A safety method in static analysis of block masonry

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ABSTRACT: Many difficulties arise in defining reliable methods of analysis for historical masonry structures. In this paper we are interested in overcoming these difficulties by working with simple models based only on a few very stable parameters, proved that they could generally be used with safety, and we refer to the class of dry block masonry structures with unilateral and frictional constraints. The setting up of a safe rigid-plastic model which also accounts for irregularities both of the shape and the laying of real masonry structures is herein proposed in order to treat these frictional materials within the framework of the standard limit analysis. A simple model related to a particular class of masonry wall, subjected to in-plane traction forces, is here analysed.

1 FOREWORD

1.1 The proposal of a new working method for the analysis of block masonry

It is well known that reliable methods of analysis and safety criteria for masonry structures cannot generally be defined with accuracy. This is because of the difficulties in finding reliable local compatibility conditions, due to irregularities and lack of continuity arising from dislocations, cracks, voids in vertical joints, etc.

Many researchers have tried to overcome these difficulties by modelling through a complex adaptation of many parameters to experimental results. However, especially for block masonry, this working method might be inappropriate for the following reasons:

1a) the validity of such experimental results holds only for particular cases, as we cannot have general standard situations;

2a) the models are nearly always based on a geometric regularity that does not generally occur in reality. We will show that for block masonry a slight variance in its regularity may give rise to a great change in the static condition of a structure.

As a consequence:

1b) it is often better to work with simpler models based only on a few very stable parameters, proved that they could generally be used with safety. And this even if they may appear at first glance to be very far from reality. It is a normally accepted rule in modern philosophy of science that repetitiveness can be achieved only by simplifying and eventually modifying the complexity of a situation, without trying to imitate it too closely;

2b) we must account for such real variances in regularity that cannot be valued by present computation tools. With this aim, and in favour of safety, we must try to identify such combinations of irregularities in order to define disadvantageous conditions. As will be seen, the achievement of this goal depends on certain choices of approximation to be made for the sake of reliability that sustains this research.

In this paper we are interested in working inside this conceptual scheme, with reference to a particular class of masonry structures, e.g. dry masonry structures made with parallelepiped blocks.

We take particular account both of the reliability and the simplification of the model.

Therefore, the lack of reliability due principally to difficulties in finding local compatibility conditions has led us to work with reference to the class of the statically admissible solutions. Thus, we will look to models that could easily be used in static limit analysis.

In this field, the simple well known Heyman's no-traction hypothesis is generally accepted. And so do we here. On the other hand we cannot add to it the hypothesis of isotropic homogenous material because it appears too limited for block masonry analysis, as it does not take into account the fundamental influence both of block shape and block organisation: a very important issue, already known to masons in ancient times, and still not completely understood.

It is however agreed that a discrete element method must be used for block masonry structures.

Masonry is often characterised by large dead loads and consequently by large and generally very reliable friction forces. Therefore these forces cannot be ignored and must necessarily be included in the model. We assume that friction obeys the cohesionless linear Coulomb's law. This is done to point out what is definitely guaranteed for these structures. Incidentally, it is evident that a right calibration of friction coefficients can suffice, though in a simplified way, to take into account the presence of mortar. Then the model we are going to define can be used to analyse a wider class of block masonry.

Once again according to the criteria indicated, that is first of all to guarantee the reliability of the model, we introduce two simplifying hypotheses into the model: of blocks regarded as rigid and infinitely resistant bodies and of joints opened along the two smaller block sides (Fig.1), so that no forces can be transmitted through them. The scheme in Fig. 1 is hereafter used as the reference model for block masonry.

We now deal with the second point concerning the geometric irregularities that often occur in real masonry. We refer mostly to irregularities both of the shape and the laying of the blocks. This means that it is somewhat difficult to find reliable solutions for the uncertainties at the block interfaces, due to the unknown application points of the normal forces and to the presence or not of shear forces. As will be shown, these factors strongly influence the static condition of dry block masonry, especially because of the large number of blocks involved.

We have then no alternative but to assume typical situations characterised by disadvantageous combinations of irregularities as our reference model for reality.

More specifically, this work is framed into the following three steps:

1. at the first level, after giving a brief state of the art, the main goal of this research is clarified and the safety criterion is set out by a few of general rules and assumptions. This criterion provides such reduced yield conditions for frictional systems in respect to which any statically admissible solution succeeds in assuring a safety state for such systems;
2. at the intermediate level, the significant role of irregularities both of the shape and the laying of the blocks are presented on a small scale. A simple model comprising just four blocks is analysed with all the possible collapse mechanisms involving sliding between the blocks and accounting for these kinds of irregularity. The translation of the results onto a larger scale implies making different choices, depending on each case;
3. at the last level, the particular case of a masonry wall subjected to in-plane traction forces is analysed. Here it is shown principally that the approximation that certain choices require to reduce the complexity of such problems, is rightly justified for the sake of reliability that sustains this research.

