On elastic models for evaluation of the seismic vulnerability of masonry churches

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ABSTRACT: The purpose of the paper is to evaluate the capability and validity of results of commonly performed linear-elastic analyses to study the seismic response of historical masonry buildings. Results from the analyses of two typical churches of the colonial era in Mexico are compared with the common patterns of damage observed in these structures after intense earthquakes. It is clear that elastic models cannot give reliable information on seismic capacity, because significant cracking occurs since early stages of lateral loading; nevertheless, the careful interpretation of the natural modes of vibration could give a good insight on basic sources of weakness of structures of this kind. Particularly useful are results about the natural modes of vibration and their corresponding frequencies, as well as about the parts suffering important amplification. Vertical accelerations are shown to significantly influence the response and vulnerability of structures of this kind.

1 INTRODUCTION

Ancient stone masonry structures frequently show extensive cracking after being submitted to earthquakes, even of rather moderate intensity. As an example, after a magnitude 7.0 earthquake in central Mexico in 1999, 1542 colonial churches were detected as damaged, in a radius of more than 100 km from the epicenter. In most cases the damage was rated as light, consisting mainly of flexural cracking in the vaults and bell towers. In situations like this, controversies commonly arise about the possible reduction of the seismic safety for future events. Decisions in this regard are taken on the basis of qualitative judgments about the sources of weakness and the possible benefits of different remedial measures. When quantitative assessments of seismic response are attempted, they are commonly carried out through linear elastic analyses of the undamaged structure, and their results are seriously questioned; estimations of the reduction of seismic capacity due to the cracking are seldom undertaken in practical cases.

A long term research program aiming at evaluating different analytical tools for assessing seismic vulnerability of historic masonry structures, has been undertaken by the Research Group on Structural Safety of Historic Buildings at the Institute of Engineering of the National University of Mexico (UNAM). As part of this program, the capability of large scale codes for linear elastic finite element analysis to provide useful information about the seismic safety of typical colonial churches in Mexico has been studied. The results obtained up to date are summarized in this paper.

Initially, the structural characteristics of typical colonial churches in Mexico are described and their most typical damage patterns are presented and discussed. Two model structures are studied: the simplest one is a single-nave temple with barrel vault and hemispherical dome; the other one has a Latin-cross floor plan, and a somehow more sophisticated vault. Their response to typical earthquake ground motions is studied through linear elastic dynamic analyses of finite element models in an uncracked stage, and also considering some basic cracked patterns.

2 TYPICAL COLONIAL CHURCHES

Thousands of churches were built in Mexico from XVI to XVIII century, and persist to date in rather good conditions; they vary in size and architectural sophistication, but follow some basic typologies. The simplest among them are rather small parochial churches which are found in every barrio of Mexican towns and
villages. One important factor that has influenced the evolution of their architectural features has been the experience of damage suffered from earthquake activity. Roughly, two thirds of the country are exposed to significant seismic hazard, but the frequency and intensity of earthquake activity has great variation, being greater along the Pacific Coast where the subduction of the Cocos plate under the North American plate generates one of the most active seismic zones of the world. It is in this area, and more specifically in the state of Oaxaca, where the recurrent destruction of the early construction produced an evolution towards low rise, heavily buttressed buildings with scarce external ornamentation. In other regions the lower concern for earthquake failures favored taller and more slender constructions.

Considering the above mentioned factors, two typical colonial churches of the Mexican southwest were chosen for the analytical study of their seismic response. The first one is taken from the highly seismically active Oaxaca state, and the other from the Puebla state, which is far from the subduction zone (around 250 km), but also suffers from rather frequent earthquake generated by normal faulting.

Oaxaca typical church has only one rectangular nave, with a simple façade and one or two small bell towers. Dimensions in plan are 15 by 28 m, and the heights of the vaults and bell towers are 11 and 14 m, respectively (Fig. 1). The roof has barrel vaults and a hemispheric dome on the apse. Trapezoidal buttresses are placed along the longitudinal walls of the nave and in the apse. In the first bay there is an intermediate floor for the chorus.

