Discrete element analysis on the Sardinian "Nuraghe"

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ABSTRACT: The "Nuraghe" in Sardinia are very fascinating architectural documents of the ancient pre-roman civilisations of the Mediterranean Sea, that still need to be deeply studied and explained. Some important problems arise regarding the conservation of these architectures from the past, that are so spread over the Sardinian Island. A deeper knowledge of the materials and of the overall structural behaviour is strongly needed to assess the stability and to choose appropriate methods for restoration. Starting from a survey on the Nuraghe "Santu Antine" in Torralba (SS), a numerical analysis was performed on the stability of the dome, considering different hypothesis on the construction process. In order to the take into account the real characteristic of the dry stone masonry, made by big and stiff blocks laying together, a discrete element approach was adopted, able to model the discontinuities, that are the relevant mechanic characteristic of this masonry. A particular approach was needed to simulate in 2D the thickness variation of the dome slice studied, neglecting the stabilising contribution of adjacent slices. The results are encouraging, and demonstrate the possibility of constructing such domes without the aid of centring, if the right shape is controlled.

1 INTRODUCTION

It is well known that the Mediterranean Sea was the scene of a large network of commercial and cultural exchanges largely before the historic age. The peoples living on his shores found in the island of the Mediterranean, as well as in the Italian and Greek peninsulas, the natural bases of their travel through the sea. So important pre-historic civilisations arise also in the Sardinian Island, and have very impressive connections with other civilisations of the Mediterranean Sea.

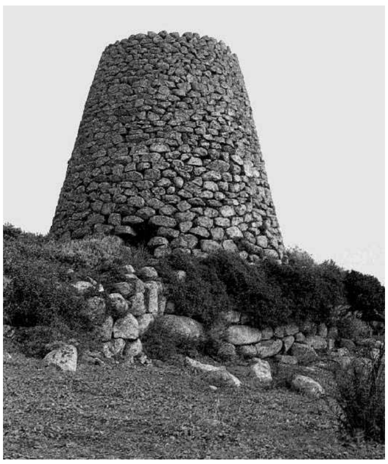

Figure1: The tholos of the Nuraghe Madrone, Silanus (NU)

One of the most impressive is the Nuraghic civilisation, that receive the name from his most important and diffused architecture, and lasts (with different fortunes) from the Ancient Bronze Age (18th cent. BC) until the roman conquest (238 BC) (see Atzeni et al. 1990).

More than 8000 "Nuraghe" are spread in the island, reaching a density of about one per square km in some zones, depending mainly on the quality and availability of the stone. They are more or less preserved in their volumetric presence, but always well recognisable almost in plan. The shape can vary from the simplest ones, in which only the *Tholos* is present, to the more complex, polylobate plan, also with external curtain walls. In any case, these massive stone monuments are characterised by a tower in the shape of a truncated cone, with an internal main room covered by a *tholos* dome that give the name to the whole tower (Fig. 1).

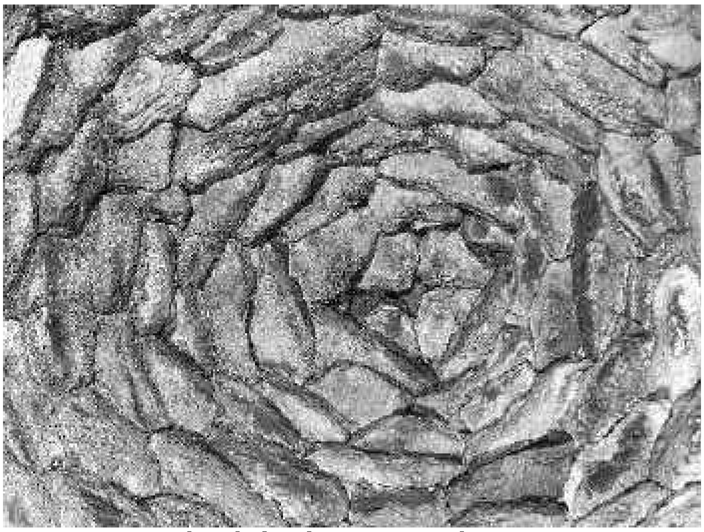

Figure 2: The Tholos dome in Nuraghe Santu Antine (view from the bottom to the top)