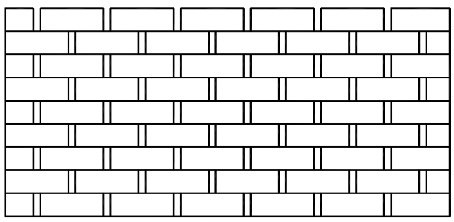


Figure 1 : Model adopted for block masonry.

2 WORKING STEPS

2.1 1st step. The model and the search for safe solutions within the limit analysis

Safety has friction to reckon with. In fact it is well known that in plastic analysis the Coulomb's law does not satisfy the normality flow rule. This means that admissible equilibrium conditions cannot with certainty assure a safety state of the structure, such as would be required. Therefore we must begin by dealing with this non-standard behaviour.

With reference to a generic non-standard material, some authors (Michalowsky and Mroz 1978, Sacchi and Save 1968, Palmer 1966, De Josselin de Jong 1964) provided the lower and the upper bounds of the exact solution by using the Radenkovic's theorems. Drucker (1954) was one of the first to deal with the uniqueness of the solution with reference to frictional materials. He pointed out the difficulty in computing dissipation due to friction in a sliding mechanism, where admissible normal forces often cannot be known, and stated a modified upper-bound condition assuming infinite friction and a lower-bound condition assuming no friction.

Later Livesley (1978, 1992), by adopting a lower-bound approach, developed a formal linear programming procedure to define the maximum load factor of two and three-dimensional structures formed from rigid blocks. Indeed, due to the non-associated flow rule, in most cases, depending on the geometry and on the friction coefficient, the computed load factor is an upper bound of the true failure load, and the associated mechanism implies dilatancy at the block interfaces.

On the other hand, Gilbert and Melbourne (1994) adopted a mechanism formulation involving dilatancy to analyse 2D multi-ring brickwork arches, under self-weight and live loads.

A further contribution, without introducing dilatancy, was given by Lo Bianco and Mazzarella (1983), followed by Baggio et al. (1991). They proposed non-standard limit analysis approaches, based on determining the minimum of a class of load factors that satisfied the kinematic and static conditions simultaneously. The associated mathematical model, however, proves rather onerous in computation, as it involves non-linear and non-convex optimisation programs.

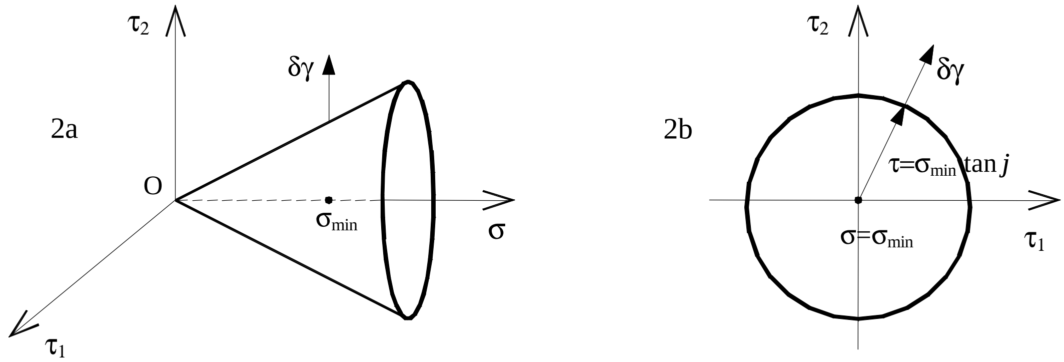
Hence in this framework it is necessary to investigate the behaviour of frictional materials better, trying to provide simple methods of analysis basically oriented to guarantee the reliability of the solutions. Indeed, the authors have done extensive work in this sense (Casapulla et al. 1996, 1998, 2000, 2001, Jossa 2000, Casapulla 1999a, 1999b, 2001).

Consider now the Coulomb's cone in Fig. 2a. It is both well known (Goyal et al. 1991) and obvious that, if normal stresses at a given point are known "a priori", in the sense that they are independent of the stress condition associated with the collapse mechanism, they can be ignored in defining the limit stress surface at this point. Hence, the cone reduces to a circle, with a constant value of limit friction stresses and with the normality rule once again guaranteed, as shown in Fig. 2b.

Obviously, in 2D problems, such as those we are interested herein, the Coulomb's cone and the circle reduce to a bilateral yield line and to a line segment, respectively.

