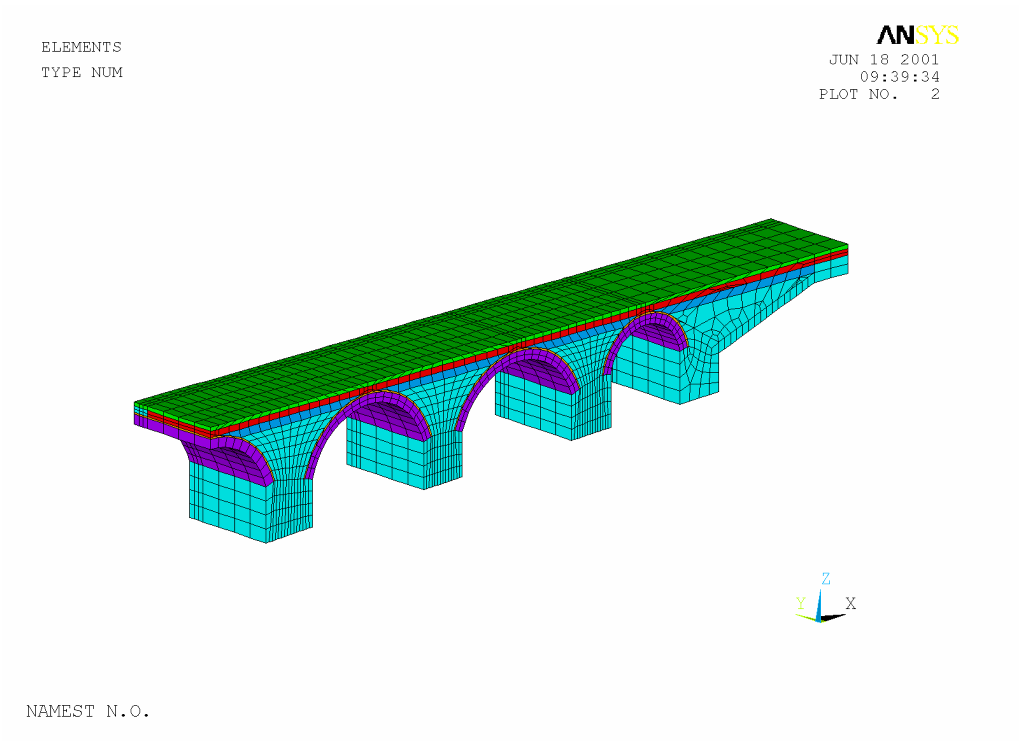

Figure 3: Model of the bridge

It is obvious that temperature changes cause the main problems. A temperature field analysis using temperature elements resulted in a temperature distribution. The calculated temperatures were converted into initial strains and then transformed into equivalent external loads for the structural analysis.

4 PROBABILISTIC APPROACH

4.1 Review of techniques

The classical reliability theory introduced the form of a response variable or safety margin (in case that the function expresses failure condition) as the function of basic random variables $\boldsymbol{X} = X_1, X_2, \ldots\ldots, X_n$

$$Z = g(X_1, X_2, \ldots, X_n) \tag{1}$$

where *g()* (computational model) represents functional relationship between elements of vector X (e.g. Freudenthal 1956, Freudenthal et al. 1966, Madsen et al. 1986, Schneider 1996). Elements of vector X are geometrical and material parameters, load, environmental factors etc., generally uncertainties (random variables or random fields). These quantities can be also statistically correlated. In case that Z is safety margin, *g()* is called limit state function and can be formulated usually using comparison of a real load and failure load. The structure is considered to be safe if

$$g(\boldsymbol{X}) = g(X_1, X_2, \ldots, X_n) \geq 0 \tag{2}$$

Then the theoretical failure probability as a measure of unreliability is defined as

$$p_f = \int_{D_f} f(X_1, X_2, \ldots, X_n)\, dX_1, dX_2, \ldots, dX_n \tag{3}$$

where D_f represents failure region where $g(\mathbf{X}) < 0$ and $f(X_1, X_2, \ldots, X_n)$ is the function of marginal probability density of random variables $\mathbf{X} = X_1, X_2, \ldots, X_n$.

Equality $Z = 0$ divides multidimensional space of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_n$ into safe and failure region. Explicit calculation of integral (3) is generally impossible therefore the application of a simulation technique Monte Carlo type is the simple and in many cases feasible alternative to estimate failure probability (e.g. Rubinstein 1967, Schreider 1967, Schueller 1998, Marek et al. 1993).