Figure 3: The stone vault of a corridor

The *tholos* construction is made by layers of stones disposed in concentric rings of decreasing diameter, overhanging from the previous ring, from the bottom to the top (Fig.2). A similar construction system was adopted for covering also long corridors forming a sort of corbelled barrel vault (Fig. 3).

There is a lot of examples of *tholos* domes in the pre-classic civilisations of the Mediterranean sea, like the well known “Atreus Thesaurus” in Mycenae, that can be taken as a proof of the connections among these ancient peoples; some of them survive also in the habits of the island inhabitants in the past centuries.

Despite a vast literature on the subject, the secret of the use of these massive stone buildings is not completely disclosed. In fact, they have been interpreted in many different ways, putting in evidence some particular aspect of the construction: they have been explained in turn as funeral monuments, or generally religious buildings, or as utility buildings, like grain silos or even bronze furnaces, or more often as military fortresses, but none of these interpretations alone is completely satisfactory. Maybe they were many different things together, but a definite proof is hard to be found.

In any case, these monuments are still the witness of the strength and the power of the people who build them, but they exhibit in the same time the toughness of stone buildings that last often more than 30 centuries and in the same time the weakness and brittleness of an archaeological site, giving rise to very delicate problems in their conservation.

2 THE NURAGHE SANTU ANTINE

One the most interesting and well-preserved example of nuraghe is in the territory of Torralba (SS). Recent archaeological excavations dated the monument before 1500 BC, although many authors supposed almost two different phases in the construction.

During a survey on behalf of the *Soprintendenza Archeolgica* (Archaeological Heritage Office) of Northern Sardinia, we take the opportunity to develop a graduate thesis on the conservation problems and on the structural analysis of this monument (Spina 2000).

The Nuraghe has a tri-lobated plan, with the main central tower on three levels (obtained by superimposed *tholos* domes) and the minor *tholos* on the three rounded corners, connected by an external continuous wall, where two covered corridors are running. An inner yard is delimited by the southern wall, where the main entrance opens, and the central *tholos*. The overall dimensions are nearly 39 m in width and 17.5 m in height (Fig. 4).

The external walls are made by big regular blocks of basalt, more refined in the upper layers, with a light slope from the vertical.

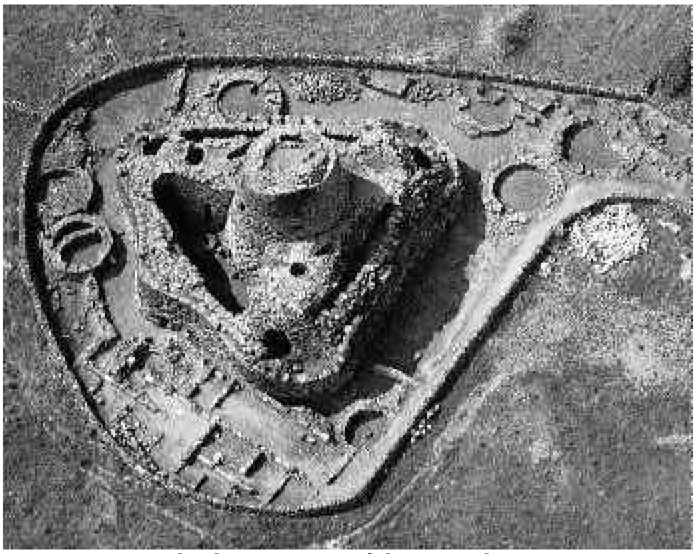

Figure 4: A bird's eye view of the Nuraghe Santu Antine

The masonry of the main *tholos* shows very regular ashlars, cut to fit the curvature of the wall. The main room has a diameter of 5.25 m and the vault is made by 21 layers of stones, reaching an height of 7.9 m. The blocks decrease in size from the bottom to the top, but are much less regular than the exterior ones. A surrounding corridor is connected to the room by four openings, leading also to the helices-shaped staircase running inside the thickness of the wall, reaching the upper levels. The second level has a circular room of 4.5 m in diameter and 5.3 m in height; the upper one has been destroyed in 1866 for building a public fountain in Torralba, and only 5 layers of stones are still in site (Contu,1993).