With reference to the general case of normal stresses dependent on the stress condition, let us suppose that at each point we have succeeded in evaluating the minimum absolute value σ_{min} that the compressive normal stress can assume, that is, a lower bound of the whole class of the statically admissible compressive stresses. The corresponding circle in the plane of shear stresses, centred in this value σ_{min} (Fig. 2b), uniquely defines the limiting value of the shear stress, that is, $\sigma_{min} \tan\varphi$ and respects the normality rule. Furthermore, supposing we extend this procedure to every point, then the block masonry becomes a standard rigid-plastic material, which can now be investigated within the standard limit analysis. This is because the internal virtual work done by these limit shear stresses, related to an equilibrated solution, and with respect to the true collapse configuration, results not greater than the internal work done by the true shear stresses. As a consequence, the load factor corresponding to any statically admissible solution is not greater than the true collapse factor and hence the solution is safer than the exact one.

However, the theoretical simplicity of this procedure does not imply a practical simplicity as well, owing to the uncertainties at the block interfaces and the irregularities and defects inside the masonry. Moreover it is almost impossible to determine the minimum value of the normal stresses in the above sense at each point of the block masonry.



Figures 2a : Cohesionless Coulomb's cone. 2b : Yield domain in the plane of the shear forces.

However, although a general procedure may not be obtainable, we can still work by defining at this step certain general rules that are helpful in the subsequent step. Let us begin by characterising frictional resistance with greater accuracy.

We agree with the hypothesis that the resultant of the frictional resistance does not depend on the size of the contact surface but depends only on the resultant of the normal stresses on this surface. Moreover, in a general 2D problem without torsion, we are not interested in the distribution of the shear stresses but in their resultant on a contact surface between two blocks. Then the above criterion might be defined for each block interface with reference to the normal force N_{min} and the shear force $N_{min} \tan j$, which are resultants of the respective stresses.

Now, if we want to extend the search for this minimum resistance to a number of block interfaces, we must first make a clarifying observation.

Concerning the model adopted in Fig. 1, if we disregard the effects of forces directly applied to a generic block, it is quite evident that the sliding without hingeing of one block on another may occur only when it involves a number of block interfaces such as to separate the masonry into two parts. This is clearly depicted in Fig. 3a.

The line passing through these block interfaces is herein called the fracture line due to sliding, meaning that it may occur only when the resultant frictional resistance, linearly dependent on the total weight resting on it, has been reached.

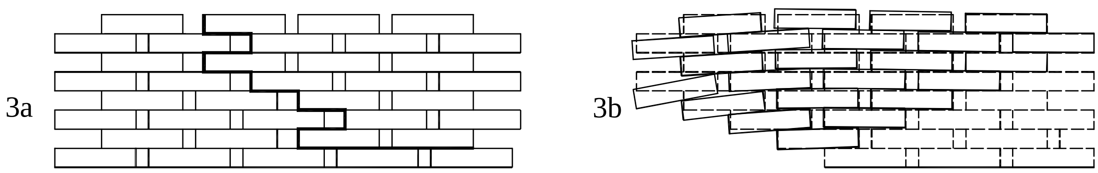
The case of sliding associated with hingeing is very different because it generally involves not a single fracture line but extended parts of the masonry, as highlighted in Fig. 3b.

Let us discuss the two cases.

In case of only sliding, we have the following two possibilities:

1) the friction coefficients are constant values along the whole fracture line. In this case the frictional resistance depends only on the resultant of the normal forces acting on all the block interfaces involved in this line;
2) the friction coefficients are variable along the fracture line. In this case, that which is more important is the distribution law of the normal forces along this line. In particular, the more disadvantageous condition might be identified by determining the value N_{min} along certain lengths of the same fracture line.

The case of sliding-hingeing mechanisms, which is more complicated because of the large number of redundancies, is currently being considered in another work. Indeed, in the following step, the case is merely outlined in the analysis of possible mechanisms of a simple pattern consisting of only four blocks.



Figures 3a : Fracture line due to horizontal sliding.3b : Sliding-hingeing mechanism.

In conclusion, with reference to the case of only sliding, the search for the minimum normal stress can be undertaken by choosing, as reliably as possible, such N_{min} as integral value on a discrete number of possible fracture lines.

To this end it might also be useful along these lines to define those values of N_{min} corresponding to equilibrated distributions of stresses not necessarily admissible everywhere in the structure. This means that one can choose to work within a class not smaller than the class of the statically admissible solutions; a limitation that is clearly in favour of safety.

2.2 2nd step. The elementary reference modules

Two reference modules of block masonry on a small scale have herein been framed to guide the subsequent analysis.

The first one is made up of four rigid blocks set out as in Fig. 4 and conceived as being extracted from the reference model in Fig. 1. The forces P_1, P_2 and P_3 represent the resultant weights of the overlaying blocks on the respective block interfaces E-F, C-D e A-B.