The aim of statistical and reliability analysis is mainly the estimation of statistical parameters of random variable Z and/or theoretical failure probability, integral (3). Pure Monte Carlo simulation cannot be applied for timeconsuming problem as it requires large number of simulations (repetitive calculation of structural response). Historically, this obstacle was partially solved by approximate techniques suggested by many authors, e.g. Grigoriu (1982/1983), Hasofer and Lind (1974), Li and Lumb (1985), Madsen et al. (1986). Generally, the problematic feature of these techniques is the (in)accuracy. Research was then focused on development of so called advanced simulation techniques which concentrates simulation into failure region (Bourgund and Bucher1986, Bucher 1988, Schueller et al.1989, Schueller 1998). In spite of the fact that they usually require smaller number of simulations comparing pure Monte Carlo (thousands), an application for complex FEM problem can be crucial and still almost impossible. But there are some feasible alternatives: Latin hypercube sampling (McKay et al. 1979, Ayyub and Lai 1989, Novák et al. 1998) and response surface methodologies (Bucher and Bourgund 1987).

4.2 Latin Hypercube Sampling – small sample Monte Carlo simulation

A special type of numerical probabilistic simulation of Monte Carlo called Latin hypercube sampling (LHS) makes it possible to use only a small number of simulations (McKay et al. 1979, Novák et al. 1998). All random variables are divided into *N* equivalent intervals (*N* is a number of simulations). This means that the range of the cumulative distribution function $F(X_i)$ of each random variable X_i is divided into N intervals of equal probability $1/N$.

The centroids of these intervals are then used in a simulation process. The representative parameters of variables are selected randomly based on random permutations of integers 1, 2, ...,j , ..., N. Every interval of each variable is used only once during the simulation. For details see referenced literature. Some improvements of LHS were published recently by Huntington and Lyrintzis (1998). A relatively low number of simulations (say a ten to a hundred) for good estimates of basic statistical parameters proved to be satisfactory. From this point of view this technique is very suitable for complex finite element calculations like ANSYS modelling.

An important task in the structural reliability analysis is to determine the significance of random variables - how they influence a response function of a specific problem. There are many different approaches of sensitivity analysis, a summary of present methods is given by Novák et al. (1998).

The relative effect of each basic variable on the structural response can be measured using the partial correlation coefficient between each basic input variable and the response variable (Iman and Conover 1980). The method is based on the assumption that the random variable that influences the response variable most considerably (either in a positive or negative sense) will have a higher correlation coefficient than other variables. In case of a very weak influence the correlation coefficient will be quite close to zero.

4.3 Stochastic model

Young modulus and coefficient of thermal expansion were selected as random variables with normal distribution for the stochastic analysis. Their values are generated in the range from X_{low} to X_{high} , $m(X)$ means the mean value with the difference $s(X)$ as shown in the table1. The above described ANSYS model was analysed thirty times for different sets of input parameters.

Table 1 : Random variables

Material	X_i	X_{low}	X_{high}	$m(X)$	$s(X)$
Sandstone	$E = X_1$ (GPa)	2,00	4,00	3,0	0,500
	$l = X_2$ $10^{-5}(K^{-1})$	0,40	0,60	0,5	0,050
arenaceous marl	$E = X_3$ (GPa)	1,00	5,00	3,0	1,000
	$l = X_4$ $10^{-5}(K^{-1})$	0,60	1,00	0,8	0,100
concrete slab	$E = X_5$ (GPa)	22,00	32,00	27,0	2,500
	$l = X_6$ $10^{-5}(K^{-1})$	1,05	1,15	1,1	0,025

5 RESULTS

Results in this chapter are demonstrated for the load case warm top surface, cold bottom surface, the temperature difference is 60C. The probabilistic comparison of the principal stress in the point 1 of the current state A with the concrete slab and hypothetical state B without slab is shown in the figure 4. It can be seen that stresses in first case are significantly higher, mean value 0.6MPa with standard deviation 0.3 MPa comparing case without slab. In probabilistic sense it can be stated based on histogram, that probability of exceedence of stress 1.13 MPa is bellow 0.04. The same comparison for compression stress in the point 2 is shown in the figure 5. The results of the sensitivity analysis, e.g. the influence of random variables to principal tension stress in the point 1 is demonstrated in the figure 6.