Puebla typical church is bigger than in Oaxaca. It has a Latin-cross floor plan, whose main nave measures 20 by 58 m, while the height of the vaults is 18 m. Its two bell towers are 28 m high (Fig. 2). Roof is constituted by a quadripartite vaulting system. A hemispherical dome is placed over the transept bay and it is supported by the drum. Small buttresses are placed along the main nave. As for the typical Oaxaca churches, the chorus is placed in the first bay.

The main material of construction is a heterogeneous masonry constituted by stones agglutinated by lime-sand mortar. Frequently, broken clay bricks or lightweight volcanic stones were added to the masonry; this heterogeneous masonry constitutes a kind of concrete whose composition varies according to the structural element; it is lighter than normal stone masonry, and has a tensile strength bigger than brick masonry, due mainly to the absence of weak planes constituted by the mortar layers.

Walls are made by stone masonry, while vaults, domes and arches are generally built with bricks. Rarely, stone ashlars were used for structural elements. In few cases brick masonry was not only used in the roof, but also in the facade, columns, buttresses and basement. Adobe was widely used for early churches, but in most cases these buildings were soon destroyed by earthquakes and replaced by stone masonry constructions.
3 PATTERNS OF EARTHQUAKE DAMAGE

Observation and evaluation of seismic damage suffered by typical stone masonry historic churches have allowed to identify their basic modes of failure and main characteristics of seismic response: behavior is governed by the low tensile strength of constituting materials, which makes it almost impossible to provide continuity within and between structural members, and leads to specific mechanisms for resisting seismic actions. The most common patterns of damage will be briefly described in the following paragraphs, following the behavior of main constituting structural elements. These patterns are schematically represented on the typical colonial temple shown in Figure 3.

Vaults, arches and domes are very efficient for resisting their own weight, but they are very sensitive
to opening of their support, which introduces tensile stresses and could produce extensive cracking. Due to the height of the temples, supporting walls and columns experience significant lateral displacements at their tops, during strong ground shaking, thus generating longitudinal cracks in barrel vaults, and cracks along the meridians in domes. In most cases this cracking does not endanger the stability of the roof, giving rise only to a redistribution of stresses; nevertheless, when large displacements take place, the shape of the elements could change to a less stable configuration, and progressively lend to collapse, especially because of the cumulative effect of subsequent earthquake motions. In this regard, domes are more sensitive than vaults to motions of their supports, which can be enhanced by the flexibility of their drums. Vertical ground accelerations, which are significant in epicentral areas, play an important role in the damage of the roof, mainly because they tend to increase the thrust of the roof elements on their supports, consequently aggravating their opening and out of plumb.

Bell towers are rather slender and weak elements, in which the ground motion is greatly amplified. Even if they are relatively low and sturdy, in temples of the most seismically active areas their failure is rather common, especially in the vertical elements and in the arches surrounding the bell fry. Additionally, their bending motion tends to separate them from the rest of the church, or to generate shear cracking in the lower body of tower.

Façade is typically a tall and heavy wall, poorly connected to the rest of the temple. Its out of plan vibration tends to separate it from the rest of the temple. Horizontal cracking in the frontispiece, or in the lower part of the façade weakened by large openings, is rather common, sometimes giving rise to the partial or total overturning of the façade. At the other end of the church, the apse has a shape that makes it generally stiffer and less vulnerable than the main façade; sometimes, it shows damage due to the thrust of its dome.

Longitudinal walls of the temple do not commonly show damage due to in plan forces; their out of plan stiffness is greatly enhanced by rather heavy buttressing; nevertheless, it is frequent to notice some out of plumb derived from the thrust of the roof during their vibration under seismic ground motion. Subsequent earthquakes produce a cumulative effect in this regard.