3 NUMERICAL ANALYSIS

The aim of a numerical analysis on a so ancient monument is on the one hand the safety assessment and the stability verification of the building itself, in order to receive some guidelines for the conservation and to avoid unnecessary interventions; on the other hand, a better understanding of the rules governing the construction, trying to contribute to the discovering of his secrets.

The first problem in modelling this masonry arises in the definition of their geometrical and mechanical characteristics. The modelling of such structures as continuous media is far from being a good representation of their real behaviour, due to the great number of discontinuities. In most of the cases, the presence of mortar in the joints is so rare that masonry can be considered as dry walls. Moreover, the difficulties of cutting regular blocks from hard stones gave origin to quite irregular dispositions.

In any case, the intrinsic nature of these construction require the use of an approach able to take into account the strong discontinuities that are present in the stone walls, so that a discrete element approach was preferred to a more classic finite element one. In fact, although both rock masses and masonry structures can be modelled in a discrete manner using FEM and BEM approaches, the description of discontinuities is usually difficult and there are often restrictions to the degree of deformation permitted. On the other hand, DEM are generally more tailored for problems in which large deformations and displacements are involved, and many material discontinuities, placing a special emphasis on how contact is handled.

3.1 Distinct Element Code

The name "discrete element method" applies to a computer program only if: (a) it allows finite displacements and rotations of discrete bodies, including complete detachment; (b) it recognises new contacts automatically as the calculation progresses.

The term "distinct element method" was coined by Cundall and Strack (1979) to refer to the particular discrete element scheme that uses: (i) deformable contacts and (ii) an explicit, time-domain solution of the original equations of motion.

In the followings, a particular commercial code originally formulated by Cundall (1971) and further developed in Lemos et al. (1985) was applied to the stone masonry of the Nuraghe.

The Universal Distinct Element Code (*UDEC*) is a 2-D numerical program based on the distinct element method for discontinuum modelling. *UDEC* simulates the response of discontinuous media subjected to either static or dynamic loading.

The discontinuous medium is represented as an assemblage of discrete blocks. The discontinuities are treated as boundary conditions between blocks; large displacements along discontinuities and rotations of blocks are allowed. Individual blocks behave as either rigid or deformable material. Deformable blocks are subdivided into a mesh of finite-difference elements, and each element responds according to a prescribed linear or non-linear stress-strain law. The relative motion of the discontinuities is also governed by linear or non-linear force-displacement relations for movement in both the normal and shear directions.

3.2 Calculation basis

In the distinct element method, a rock mass is represented as an assembly of discrete blocks. Joints are viewed as interfaces between distinct bodies (i.e., the discontinuity is treated as a boundary condition). The contact forces and displacements at the interfaces of a stressed assembly of blocks are found through a series of calculations that trace the movements of the blocks. Movements result from the propagation through the block system of disturbances caused by applied loads or body forces. This is a dynamic process in which the speed of propagation depends on the physical properties of the discrete system.

The dynamic behaviour is represented numerically by a timestepping algorithm in which the size of the timestep is limited by the assumption that velocities and accelerations are constant within the timestep. The distinct element method is based on the concept that the timestep is sufficiently small that, during a single step, disturbances cannot propagate between one discrete element and its immediate neighbours. This corresponds to the fact that there is a limited speed at which information can be transmitted in any physical medium.