The possible mechanisms of only blocks 3 and 4 are briefly distinguished into the following three cases:

1) Horizontal displacement of block 3:
   1a) $P_1>P_2$; block 4 does not move: sliding occurs at the interfaces AB and CD;
   1b) $P_1<P_2$; block 4 moves integral with block 3: sliding occurs at the interfaces AB and EF.
2) Hingeing of block 3 about point B:
   2a) $P_1>P_2$; $P_1(d_1+s)>P_2d_2$; block 4 touches blocks 2 and 3 in E and C; block 4 hinges about E; sliding occurs in C;
   2b) $P_1>P_2$; $P_1(d_1+s)<P_2d_2$; block 4 moves integral with block 3 and becomes separated from block 2; absence of sliding;
   2c) $P_1<P_2$; $P_1(d_1+s)>P_2d_2$; block 4 touches blocks 2 and 3 in E and C; block 4 hinges about E; sliding occurs in E;
   2d) $P_1<P_2$; $P_1(d_1+s)<P_2d_2$; block 4 moves integral with block 3 and becomes separated from block 2; absence of sliding.
3) Hingeing of block 3 about point A:
   3a) $P_1>P_2$; $P_1(d_1+s)>P_2d_2$; block 4 touches blocks 2 and 3 in E and D; block 4 hinges about E; sliding occurs in D;
   3b) $P_1>P_2$; $P_1(d_1+s)<P_2d_2$; block 4 touches blocks 2 and 3 in E and D; block 4 hinges about E; sliding occurs in D;
   3c) $P_1<P_2$; $P_1(d_1+s)>P_2d_2$; block 4 touches blocks 2 and 3 in E and D; block 4 hinges about E; sliding occurs in E;
   3d) $P_1<P_2$; $P_1(d_1+s)<P_2d_2$; block 4 touches blocks 2 and 3 in E and D; block 4 hinges about E; sliding occurs in E.

In conclusion:

a) two possible mechanisms, each with two sliding interfaces, correspond to displacement 1);
b) three possible mechanisms correspond to displacement 2). Two of them report sliding at C or E and the third one (2b) or 2d)) does not involve sliding. However, the latter mechanism implies a strong increase of potential energy of the weights acting on block 4 and therefore it is rather unlikely to occur. This is because the collapse is defined by the minimum load factor and hence by the minimum resistance of the system;
c) two possible mechanisms with sliding at E or D correspond to displacement 3).

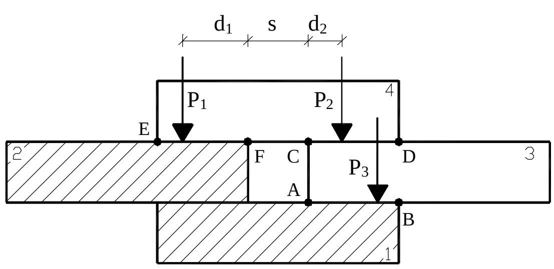


Figure 4 : First elementary module for block masonry.

With the kinematic analysis of this elementary module we wish to underline that the larger number of blocks is in a real block masonry structure the wider range of possible mechanisms may be involved. Moreover the irregularities discussed may also bring about great changes in the mechanisms, as will be shown later. Therefore, generally we have no hope of obtaining unique solutions to represent real masonry and consequently procedures of analysis finalised to find the true collapse factors are methodologically wrong.

On the other hand, the second elementary module represented in Fig 5a has appropriately been chosen in order to handle the uncertainties at the block interfaces and therefore to find the discussed N_{min} along certain lengths of a possible fracture line.

This module is a triangular scheme of blocks once again conceived as being extracted from the reference model in Fig. 1. It is based on the assumption that the blocks interact only at their outermost points of support within the horizontal joints. It is used to study the possibilities of maximum diffusion of a load Q acting on its top.

We can have two limit situations: one corresponding to zero and the other to limit shear forces.

In case of absence of shear forces the law of the spreading of Q is referred to the well known Tartaglia’s triangle. The load Q is distributed so that the generic normal force acting on the k^{th} block of the i^{th} row can be written in this form:

$$\mathrm{N}_{i,k} = \frac{\begin{pmatrix} i-1 \\ k-1 \end{pmatrix}}{2^{i-1}}\mathrm{Q} = \frac{\frac{(i-1)(i-2)...(i-k+1)}{(k-1)(k-2)...1}}{2^{i-1}}\mathrm{Q} \tag{1}$$

On the other hand, the case of limit shear forces has been analysed in a simplified way by adding the leftward shear forces related to the triangular sub-scheme of blocks 2-4-5-…11…-14 in Fig. 5a to the correspondent rightward shear forces related to the analogous sub-scheme 3-5-6-…12…-15. To this end the limit equilibrium of the single block in Fig. 5b has been assumed as reference for each block of each sub-scheme analysed. As a consequence, the total forces acting on each block will have angles $\underline{j}$ with the vertical minor or equal to friction angle j.

