Some considerations on the shape of the caps of vaults

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ABSTRACT: The ribs or groins of vaults are usually carefully designed in their geometry, and their geometrical layout is normally well analyzed. Less attention is usually given to the shape of the caps in between, as its understanding is not crucial for the interpretation of the architecture. The information available about the shape of the caps, either from historical building manuals or from secondary literature, in many cases appears not to be corresponding to the reality as it can be observed in historical vaults, or not describing it sufficiently.

This study is an attempt to describe their shape in regard to their construction process. A better understanding of the shape of these caps can be useful for their numerical modelling, as its shape has influence on the structural behaviour of a vault. Beyond that, such knowledge may be applied in the restoration, repairing or rebuilding of traditional or historical vaults.

1 INTRODUCTION

The basic elementary description of a cross vault is that of two intersecting cylinders. In such a first approach, the surfaces of its portions would be considered cylinders whose axes run parallel to the spring line of the vault. This idea can be also found extended to cross vaults with a pointed profile, conceiving the vault as composed of eight cylindrical portions (Fig. 1).

A comparison with the design procedures proposed in the historical technical literature and with the built reality, however, reveals that this approach, if not in every case totally wrong, at least is far from being precise. Since scholars like Porter (1911) and Rave (1955) drew their attention on the shape of the caps of vaults, it is known that the problem is less trivial that it may seem at first glance, especially regarding those vaults that present a pronounced double curvature in their caps, so common in gothic architecture. Barthel (1991) elaborates a typological order and a systematic approach to the description of vault geometry which enables to model continuous surfaces on their webbing that correspond well to the reality as it can be observed in historical constructions.

Figure 1. Elementary composition scheme of a pointed cross vault, conceived as being composed of cylindrical portions, after Breymann/Warth.

Figure 2. The extrados of a vault constructed without formwork. Note the difference of the shape respect to Fig. 1.
2 THE CONSEQUENCES OF THE GEOMETRIC DESIGN OF THE RIBS AND GROINS FOR THE CAP GEOMETRY

The first reason for the geometrical complexity of the vaults' surfaces is the common practise to design preliminarily the "lines of discontinuity" of the vault, i.e. its formeret and transversal arches (those arches that confine the bay) and the groins, as these during construction are supported by centering frames which have to be drawn and pre-manufactured, and as these have the strongest impact in the visual appearance of the vaulted ceiling. These centering frames are usually designed according to elementary geometric patterns, sometimes as arches with more than one center but in most cases simply as circle segments; in every case with two-dimensional curvature and always arranged in vertical planes (leaving apart the groins of barrel vaults with lunettes built on a formwork, or the particular architectural features of the Guarinian tradition).

From this fact arises the difficulty to describe the vaults' surfaces as cylinders. This can be shown in vaults with elementary design. If, for instance, a barrel vault with large lunettes is conceived as consisting of intersecting cylinders with different radii, their curve of intersection must present a double curvature (Fig. 3). If, in the contrary, as usually practised, the groin is built with a single curvature (as circle segment or arch with more than one center) in a vertical plane, the surfaces of the vault in consequence cannot be cylinders (Figs 4, 5). The same is the case in cross vaults with semicircular profile but on rectangular plan where the groins would result double-curved, or even in "regular" cross-vaults on square plan, where usually the groin curves are circle segments.

In some cases the caps can be described by ruled surfaces between the arches which are generated with straight lines. This is the case where vaults have been built on a wooden formwork. An example can be found in the Münster at Freiburg (Barthel 1991: 28). In some cases, traces of the wooden planks or laths of such formworks arranged on the stone ribs could be found in medieval buildings (Nussbaum & Lepsky 1999: 65).

3 THE SHAPE OF DOMED CAPS BUILT WITHOUT FORMWORK

In many cases cross vaults have been constructed without formwork. Where this is the case, no guidance is
available for the shape of the caps. Moreover, these vaults usually present a pronounced double curvature, as the shape was not restricted by the constraints of a wooden auxiliary structure, and as such a double curvature is extremely helpful during the construction process.

Some building manuals (e.g. Körner 1901) suggest that the shape of these domed caps could be spherical, thus describing them as spheres truncated by the circular groins and arches that confine the cap. In this case, the surface of the cap would have to be clearly discontinuous at its ridge, presenting a re-entrant groin along the ridge line. The profile of this ridge line, produced by the intersection of two similar spherical segments, would necessarily be circular.