The calculations performed in the distinct element method alternate between application of a force-displacement law at all contacts and Newton's second law at all blocks. The force-displacement law is used to find contact forces from known (and fixed) displacements. Newton's second law gives the motion of the blocks resulting from the known (and fixed) forces acting on them. If the blocks are deformable, motion is calculated at the gridpoints of the triangular finite-strain elements within the blocks. Then, the application of the block material constitutive relations gives new stresses within the elements.

To better explain how discretization proceeds, start considering the one-dimensional motion of a single mass acted on by a varying force, $F(t)$. Newton's second law of motion can be written in the form:

$$\frac{d\dot{u}}{dt} = \frac{F}{m} \tag{1}$$

where $\dot{u}$ is the velocity and m is the mass. Using a central finite difference scheme, velocity is calculated as:

$$\frac{d\dot{u}}{dt} = \frac{\dot{u}^{(t+\Delta t/2)} - \dot{u}^{(t-\Delta t/2)}}{\Delta t} \tag{2}$$

so that, substituting left hand side of Eq. (1) and rearranging terms, the new velocity can be calculated as:

$$\dot{u}^{(t+\Delta t/2)} = \dot{u}^{(t-\Delta t/2)} + \frac{F^{(t)}}{m}\Delta t \tag{3}$$

With velocities stored at the half-timestep point, it is possible to express displacement as

$$u^{(t+\Delta t)} = u^{(t)} + \dot{u}^{(t+\Delta t/2)}\Delta t \tag{4}$$

so that new contact forces can be calculated based on the updated position and a new timestep can start.

4 SELECTED VERIFICATIONS

Some selected problems were chosen in order to verify the possibility of analysing these massive stone buildings with modern calculation techniques, and to compare the numerical prediction with the physical evidence.

The first problem is the stability analysis of a stone wall with openings, that gave the opportunity of testing some general features of the code, like the use of deformable vs. rigid blocks, the choice of material properties and of the appropriate time step. A particular aspect was investigated, that is the capability of reproducing also some details of stress concentrations inside the blocks, giving rise to the cracking behaviour of the stones.

The second problem afforded, structurally more complex, was the stability analysis of the *Tholos* dome, taking into account in particular the different phases of the construction process, simulated with modifications both of the model and of the loads.

4.1 The north-east wall

A photographic survey of the north-east wall was used as a basis for drawing a wire-frame representation of the stones forming the blocky structure (Figs. 5,6) that also constitutes the 2 D model studied in the analysis, loaded by the dead weight of the same stones.

Figure 5: Photographic survey of the north-east wall.

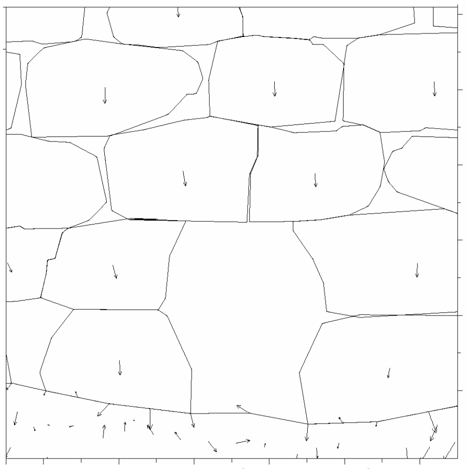
Figure 6: Cracked configuration of the lintel and velocity plot at the end of the analysis.