Such being the case, calling $S_{i,k}$ the normal force acting on the k^{th} block of the i^{th} row, when acting leftward shear forces, the total compressive force due to the symmetry will be:

$$\underline{\mathrm{S}}_{i,k} = \mathrm{S}_{i,k} + \mathrm{S}_{i,(i-k+1)} \tag{2}$$

Given then:

$$h = 2\frac{b}{a}\tan j \tag{3}$$

where *a* and *b* are the block length and height respectively (Fig. 5b), taking into account eq.(1), we have:

$$\mathrm{S}_{i,k} = \frac{1}{2}\mathrm{N}_{(i-1),k}\left(1+h\right)^{(i-k-1)}\left(1-h\right)^{(k-1)} \tag{4}$$

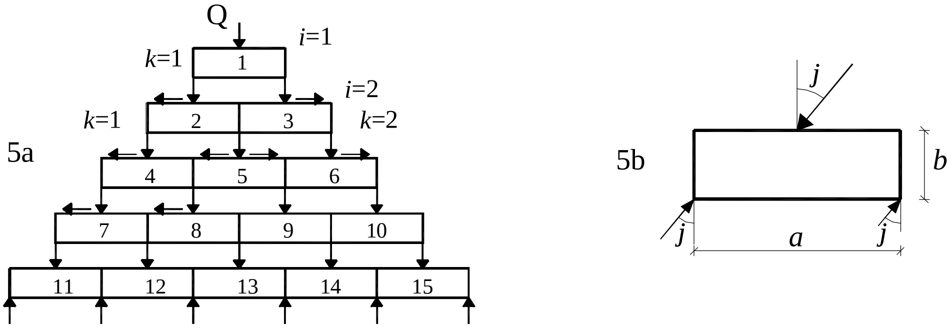


Figure 5a : Second elementary module. 5b : Limit actions and reactions on the single block.

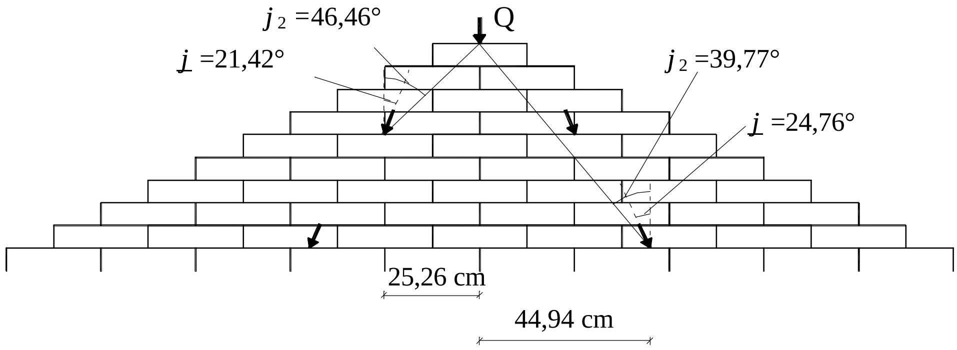


Figure 6 : Representation of angles j_2 and $\underline{j}$.

It is anyway evident that the practical assumption often used of spreading at 45° cannot generally be accepted for block masonry structures.

As a numerical example of the case of limit shear forces with j =30°, Fig. 6 sketches the resultant forces acting on the half fifth and tenth row of a scheme made from UNI blocks (6x12,5x25 cm). In the same figure, angles $\underline{j}$ defined by these resultants with the vertical are indicated together with angles j_2, defined by the line linking the application points of Q and the resultant normal force acting on a half row. Angles $\underline{j}$ and j_2 will be used in the 3rd step of this work.

2.3 3rd step. A particular case: a masonry wall subjected to its own weight and to a horizontal traction force

Consider the masonry wall in Fig. 7, simply placed on a rigid plane and subjected to a traction force F applied to a point such as to involve only sliding between the blocks. In this figure the lined blocks are assumed to be rigidly jointed to each other and to represent the transmission element for the external force (e.g. the section of an orthogonal wall subjected to seismic actions).

In this case, if we assume that the self-weight of the wall is uniformly distributed, e.g. within the hypothesis of regularity of its shape and laying, all the fracture lines shown in Fig. 7 between the vertical and line 3 will have the same probability of occurring, however much the friction coefficient changes from row to row. In particular, if the friction coefficient is constant, we can easily compare the resistances along the two lines 1 and 2, with reference to a masonry wall with *n* rows and one block in thickness. In fact, being *a*, *b* and *c* the block length, height and width respectively, *g* the specific weight of masonry, the limit traction force for both lines will be:

$$\mathrm{F} = g\,\frac{a}{2}\,bc\tan j\sum_{i=1}^{i=n-1} i = g\,abc\tan j\,\frac{n(n-1)}{4} \cong g\,abc\tan j\,\frac{n^2}{4} \qquad (5)$$

and consequently also its application point will not change (Casapulla et al. 1998).