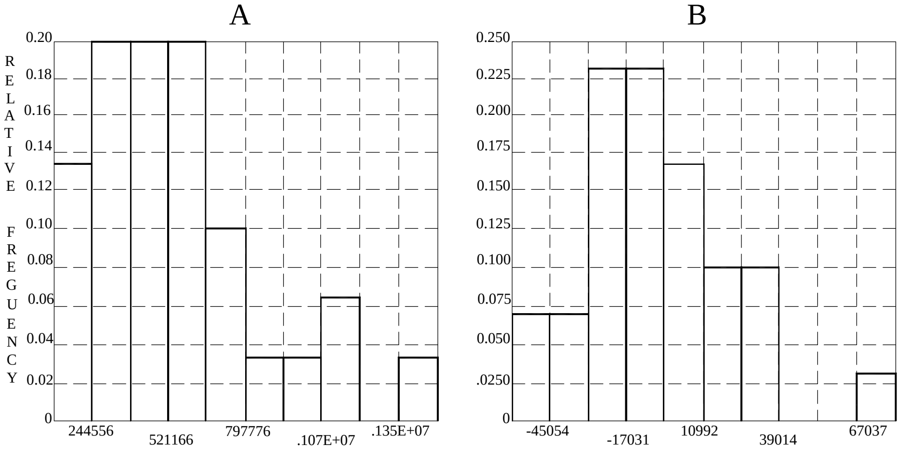

Figure 4 : The principal tension stress in the point 1 – A current state with the slab, B without slab

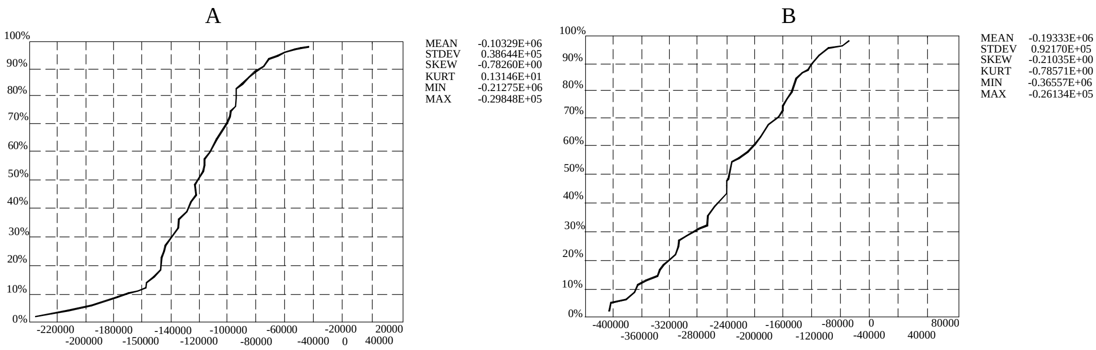

Figure 5 : The principal compression stress in the point 2 – the cumulative distribution function

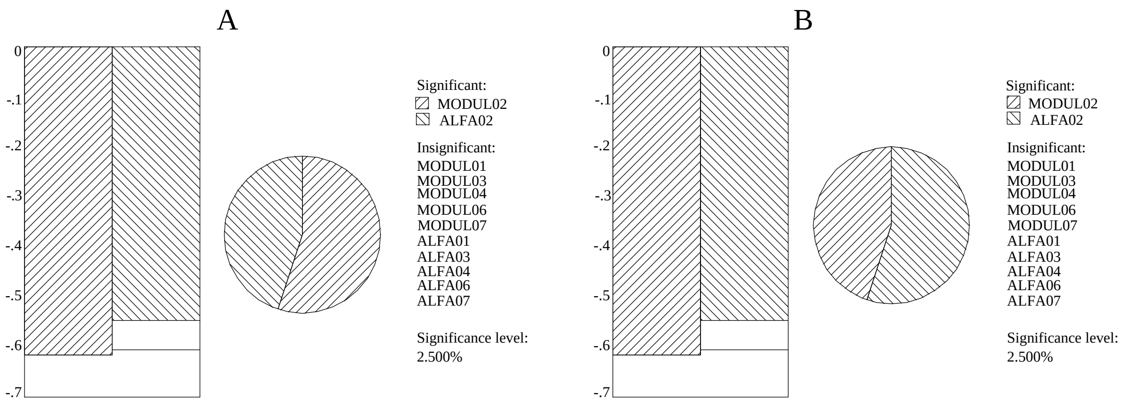

Figure 6 : The principal tension stress in the point 1 – sensitivity analysis

Numerical results confirmed the different behaviour of concrete slab and masonry structure especially under the temperature load and signified that the way of the last reconstruction was not suitable and appropriate. This is also proved by many defects that appeared only few years after the reconstruction.

6 CONCLUSIONS

The present probabilistic simulation based on the 3D finite elements model can produce accurate comprehensive results and can be directly used for the assessment of the structure and proposals for further interventions. Careful in situ measurements and material tests are, of course, necessary preliminary steps. Stochastic approach was found to be an effective and suitable way, how to deal with uncertainties of input parameters that vary within a wide range. Probabilistic approach can provide information on scatter of structural response which supports decision-making process concerning the selection of proper repair of a structure in quantitatively higher reliability level.

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