Table 1: Material properties

stone (basalt)	
density	2100 kg/m^3
compression strength	17 MPa
tension strength	1.7 MPa
bulk modulus	27.77 GPa
shear modulus	20.83 GPa
soil	
density	1600 kg/m^3
bulk modulus	4.5 GPa
shear modulus	4.0 GPa
joints	
friction angle	30°
normal stiffness	50 GN/m^3
tangential stiffness	20 GN/m^3

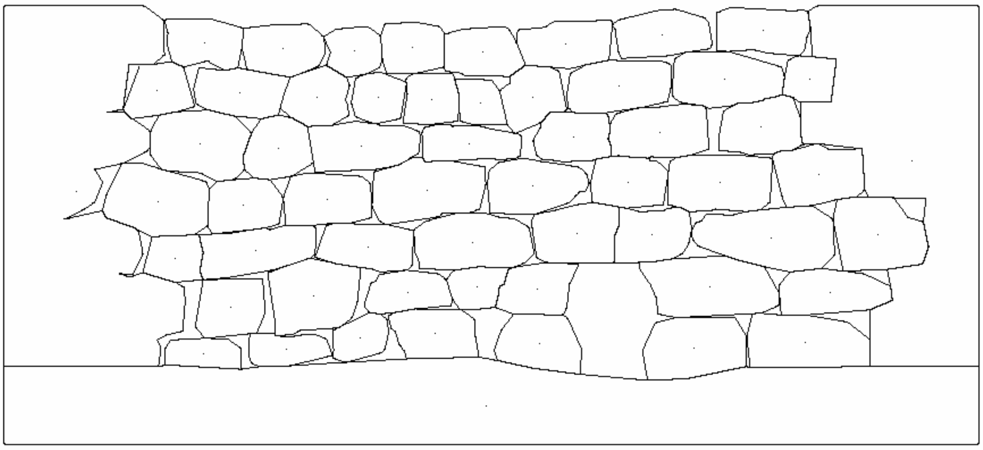
Figure 7: Wire-frame model of the north-east wall.

In the analysis, three different approaches were adopted: in the first, the blocks were considered perfectly rigid; in the second, all the blocks were assumed deformable; in the third, only selected blocks were assumed deformable. The soil beneath the wall was always assumed deformable, in order to allow some movement of the blocks. The mechanical properties of the materials (taken form the literature) are reported in Table 1.

The two parts of the cracked lintel were considered first stuck together, assigning very high tension strength to the joint representing the crack; then the strength was reduced until the joint opens under the gravity loads.

The crack open for a tension strength of 21.4 MPa for the first model and 20.7 for the second. The third model (partially deformable, to reduce computational time) gives an intermediate result of 21 MPa. These values are too high, compared to the supposed properties of the stone, indicating that the lintel can not stay integer in the actual situation of the wall, or else that this kind of analysis is not enough accurate at the stress level to catch the phenomenon of crack propagation. In any case, in the cracked configuration the lintel (Fig. 7) results stable, excluding the necessity of a structural intervention.

4.2 The Tholos Dome

Perhaps the most appealing structural problem of the “Nuraghe” is the study of the *tholos* dome.

Many authors suggest that the tholos construction can exploit the ring effect that the stones exert one against the other on the same layer when they are trying to fall inside (Cavenagh &

Lauxton), although this effect has been somewhat exaggerated. In any case, it is quite difficult to imagine the system adopted by the ancient builders to erect these monuments. The cantilever effect, if cleverly controlled, can allow to form the dome shape without the aid of scaffolds?

The first problem in the model was the simulation of the varying thickness of the dome in a 2-D model, also neglecting the contribution of the ring effect, as pointed out already the Poleni's study of on the St. Peter's dome (Fig. 8). The solution chosen was to assign different mass densities to the blocks, in proportion to the distance of their gravity centre from the axis of the dome (and so to the thickness of the dome slice in this point).

Many other aspects are to be considered: only the inner geometrical shape of the dome can be reproduced, being unknown the deepness of the stones; the same problem arise in the thickness of the external leaf and in the presence of the backfill. So that the geometrical model can be only a trial reconstruction of the cross-section, based on the examination of similar tholos partially collapsed, with a much more regular blocky structure represented in Fig. 9 (where the backfill is made by smaller deformable blocks of reduced density).

The first model analysed shown that the dome slice remain stable when the gravity load is applied, without the help of the ring effect (Fig. 9). This result can be confirmed by many examples of tholos domes partially collapsed and still standing.