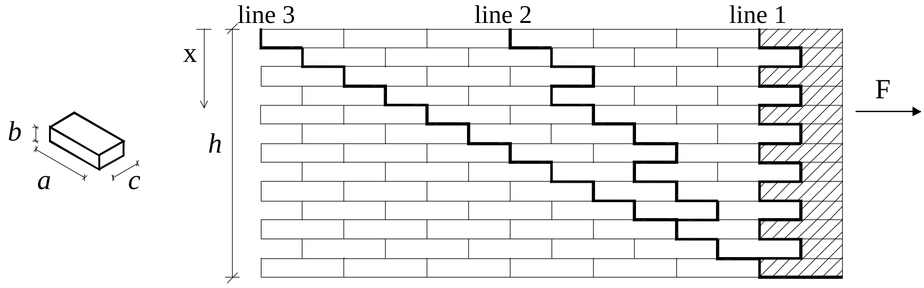


Figure 7 : The masonry wall with possible cracks.

It is worth noting that the force F is also obtainable as $Q_o \tan j$, Q_o being the weight of that part of the wall resting on fracture line 3. In other words, being:

$$\Omega = \frac{h^2 \tan a}{2} \qquad (6)$$

the area of the triangle of masonry bounded by the maximum slope (line 3) and the vertical (forming angle a, better represented in Fig. 9a), the said frictional resistance will be:

$$F = Q_o \tan j = g\, c\, \Omega \tan j \tag{7}$$

However, the real situation might be quite different. In fact, the application points of the resultants of the normal stresses between the blocks could change from interface to interface because of possible irregularities of the shape and the laying of the blocks. Fig. 8 illustrates a case of very disadvantageous irregularity that regards only the laying of the blocks. In fact, it can be observed that the weights of the lined blocks do not interact with those of the non-lined blocks and hence the shear resistance along the sub-vertical fracture line is zero. However, the same figure also shows that, in this case of only sliding, the fracture line separates the wall into two parts, as already discussed, and hence it also includes the segment B-B’, nonetheless subjected to a normal force. This segment represents the supporting base of the lined transmission element in Fig. 7.

On the other hand, these geometric irregularities being equal, the situation in Fig. 8 could be improved by the deformability of the blocks, which might allow more regular supports. Conversely, even with a good geometric regularity, an opposite situation might occur when the cohesion between the blocks implies their total separation along the fracture line in respect to equilibrium conditions.

On the bases of what has been observed so far, we may now discuss the possible cases.

Firstly let us imagine that collapse is caused by the activation of the fracture line 2 in Fig. 7, represented with line C-B in Fig. 9, inclined at $\underline{a}$ in respect to the vertical. We are interested in discussing the part of the masonry on the right-hand side of this fracture line.

In the event of regularity of contacts and of constant friction coefficients, the resultant frictional resistance along this line is still given by eq.(7), but the area of masonry now concerned will be:

$$\underline{\Omega} = \frac{h^2 \tan \underline{a}}{2} \tag{8}$$

Therefore, to determine the frictional resistance as function of the area Ω (and the corresponding weight of masonry) we increase $\underline{\Omega}$ by the coefficient $d = \Omega/\underline{\Omega}$.

This coefficient, since it expresses equivalence justified only for geometrical reasons, is still valid when we take into account irregularities in the dead-load distribution. In this case, in fact, the search for N_{min} to define a safe resistance criterion does not alter the number of interfaces involved in the fracture line, but refers only to the reduction of weight resting on them.

Such being the case, we must refer back to what was discussed in the first step of this work concerning the case of only sliding mechanisms. Here, if the friction coefficients between the blocks are constant, the irregularities count for nothing and the resistance will depend only on the value of Ω. Conversely, if the friction coefficients are variable, we must determine such a distribution law of the normal forces as to provide safer conditions, in function of this variability.

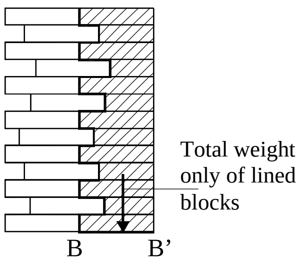


Figure 8 : Irregularity of the laying of the blocks causing a vertical fracture line.

