A comparative analysis of some historical stone arch bridges in Greece by two new numerical approaches

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ABSTRACT: The paper deals with a numerical comparative study for the analysis of old stone arch bridges in Greece by the use of two new numerical procedures: the first procedure is based on a new space-frame simulation (finite-element model) developed by the first two authors (M.K. & E.S.), whereas the second one is based on the inequality numerical approach, developed by the fourth author (A.L.). The comparative study of four cases (bridges in the Greek regions of Epirus, Thessaly and Thrace) shows that both approaches can be used effectively for the numerical analysis of historical stone bridge-structures, and in general of monumental structures, although the second one, as more sophisticated, is more computer-time and -space consuming.

1 INTRODUCTION

As well known, the effective static and dynamic analysis of historical masonry structures, especially for the case of the seismic excitation, requires the use of advanced computational methods and the cooperation of scientists from different disciplines, see e.g. Moropoulou et al. (1995), Durukal et al. (1998), Papastamatiou et al. (1997) and Tasios (1988). This is obviously necessary to obtain the realistic determination of the stress distribution in the masonry parts of a historical structure under static and dynamic loading, taking into account the particular characteristics of the construction, the complexity of its geometry and the foundation. However, in each case of calculation, there are various difficulties, which demand some drastic simplifications in order to idealize complicate monumental structures as relatively simple models, e.g. frames consisting of line or curved members with constant or variable cross-section.

Especially in the analysis of stone masonry bridges by the use of 2D or 3D finite element modeling, walls, piers and arches, which are the main construction parts of old bridges, are commonly described by shell or beam elements. The achievement of a model lies on the possibility to reflect the true behaviour of an old bridge without a useless increase of the computational time due to complexity of the model, and/or manual interventions for a better understanding of the bridge behaviour.

In the present paper, two numerical methodologies for the structural analysis of some historical stone bridges in Greece are briefly presented and compared.

So, first an analytical model, developed by the first two co-authors Karaveziroglou and Stavrakakis (2000) is presented, which is specific for the investigation of the static structural behaviour of old stone bridges. The proposed model is a space-frame simulation consisting of line beam elements representing the stone masonry construction. The material behaviour is initially assumed as a linearly elastic one. The interaction between soil and structure can also be taken into account by minimal modifications in the model geometry. Three examples of applications (stone bridges in Thessaly and Epirus) verify the effectiveness of the proposed model, as the numerical results are in very good agreement with the present state and the cracking pattern of the historical bridges.

Secondly, the inequality numerical approach, developed previously by the fourth co-author Liolios (1998) for the seismic response of monumental structures, is used by the remaining group of the co-authors (P.L., M.Y., Y.R. & M.Y.) for the comparative analysis of the same historic bridges, and moreover for another bridge in Thrace, (the Kompsatos bridge). In this second approach, monuments and old masonry structures containing structural elements with unilateral behaviour are treated numerically. The behaviour of these elements is considered as a non-convex and non-monotone one. So the problem formulation leads to a set of equations and inequalities, which is equivalent to a hemivariational inequality in the way introduced by P.D. Panagiotopoulos (1993). The problem is treated incrementally by double discretization: in space by finite elements and piece-wise linearization of the unilateral behaviour, and in time by the Houbolt method. Thus, in each time-step a non-convex linear complementarity problem is solved with a reduced number of problem unknowns.

Finally, some concluding remarks useful for the civil engineering praxis are discussed.

2 THE INVESTIGATED ACTUAL CASES OF FOUR GREEK STONE BRIDGES

The first example is the *Krania* bridge in the mountainous region of Chasia (Thessaly), which was built in the 19th century. The total length of the bridge is 28.0m and the superstructure overall width expands in 4.0m. Its arch has a span of 15.0m and a height of 6.0m over the mean water level. The natural ageing and weathering caused damages in the structure such as: loss of the mortar in the joints of the masonry as well as some longitudinal and transverse cracks in the intrados and the key. Masonry damage has been locally repaired in the past but no serious attempt for restoration has been ever made. It is also worth indicating a particularity, which is observed at a part of the intrados of the bridge. The erosion of the limestone construction material has led, in the course of time, to the creation of stalactites hanging from the arch like ice crystals.