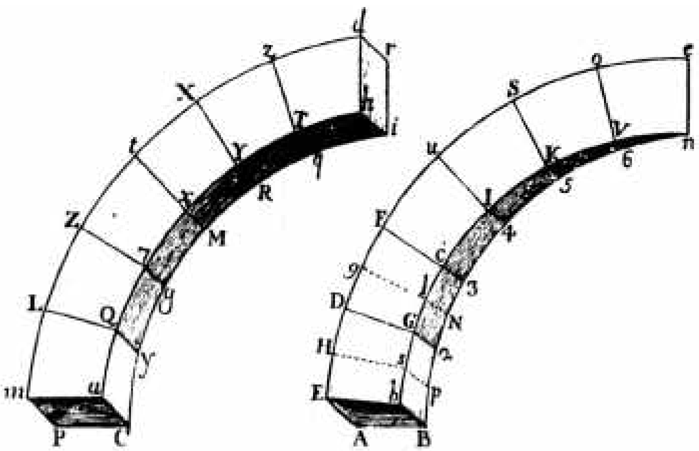

Figure 8: Half an arch compared to a dome sector, from Poleni (1748)

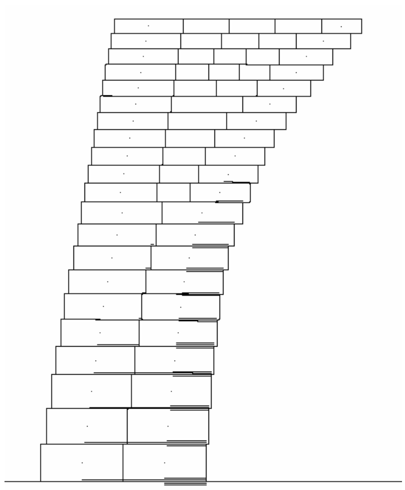

Figure 9: Contact forces for the dome slice after gravity stabilisation.

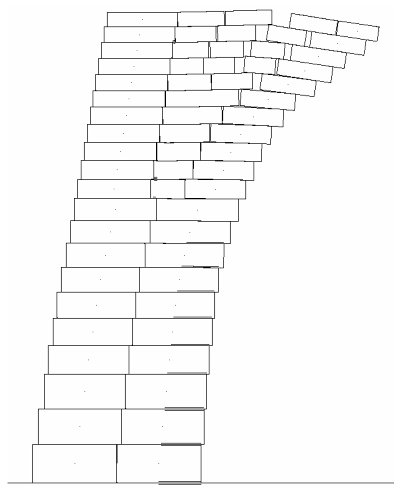

Figure10: Collapse mechanism for the model of barrel vault with friction angle <3°

Another aspect of the problem is the effect of the friction between blocks. For the dome slice, decreasing the value of friction angle from the nominal value of 30° to a null value do not produce any remarkable effect on the stabilisation; this is not completely true for a barrel vault of the same cross section, that reach the collapse for a friction angle of 3° (in this test the blocks of the same material have constant density).

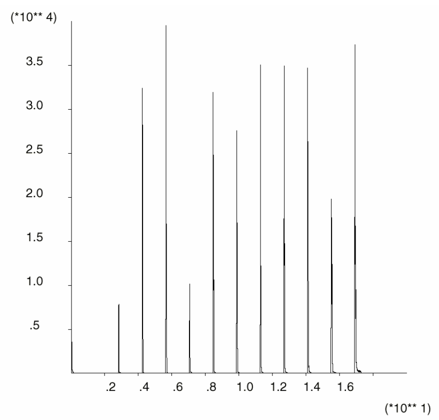

Figure11: Unbalanced forces during the layer's superposition

Finally, the superposition of the blocks during the construction was simulated, activating one layer at time (starting from the 10th) and waiting until complete stabilisation under gravity force before the activation of the next layer. The history of the "unbalanced force" (that include inertia forces) during the analysis is reported in Fig. 11. The contact closure vary during the analysis as shown in Fig 12 and the final configuration is lightly different from the one in Fig. 9, but the dome is still stable, provided that the backfill and the external wall are built together with the internal dome.