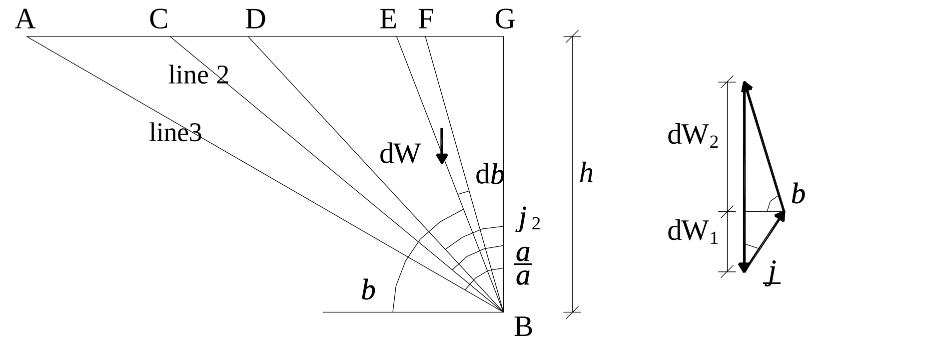


Figure 9a : Scheme of possible cracks. 9b : Distribution of dW according to angle $\underline{j}$.

Let us, then, examine the simple case of friction coefficient between the transmission element and the supporting plane less than that between the blocks. In this case the fracture line includes the inclined length C-B in Fig. 9a, with friction angle j, and the horizontal length B-B' in Fig. 8 (represented by point B in Fig. 9a), with friction angle $j*$ (with $j* < j$).

Calling $N_{C\text{-}B}$ e $N_{B\text{-}B'}$ the resultant normal reactions on the respective lengths of the fracture line, the total frictional resistance will be:

$$F = N_{C\text{-}B} \tan j + N_{B\text{-}B'} \tan j* \tag{9}$$

whose minimum value in favour of safety will clearly be given by minimising $N_{C\text{-}B}$. This minimisation can be obtained by maximising the spread of the weights of all block involved in the triangle of masonry C-B-G on the left-hand side, and with actions on the length C-B, so minimising $N_{C\text{-}B}$ and maximising $N_{B\text{-}B'}$

Then, if we suppose that the distributions of these weights do not interact with each other approximately, the two distribution laws described in the second step of this work can now be used and compared with each other, once again in favour of safety. In this way, we can now demonstrate that, for this particular case of only sliding mechanism, the distribution including shear forces results safer than the Tartaglia's distribution.

To this end let us consider the total weight W of the triangle of masonry C-B-G and the two spreading angles j_1 and j_2 (with $j_2 > j_1$), according to the Tartaglia's law (Fig. 10a) and to the other distribution (Fig. 10b) respectively. The Tartaglia's distribution gives only the vertical reactions N_1 and N_3 on the fracture line, while the other provides inclined reactions with components N_2, N_4 and $N_2 \tan \underline{j}$. It should be noted that angle $\underline{j}$ is always less than or equal to friction angle j, as previously explained, and according to the distribution law it always has such values that $N_2 \tan \underline{j}$ never exceeds the shear resistance at point B (that is $N_4 \tan j*$).

The two limit external forces F_1 e F_2 corresponding to the two schemes in Figs. 10a and 10b are:

$$\begin{aligned} F_1 &= T_1 + T_3 = N_1 \tan j + N_3 \tan j* \\ F_2 &= \underline{T}_2 + \underline{T}_4 = N_2 \tan j + N_4 \tan j* \end{aligned} \tag{10}$$

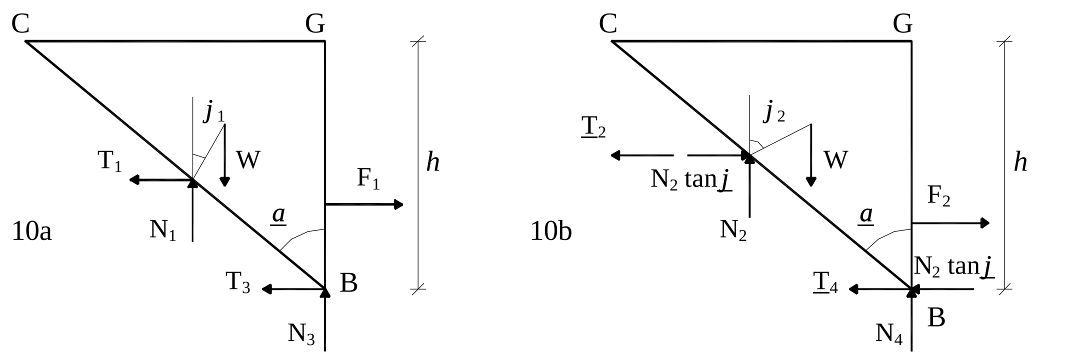


Figure 10a: Resistance along the line C-B according to the Tartaglia's distribution. 10b: Resistance along the line C-B according to the distribution including shear forces.

which, always being $F_2<F_1$, allows us to say that the distribution including shear forces is safer than the Tartaglia's. Note that in case of $N_2 \tan\underline{j} = N_4 \tan j^*$ we will have:

$$F_2 = \underline{T}_2 + \underline{T}_4 = N_2\left(\tan j + \tan\underline{j}\right) + N_4 \tan j^* \tag{11}$$

which does not change, but fully respects what has just been said.