Existing vaults with pronounced re-entrant groins in the ridge line, however, in contrast to statements that can be found in the historical technical literature since Ungewitter (1859), present in a vast number of cases a profile which is not a circle segment: in fact, the characteristic curve of such a ridge line usually has a curvature which is smooth but variable (see below, section 3.4). The surfaces of the caps that intersect in the ridge line therefore cannot be spherical. Moreover, besides those vaults that present such a re-entrant groin at the ridge, many cases can be observed where the ridge is smoothly curved, or where part of the ridge is smooth and part is a discontinuity of the caps’ surface. Therefore we must consider the possibility of a variable curvature in the vaults’ surface, and also different radii of curvature corresponding to the different directions at any given point of the surface. In reality, such a situation is far more frequent than a constant uniform double curvature as it would be the case in a spherical portion of a vault’s surface.

Rave (1955), who systematically analyzed different vault typologies observing their level curves (looking at the intersection profiles of the vaults’ surfaces with 5 horizontal planes – Fig. 6), compares the highest level curve of such “domed” caps with the shape of an almond or egg. The formula for a “simple idealization” introduced by Barthel (1991: 29–33) is that of a bundle of 2nd order parabolas which have their apex in the highest point of the cap (Fig. 7). This formula has the benefit of being simple while avoiding the constraints of Euclidean primitives; it is easy to adapt to a given geometry of groins and arches, defines a continuous surface and is still a good approximation to the reality of domed cross vaults.

3.1 The procedure of free-handed vault construction

Free-handed construction of vaults, as regards the current typology of half-stone vaults (i.e. those vaults where the units are placed in a running bond normal to the vault’s surface, the resulting thickness of the shell equaling the width of the masonry unit), is mentioned in the technical literature since the end of the 18th century (e.g. Gilly 1805, first edition in 1795) – but it has certainly been practised since long before. Today it is still practised by few specialists. The basic principle has been formulated for the first time by the German architect J. C. von Lassaulx, whose essay (1829) had a considerable influence to the development of the topic in the technical literature (Wendland 2003). It consists in building the vault with self-supporting courses that are stable through their shape by forming arches. Every new course of the vault’s masonry must be held in place by the adhesion of the mortar only until it is closed; once the course is complete, it is stable by having the form of an arch and it offers reliable stability for the units of the next course to be laid on top of it.
In cross vaults, these arch-wise curved courses can be spanned conveniently between the centering frames of the arches and groins (or their respective ribs), which will offer sufficient abutment. The curvature of the courses necessary for their stability can be obtained by providing a double curvature to the caps ("domed" caps), by tilting the planes of the bed joints, or both in combination.

The caps can be built up independently without any continuity of the masonry fabric between them, connecting the single caps over the diagonal ribs with a mortar joint. In this way, in every single portion the best curvature is obtained by a modest doming and some tilting of the bed joint planes. Their spatial inclination can be corrected if necessary to optimize course curvature. The single caps should be built up more or less at the same time just not to cause asymmetric loads to the centering frames. This is the way Lassaulx, who has been mentioned above, built his neo-medieval vaults (Wendland 2003), and that has been commonly used in such cases where the groins were provided with stone ribs set on the centering before constructing the caps.

The most frequent masonry apparatus in cross vaults, however, is the dovetail pattern (Fig. 9). It consists in arranging the courses on diagonally tilted planes perpendicular to the groins. Hence, the masonry fabric is continuous through the neighbouring caps over the groin, avoiding the joint behind the groin rib. The diagonally tilted courses are seamed in the ridge of every cap. The continuity over the groins is beneficial in those vaults which have no ribs or those where the groin ribs consist of small units within the masonry fabric of the caps.

3.2 The masonry apparatus in the dovetail pattern: inclination of courses, course curvature and corrections

In order to study the building process and its consequences on the shape in detail, a series of small-scale models simulating different typologies and vaulting patterns (e.g. Fig. 9) and a prototype in scale 1:4 (Fig. 10) were built.

From these trials it became obvious that the layout of a regular masonry apparatus, apart from its formal and structural benefits, is necessary in order to guarantee the stability of every new completed course during construction, and to facilitate an efficient geometric control to achieve a similar shape in all caps of the vault and to make sure that the portions rising from
the different spandrels come to meet correctly in the vertex.

In principle, the bed joint planes are lying normal to the curve of the groin. However, to follow strictly this rule would lead to a radial inclination of the bed joints and consequently to an angle of every bed joint plane respect to the preceding one. Such an arrangement is usually described in the building manuals since Ungewitter (1859: 104–114) (Fig. 13), although this author already admits that it could be debatable. Actually, in practise it is problematic to set the bed joint planes in an angle to each other, as this would lead to a variation in the thickness of the bed joints because the distance between the successive bed joint planes varies according to the local distance of the course to the turning axis of the planes (Fig. 12).