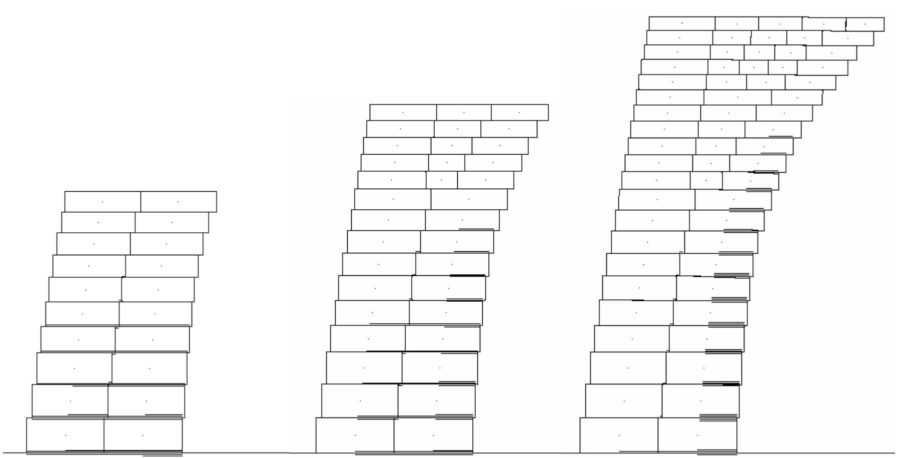

Figure 12: Contact stresses during construction phases

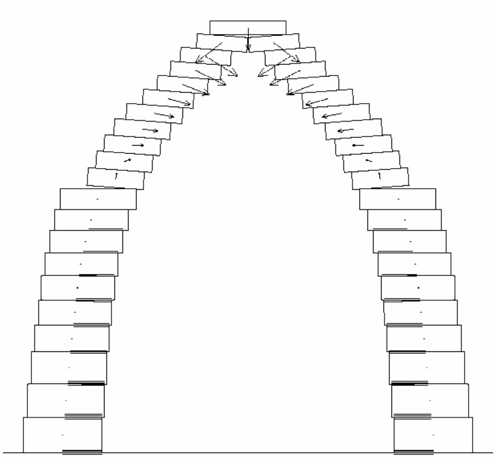

Figure 13: Velocity vectors and contact closures at collapse for the dome built without backfill.

The importance of the backfill is demonstrated by another simulation, where a dome sector was analysed together with its symmetrical counterpart in the absence of the backing structure.

If the ring effect is neglected, (or not effective due to the irregular cut of the blocks) the dome collapse, also with high friction angle values, as shown in Fig.13. The same experiment repeated increasing the dimensions of the blocks of the upper layers (from 10th to 21st) gave similar results, suggesting that the stability is not governed by the shape of the blocks but by the total load of the layers, and the external wall give a fundamental contribution to the stability.

5 CONCLUSIONS

Although limited to a 2-D analysis (mainly for economical reason, because a 3-D code is on the market) the Distinct Element Method seems to be an effective tool for the stability analysis of these ancient construction, focusing more on block stability and equilibrium than on accurate stress determination inside the blocks. This aspect can be critical on the study of block cracking due to changed boundary conditions, but much work is needed on this subject.

Some general consideration can be drawn on the stability of the tholos construction:

- a sector of the tholos dome can stand up alone, provided that the section of the wall is complete, including the external wall and the filling;
- the out-of-plane ring effect can be helpful, but is not necessary to the stability;
- the friction plays a role only in the case of vault of constant thickness (barrel vault cross-section) although a very small friction angle is sufficient to ensure the stability.

Further research will deal with the problem of cracking detection and with a better simulation of the tri-dimensional effects in the dome.

ACKNOWLEDGEMENTS

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