The second example is the *Psiras* bridge, which is a double arched one located near Meteora. This structure having a very elegant form, is one of the most picturesque stone bridges in Central Greece. It was built in the 18th century by Psiras, who was a Greek guerrilla fighter against the Ottoman rule, (Spanos, 1950). The total length of the bridge is 31.0m. The main arch of the bridge has a span of 12.5m and reaches a height of 6. 2m approximately. This bridge shows local damage of its stone masonry. Some unwise interventions in the past, -see Karaveziroglou & Stavrakakis (2000) -, by the use of reinforced concrete at the top of the parapets and the deck surface, caused more damage (structural and aesthetic).

The third example is the *Porta* bridge-see Karaveziroglou & Stavrakakis (2000), and Karaveziroglou et al. (1997) for dimension details-, which was built in the 1514 AD and till 1936 it was the only link between the Thessaly fields and the mountainous villages of Pindos. The imposing beauty of this bridge, owing to the lofty height and its superb location, almost overshadows its remarkable technical character. The total length of the bridge is 68m. The circular arch of it has a span of 28.73m and reaches a height of 12.7m over the mean water level. The superstructure overall width expands in 2.65m. Deterioration of the mortar in the joints of the stone structure, falling of stones in the intrados of the arch, moisture effects on the stones, growth of plants and algae in the joints between stone blocks causing cracks or splitting of the bed-joints and a large longitudinal crack in the intrados of the southern part of the bridge, at a distance of 2.0m from the basis of the arch up to 9.0m approximately, consist the damage of the bridge, see Karaveziroglou & Stavrakakis (2000). An enlargement of the foundation of the southern abutment, repair of the parapets and some unwise filling of a part of the large longitudinal crack in the south of the arch with cement mortar are the main intervention works, that have been undertaken in the past.

Finally, the fourth example is the *Kompsatos* bridge in Greek Thrace (Zacharopoulos et al. 1996). River Kompsatos is between Xanthi and Komotini (north-east part of Greece). The bridge is located in a distance of 32km in the east of Xanthi and had initially three arches with a total length of about 68m. Today the western arch is destroyed. The middle arch has 12m height and 21,80m length, and the east arch has 17m length. The total structure is very impressive due to its resistance and the perfect art of stone building. It has been builded probably in the beginning of the 18th century , on the ruines of an ancient Thracian bridge, which had been also used during the Roman and Byzantine periods.

For the finite element modelling of the above first three stone bridges (Krania, Psiras, Porta) the following general remarks have been taken into account in Karaveziroglou & Stavrakakis (2000):

A. In the intervention project for the restoration – strengthening of a stone bridge they must be estimated, among others: 1) the physical and mechanical characteristics of the building materials, 2) the state of masonry degradation, 3) the cause of cracking and deformation of the superstructure, 4) the properties of the foundation soil, 5) the intervention works undertaken in the past, and 6) the structural analysis.

B. As far as it concerns the analysis of a bridge, it can be carried out by the use of proper approaches to represent the bearing system and by the choice of suitable techniques to convert the actual reality of the structure into computer codes. As it is already mentioned above, the appropriate model in the finite element method should describe as much as possible structural and architectural details of a building. Stone bridges, always of arch type, have an architectural form in which the structural conception is very clear when comparing with the one of other historical buildings.

C. The geometry of a stone arch bridge consists commonly of the following parts: 1) the arch ring, 2) the spandrel walls, 3) the fill, 4) the deck surface, 5) the parapets, 6) the abutments, and 7) the piers (in case of multi-arch bridges).