According to the observations on existing vaults and the trials on the prototype, such a turning of the bed joint planes is feasible only in those portions of the cap where the courses are very short and a manifest angular divergence can therefore be produced within the normal thickness of the mortar joint. This is the
case only in the lowest and highest portions of the cap. In the main part of the cap, in contrast to what has been stated before, the bed joints are always parallel, tilted in space with a constant angle.

For the curvature of the single courses, no precise indication can be found in the historical technical literature. Generally, a pronounced curvature serves to stabilize the courses during construction; Hörning's manual (1836) says that less experienced masons will tend to give more curvature to the courses. Within a course, the curvature can be assumed as being constant, the resulting form of the courses therefore as circle segments. Over the entire cap, the curvatures need not be constant, but may vary from course to course or in different portions of the cap.

The area of the cap which is crucial for the choice of the course curvature is right above the summit of the transversal or formeret arch. Here, the courses have the greatest length and therefore most deserve to be stabilized through their curvature. The limit of possible curvature, i.e. the smallest possible radius, is defined in this area by the tangential continuity of the course with the surface of the opposite half of the cap (arising from the neighbouring groin), which causes a completely smooth vault surface at the ridge (Fig. 16). A greater radius, or minor curvature, in fact, leads to a re-entrant groin in the ridge, which is quite common and formally satisfactory, while a smaller radius, or greater curvature, would lead to an inverse discontinuity of the vault surface in the ridge that would appear as projecting to the inside, which would be a flaw. Hence, the limitations of the curvature are the stability of the longest courses and the disturbing of the surface continuity at the ridge of the cap.

It is rather difficult to carry out corrections in the curvature of the courses while proceeding towards the zone just mentioned. Such corrections are first of all limited by the obvious necessity of laying one course on top of the other. Also, they easily disturb the surface continuity, leading to an outward curvature of the cap which must be avoided for formal and structural reasons.

Actually in many points of the cap masonry the courses are not exactly superposed upon each other (Fig. 17) – this happens because even if a radial inclination of the bed joint planes is achieved respect to the diagonal groin, these will not intersect the vault surfaces orthogonally in every point. As result, especially in the lower half the courses are shifting outwards in the part which is more distant from the groin. An attempt to exactly superpose them would be a mistake as it would successively reduce the curvature of the courses because the fleche would remain constant while the length is increasing – moreover, the starting point at the formeret or transversal arch itself is progressively shifting outwards. Such shifting of the courses must be restricted to a sufficient superposition of the courses, as basic principle of masonry construction.

3.3 The continuity of the slope

The rule for the building of the cap masonry, and therefore for the modelling of the cap's surface, is provided by the necessity of building curved courses on tilted bed joint planes between the diagonal groin and the formeret or transversal arch that confine the cap, i.e. between two continuous curves (usually circle segments), up to the longest course at the summit of the arch.

Above that point, one of the track curves is lacking, and the question is how to continue.

In a first hypothesis, it could be assumed that the surface continues as a translation surface, extending, as track curve, the curve of the formeret or transversal arch either with constant curvature or tangentially. However, such a surface generation is problematic because the length of the generating curves (coherent with the upper courses) continues to increase while rising upwards. If a constant curvature is assumed (Fig. 18), there is a high risk that the distance spanned by the course might exceed the diameter of the
circles which the courses describe, which is impossible, and this modelling procedure therefore inconsistent. But even if the course length remains less than the diameter, the longest courses would project outwards, generating a surface which in normal direction to the courses is bending outwards, which is not acceptable in a masonry vault and not realistic.

In order to make this procedure consistent, a gradual reduction of curvature of the courses could be assumed. However, in reality such reduction does not happen. In the prototype, for instance, the measuring data show that the curvature of the courses within the region in question is nearly constant, with a radius of curvature considerably shorter than the length of the longest courses (Figs. 14, 19).

In the practical realization, building up the cap masonry, the way to continue beyond the summit of the arch is given by the demand of continuity in the slope of the cap. Any bending outwards is prohibitive, but also an abrupt change of the slope towards the inside would be problematic both under structural and formal aspects. Besides that, such a change in the slope would demand a drastic reduction of the course curvature and a strong shifting of the courses reducing their superposition; it would be feasible only by strongly increasing the inclination of the bed joints planes, which due to the length of the courses in this area is impossible but by interrupting the masonry apparatus.