On these bases we can now determine the minimum value of $N_{C\text{-}B}$, from now onwards referred to as N_{min}.

With reference to Fig. 9a, we may distinguish two cases:

1) $\underline{a} > j_2$

In this case all the weight of the region C-B-G rests on the interfaces of the fracture line C-B, while the weight of the region D-B-G, defined by spreading angle j_2, may result in the chosen distribution. We must also take into account that the total weight of the region C-B-G, defined by the fracture line, must be increased by coefficient ***d*** previously described. Therefore, being:

$$dW = \frac{g\, c\, h^2}{2\sin^2 b}\, db \tag{12}$$

the weight of the elementary cone of masonry E-B-F, the smallest value of dW_1 acting on the length C-B will be given by the force polygon in Fig. 9b, defined by angle $\underline{j}$: We have:

$$dW_1 = dW \frac{1}{1+\tan b \tan j_2} \tag{13}$$

and consequently, the minimum resultant normal force acting on the interfaces involved in the fracture line C-B will be:

$$N_{min} = \left[\frac{g\, c\, h^2}{2} \int_{p/2-j_2}^{p/2} \frac{1}{\sin^2 b\,(1+\tan b \tan j_2)}\, db + W_{(CBD)}\right] d \tag{14}$$

where $W_{(CBD)}$ is the weight of the region C-B-D and ***d*** is the amplifying coefficient ($d = Area_{(ABG)}/Area_{(CBG)} = \tan a / \tan\underline{a}$).

By integrating function (14) we get:

$$N_{min} = \left[\frac{g\, c\, h^2}{2}\left(1+\ln(\cos j_2)+\ln(\sin j_2)-\ln(\sin(2 j_2))\right)\tan j_2 + W_{(CBD)}\right] d \tag{15}$$

2) $\underline{a} < j_2$

In this case the part of the masonry resting on the interfaces of the fracture line C-B lies within the slope of the spreading angle and so the integral in eq.(14) must be determined for the range (p/2-$\underline{a}$) – (p/2). We get:

$$N_{min} = \frac{g\, c\, h^2}{2}\left[\tan\underline{a} + \left(\ln(\cos\underline{a})+\ln(\sin j_2)-\ln(\sin(\underline{a}+j_2))\right)\tan j_2\right] d \tag{16}$$

In conclusion, the procedure described above allows us to obtain safe values of frictional resistance such as:

$$T_{min}=N_{min} \tan j \tag{17}$$

for each fracture line and the analysis falls within the framework of the plasticity theory. This means that any statically admissible solution related to a given fracture line means that the shear resistance along such a line is not greater than the frictional resistance $T_{lim}=T_{min}$.

Let us now apply this procedure to the analysis of a masonry wall that contributes to opposing the overturning mechanism of an orthogonal wall connected to it and subjected to seismic actions.

The model is still represented in Fig. 7, but in this case the seismic force has a fixed application point. This means that the law of distribution of the normal stresses along the possible fracture line now influences the equilibrating moments too. A rigorous analysis is then quite difficult. However heuristic assessments are possible with reference to the working method described above. Moreover this analysis should also account for the rotational effects of the blocks, which produce increase of resistance due to an increase in the potential energy of the weights. In this way, if we disregard these rotational effects, the results obtained due to only sliding mechanisms are extendible to this problem, as they are largely safe.

As a numerical example consider a masonry wall consisting of UNI blocks (6x12,5x25 cm) and with specific weight g = 18 KN/m^3, in which a crack inclined at angle $\underline{a}$ = 50° in respect to the vertical, involving a height h = 3 m (equal to 50 rows), is generated.

j_2 = 29,67° is the relative spreading angle found with the procedure described in the second step of this work, while j = 30° is the friction angle. The frictional resistance along this line according to eqs.(15) and (17) is T = 8,14 kN, equal to about 67% of the resistance corresponding to the hypothesis of regular and uniform weight distribution of the wall ($Q_o \tan j$).

3 CONCLUSIONS

A new working method for the analysis of dry block masonry has been proposed in this paper in order to account for possible irregularities both of the shape and the laying of real masonry structures.

The method is based on a heuristic procedure that implies step by step some strategical choices to be made in favour of safety. In this way a safe rigid-plastic model for this frictional material has been set out to be analysed with the standard limit analysis.

The procedure shown for a masonry wall subjected to in-plane traction forces cannot easily be generalised as it is strictly related to certain particular choices, but it affords an example of the new methodology described, oriented to reduce the complexity of frictional systems for the sake of reliability.

Further development is currently in progress.

ACKNOWLEDGEMENTS

This research has been sponsored by M.U.R.S.T. 98.

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