D. Live vertical loads (for pedestrian crossing) had been considered as uniformly contributed forces due to the height of the structural part (masonry and fill) existing between the extrados of the arch and the deck surface of the bridge. The seismic effects were computed according to the current Greek code on seismic actions, which is in agreement with the international recommendations in the field.

3 THE SPACE-FRAME APPROACH BY KARAVEZIROGLOU & STAVRAKAKIS

3.1 General

The space-frame approach, based on the finite element method, is fully presented in the paper of Karaveziroglou & Stavrakakis (2000) and has been initially developed for the analysis of three old stone bridges in Greece. These bridges are representative of a large amount of similar old stone structures scattered throughout the mountainous Greece. They survived till today bypassing floods, earthquakes and wars. However, there are a variety of defects reflecting to their considerable age. The importance of these bridges, as part of the transportation infrastructure in the country, has been lost forever and nowadays they are only used for pedestrian crossing. However, for their form and their remarkable technical character they have gained recognition as masterpieces of the architectural heritage in Greece, see Galeridis et al. (1995), Karaveziroglou et al. (1997), Mantas (1984), and Zacharopoulos et al. (1996).

The proposed analytical model by Karaveziroglou & Stavrakakis (2000) is based on beam or shell finite elements, which are commonly used to represent the bearing system of stone bridges. The application of such a modeling in the study of some historical bridges is discussed in Karaveziroglou & Stavrakakis (2000) for the case of two-dimensional finite element modeling. The use of linear elements in a 2-D computer code leads to an uncomplicated idealisation of the structure and facilitates its strength analysis. Assuming that the thickness of its element is constant – equal to the whole thickness of the cross-section of the bridge, a pseudo 3D- analysis can be obtained. However, such a computational model approaches the behaviour of the bridge, in any way more close than the beam element plane frame model, presented in the foregoing pages, but it cannot describe the stress distribution in the interior of the stone masonry. For this purpose a refined evaluation of the bridge structure can be applied as the one proposed in Karaveziroglou & Stavrakakis (2000).

3.2 Application of the space-frame approach to the Porta bridge

Due to space limitations, the application of the space-frame approach to the Porta bridge only, in comparison to the inequality approach, will be briefly presented in the present paper. For the complete space-frame analysis of the historical stone bridge of *Porta*, , as well as for dimension details, see Karaveziroglou & Stavrakakis (2000) and Karaveziroglou et al. (1997). The typical construction style of the Porta bridge is shown in Fig. 1.