The demand that the longest courses should not project outwards due to their circularity can in some cases even oblige to reduce their curvature (increase the radius) respect to the preceding ones (which is not desirable if we remind the benefit of the curvature especially in the longest courses). In such a case, the slope of the cap would result straight in this portion – i.e. avoiding to curve inwards below the longest courses in order to reduce curvature the less possible, and in continuation aiming to regain the stronger curvature in the following courses. Otherwise, the slope normal to the bed joints will be very gently curved to the inside in the central part of the cap, while its curvature would increase in the lower part and near the keystone of the vault where the bed joint planes are radially turning as permitted by the short length of the courses in these areas.

Building the highest part of the cap is not very difficult, as the courses in this area are rather short. Therefore, the slope of the cap can easily be bent to the inside by shifting the courses, by inclining the bed joint plane and even by allowing some double curvature in the bed joint. Also the curvature of the courses can be easily reduced at one's convenience.

3.4 Surface models for the shape of the cap

The shape of the cap, described in these terms, is directly connected to the basic principles of masonry and their application in the building process of the vault. For the formulation of a geometric description, the profile of the slope, assumed to be straight or
gently curved in the main part of the cap, could be used for the procedure of surface modelling.

In some cases a torus may be considered a fair approximation for every half cap, although this would be accurate only in case that the bed joint planes were radial, which normally is not the case.

In the case of the prototype, a translation surface generated by the curves representing the courses sliding on the curve of the slope normal to the bed joints, proved to be rather accurate in the “critical” part of the cap surface (Fig. 20). In any case, the assumption of a translation surface is rather close to reality as it takes in account both the fact that the bed joint planes are parallel and the fact that over the central part of the cap the radii of the curves described by the courses are more or less constant. Therefore, we may assume that the parallel translation of a constant curve sliding first on the curves of the groin and the formeret or transversal arch and then on a curve that running normal to the bed joints expresses the continuity of the slope, will lead to an accurate surface model, perhaps except for the highest portion of the cap. The generator curve can be assumed as being plane and circular, the inclination in space constantly normal to the central portion of the groin curve (this could be relevant in the cases of high pointed or surbased cross vaults) or simply by 45°.

The “theoretical” translation surface shown in Figure 21 could also explain the typical profile of the ridge line which, as mentioned, can be seen in many existing vaults (Fig. 22). In fact, the curve generated by the intersection of this surface with a vertical plane (analogous to an intersection with the symmetric counterpart of the opposite half cap) is quite similar in its shape to these ridge profiles: the curvature is stronger in the portion just above the summit of the formeret arch and decreases in the central part towards the keystone of the groins.
Figure 24. The vault shown in Figure 22 (a stellar vault with radial caps), reproduced in a model. In the head joints, the straight slope of the central part of the cap is visible.

4 CONCLUSION

In the considerations noted above, the terminology that has been employed is near to that of current CAD tools, as the problem discussed is essentially practical.

However, the formulation of an unequivocal answer of how to model the surfaces of a vault has been avoided. This is because, to the author’s conviction, the solution to be adopted in a particular case cannot be generalized, first, as it depends on the tools to be used for modelling, the quality of the available information (the survey data) and the degree of accuracy needed (a high accuracy often implies a high complexity). Neither can the particular case be generalized beyond a certain point: Apart from the boundary conditions of a vault, like the layout and geometrical design of its arches and groins, which may present a great variety, the influence of the mason on the final shape of the cap is strong – it regards his choices, like that of the radii of curvature in the courses and the manner of closing the highest part of the cap, his strategy of correcting defective tendencies in the shape, and certainly his ability (including his manual dexterity and his spatial imagination), and also the demand of perfection and pleasant appearance of the building in question.

Therefore, the way which the author has chosen to approach the problem is that based on a close view at the construction process, to formulate principles which must be adapted from case to case. The basic approach, starting from an elementary geometric design of the arches and ribs, is definitely according to the usual design process, which may be considered as a precondition for its accuracy. The modelling of the caps according to the basic principles of the construction process, as they have been pointed out, will lead to an accurate result.

NOTE

The present study is part of the author’s doctor thesis in preparation, which is supervised by Prof. D. Kimpel, University of Stuttgart. Part of the research on traditional vault construction has been performed with the Fraunhofer-IPK Berlin within the EU project “iso-brick” (EU CRAFT Research Project GSST-CT-2001-50095). The models have been built by María José Ventas Sierra and the author. The practical
experimentation has taken place at the Chair of Structural Design (Prof. W. Jäger), TU Dresden, under direction of the author. An acknowledgement is due to María José who gave strong support to the development of the contents and the text.

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