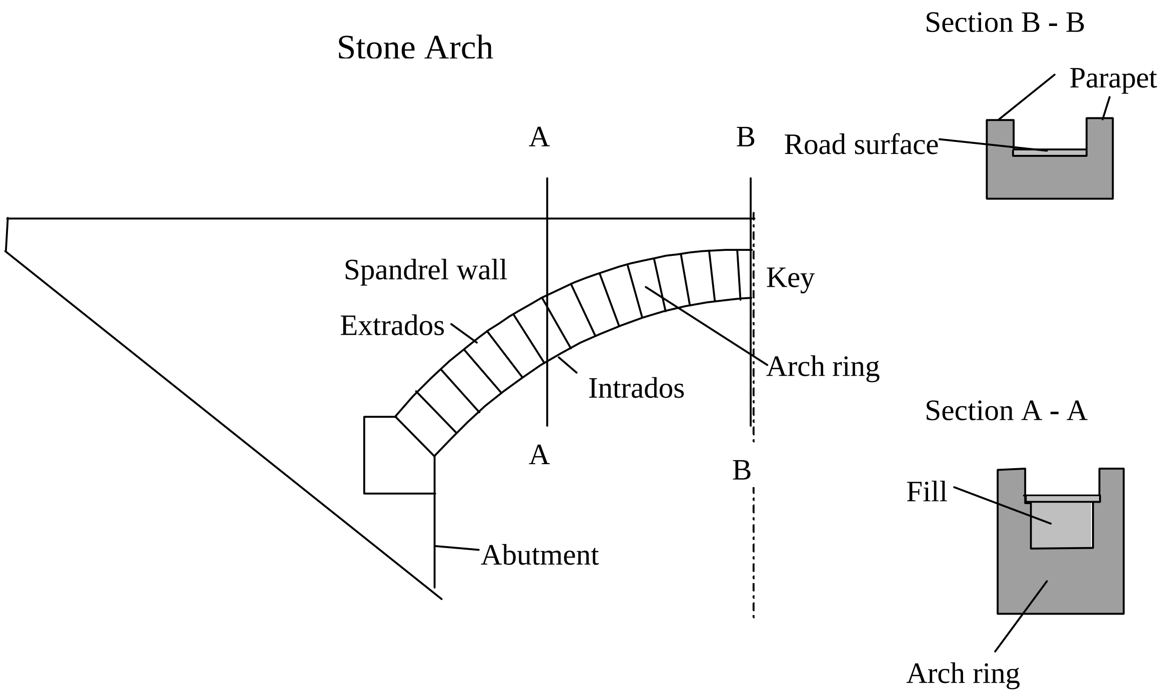

Figure 1 : Typical stone arch bridge (Porta) construction

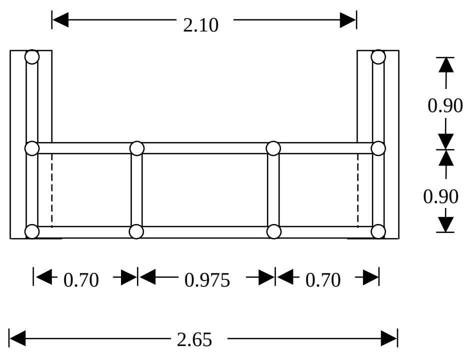

Figure 2 **:** Plane frame idealisation of the cross-section at the arch key of the Porta bridge

The space frame consists of 427 joints and 1107 beam elements. The vertical beams at the boundaries of the stone masonry are fixed in the ground. Figure 2 shows the arrangement of the beam elements in the cross-section at the key of the bridge. The space frame model of the bridge shows no remarkable tensile stresses in any of its beams where it represents the undamaged state of the bridge under vertical loading (combination of dead and live loads). The model under these conditions reflects the state after the completing of the bridge construction (16^{th} century).

An attempt to check the influence of the scouring of the original foundation of the bridge on the superstructure led to a modified model, in which the boundary conditions have been changed. It should be noted that nowadays the lowest northern part of the arch ring lays under water. In the new model the external two of all four vertical beams, that connect the intrados of north part of the arch ring with the ground, do not exist. The analysis results are similar to these obtained by the first space frame model (no significant tensile stresses in the model beams). The influence of a change in the boundary conditions at the southern part of the arch ring can be expressed in a further modification of the model by removal of the external two vertical beams connecting the southern part of the intrados with the ground. An enlargement of the foundation at the southern part of the bridge that has been undertaken in the past, assures that scouring problems existed there for several decades.

3.3 Some results of the Porta bridge analysis

The new model for the analysis of the bridge under vertical loading has shown that a new stress distribution occurs in the structure. So, by this assumption some horizontal beams of the space frame, representing the central third of the intrados in the southern part of the arch ring (Fig. 3a), develop significant tensile axial forces.

All the plots cannot be shown here due to lack of space, but representative results are shown. In Figures 3b and 3c the axial forces of the horizontal beams representing the central third of the southern part of the arch in the section A-A as well as along its intrados are shown. It is worthy to note that behind the section A-A no tensile axial forces occur in the frame beams, showing that the extension of the crack in the interior of the masonry is quite large.

The results of analysis of the bridge, using the space frame model indicate that the scouring of the foundation of the arch ring at the south of the bridge caused the structural cracking along its intrados without any change in the loading conditions of the bearing structure. It can be clearly seen in Fig. 3d in which the longitudinal crack in the intrados of the arch ring is depicted.

Finally, photographic documentation of the longitudinal crack shows that the model has the potential to describe the bridge behaviour.

4 THE INEQUALITY NUMERICAL APPROACH BY LIOLIOS (1998)

4.1 General

The inequality numerical approach for the analysis of historical structures has been developed by the third co-author Liolios (1998) and is based on the finite element method (discrete models, see also Lemos, 1998) by using concepts of the convex (or in general non-convex) analysis and optimisation algorithmes - see also Panagiotopoulos (1993). By this approach is taken into account, in a realistic way, the unilateral and the non-smooth behaviour of structural elements contained usually in historical structures, such as diagonal struts simulating masonry walls, rod (or cable) elements strengthening the aseismic capacity, interface elements simulating the soil-structure interaction and the seismic pounding between adjacent parts of monumental complexes (see for further details Liolios, 1991).

4.2 Mathematical formulation of the inequality approach

As it is fully described in Liolios (1998), by piece wise linearizing the unilateral and the non-smooth behaviour, linear complementarity constitutive relations of the following form hold for the unilateral elements:

$$w_j \geq 0 , \qquad r_{jN} \leq 0 , \qquad w_j . r_{jN} = 0 , \qquad \text{(1a,b,c)}$$

where w_j are non-negative multipliers. Further, for the assembled discretized structure, a nonconvex linear complementarity problem of the following form is eventually formulated:

$$\underline{v} \geq \underline{0} , \quad \underline{D}\,\underline{v} + \underline{d} \leq \underline{0} , \quad \underline{v}^{T} . (\underline{D}\ \underline{v} + \underline{d}) = 0 . \qquad \text{(2a,b,c)}$$

This problem is solved by using available optimization codes of non-linear mathematical programming, see e.g. Panagiotopoulos (1993) and Liolios (1991, 1998).

4.3 Some applications of the inequality approach

The above inequality approach, which is mathematically more sophisticated in comparison to the space-frame approach, has been applied, among other cases (see e.g. Liolios 1991, 1998 and references given therein), for the static and dynamic analysis of the Porta bridge (Galeridis et al., 1995) and of the Kompsatos bridge in Thrace (Zacharopoulos et al., 1996). Due to space limitations, it is here commented only that the computation has given results that are in a very good agreement with those obtained by the space-frame approach. More specifically, as concerns the stress distribution shown in Fig. 3b,c and the crack distribution shown in Fig. 3d, the results of the inequality approach are about 10-15% greater than the corresponding ones of the space-frame approach. But it has to be stressed that the computation time and the required memory space for the inequality approach are much more (about 80%) than those of the frame-approach. The same holds as concerns the comparison of the inequality approach to other trial-and-error approaches used for similar problems, see e.g. Lemos (1998) and Pegon and Pinto (1998).

5 CONCLUDING REMARKS

The usual 2D representation of bridges by the use of beam or plane finite elements leads to bearing systems, which, inspite of their complicate calculations, cannot describe sufficiently the real state of old masonry bridges. On the contrary, the analytical model proposed by the frame-space approach in Karaveziroglou & Stavrakakis (2000) could be taken as a contribution to the realistic static analysis of stone arch bridges. This modeling has already given a satisfactory understanding of the real state of a stone bridge.

More specifically, the results obtained from the comparative analysis of the historical arch stone bridges of Porta and Kompsatos by both, the space-frame approach and the inequality approach, are very encouraging. It must be stressed that these results are in a very good agreement with the in situ observed cracks. Finally, the inequality approach in the one hand is more sophisticated from the mathematical point of view, but on the other hand, as the comparison study has shown, is more computer time and space consuming.

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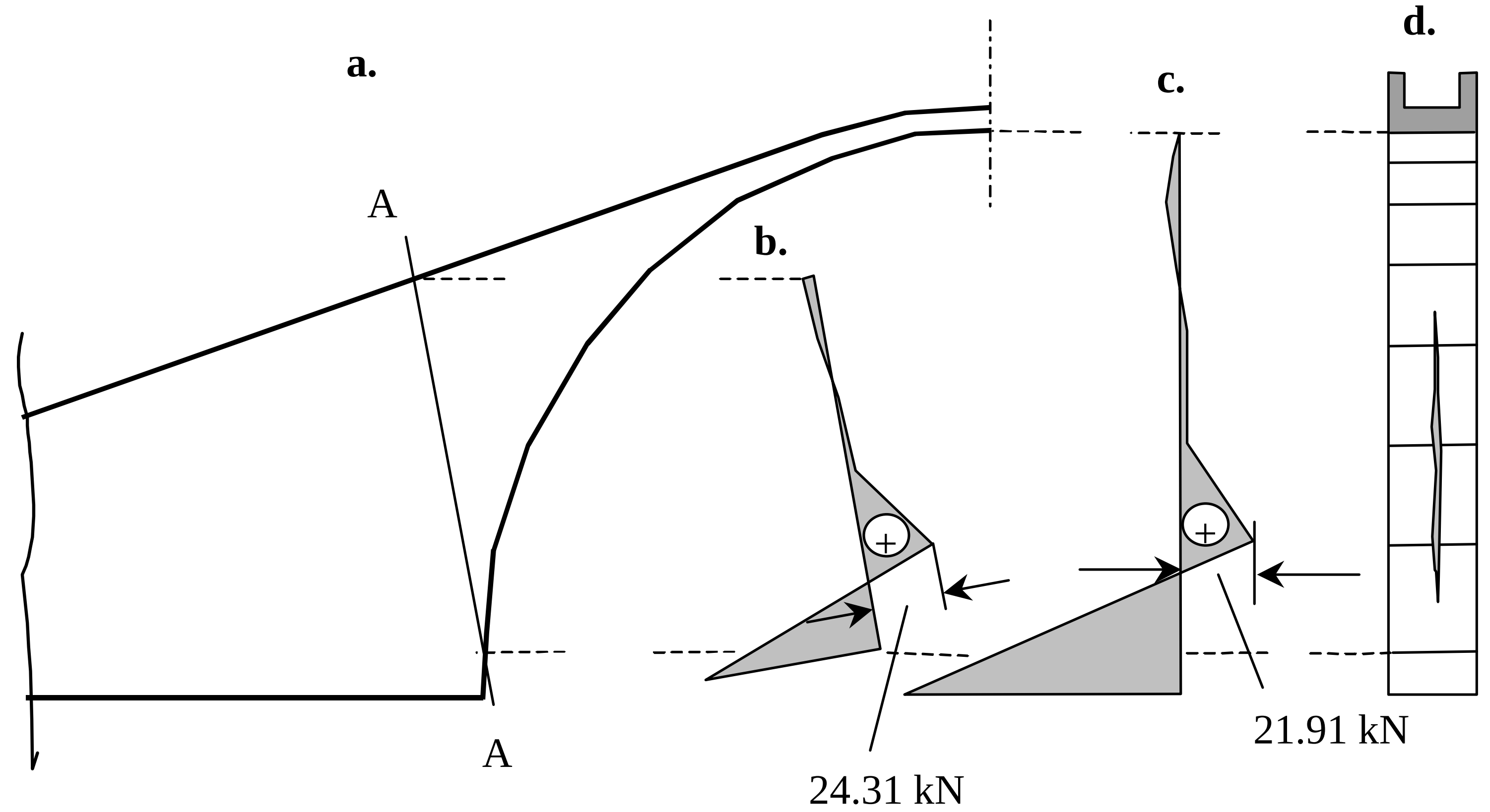

Figure 3   Some results of the analysis of the Porta bridge