4 FINITE ELEMENT MODELS

In recent years, different analysis methods have been developed to study the seismic behaviour of historical structures (Lemos 1998). Some are very sophisticated, like finite element methods (Mola & Vitaliani 1997) or discrete element method (Azevedo et al. 2000). Other methods are simpler, as macroelement method (Brencich and Lagomarsino 1998), rigid element method (Casolo & Peña 2004) or limit analysis (Orduña & Lourenço 2002). All these methods have their advantages and disadvantages and can be useful for some specific purposes.

When fed with reliable data, these sophisticated methods give results with good agreement with the reality, but the computational effort they require sometimes makes them unfitted for the analysis of complete structures or for parametric analyses, especially if non-linear behaviour is considered. On the other hand, simplified methods are useful to analyse large structures or to make parametric analyses, but they are limited by their simplified hypotheses.

Finite element method is a widely used tool for seismic analysis of masonry structures, since its theory and limitations are well-known and some strong commercial computational codes are available. However, most FEM codes do not consider non-linear behavior, because of the higher computational effort required. On the other hand, the particular mechanical characteristics of masonry make it necessary to use specific constitutive models that sometimes are not available in the libraries of the computational codes. Thus, elastic analyses become the only available approach, mainly for the practitioners.

The complex geometry of most historical structures leads to the use of 3D solid finite elements. Shell elements, are not convenient, since the assumptions in which they are based, are not accomplished for the massive parts of the structure, such as piers, walls or buttresses.

For performing finite element linear elastic analyses of the two prototype churches previously described, two models were built, one for Oaxaca (Fig. 4), and the other for Puebla (Fig. 5). The same material properties were used for both models; they were derived from

Figure 4. Oaxaca model.
testing of similar materials and are shown in Table 1 (Luna 1995). Vaults were considered as built with brick masonry, while walls, arches and towers with stone masonry.

Figure 5. Puebla model.

Table 1. Elastic properties of the materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick masonry</td>
<td>525</td>
<td>0.2</td>
</tr>
<tr>
<td>Stone masonry</td>
<td>2000</td>
<td>0.2</td>
</tr>
</tbody>
</table>

In order to study the influence of cracking pattern in the dynamical response of the churches, for the Oaxaca church an additional model was built, in which three longitudinal cracks were considered along the vaults, in the sections of maximum tensile stresses (Fig. 6). It must to borne in mind that the cracking pattern is considered as part of the initial geometry and that the development of the cracks is not followed by the analysis; the purpose of this simplified cracked model is to evaluate the effects of pre-existing cracking on dynamic response, no to consider the non linear behavior produced by the cracking.

Cracks are modeled by non-linear spring elements called “gaps”. These springs are linear in compression, but they are unable to take tensile forces. Mohr–Coulomb criterion, relates their shear strength to axial behavior, and thus allows the modeling the opening and closing of the cracks.

5 DYNAMIC PROPERTIES OF THE TWO MODELS

Table 2 shows the first five natural periods of vibration for both uncracked models. The shorter periods of the Oaxaca model reflect mainly its lower height and stronger buttresses, which give it a greater lateral stiffness than for the Puebla model. The closeness of the first five periods for each model is worth nothing, as well as the reduced participation of each specific mode to the overall response, as reflected by the respective participating modal masses (PMM) also reported in Table 2. As a matter of fact, about 250 modes are needed to reach a PMM of 90% for the Oaxaca model, thus giving an indication of the complexity of its response to ground motions.

The first mode of the Puebla model has a period of 0.396 s (Fig. 7a), while the Oaxaca model has a first period of 0.152 s (Fig. 8a); modal shapes are similar for both models, and are governed by the lateral movement of the nave. The second modal shape of the Puebla model involves a vertical vibration of the vault and an alternate opening and closing of its supports (Fig. 7b), while for the Oaxaca model this mode involves a lateral

Figure 6. Localization of initial cracks.
movement of part of the nave (Fig. 8b). On the other hand, the third mode of the Puebla model presents a lateral vibration of the nave (Fig. 7c), while for Oaxaca model this third mode presents the same pattern of vertical vibration of the vault than mode two for Puebla (Fig. 8c).

It is interesting to know that among the first ten modes of the Oaxaca model, none presents a shape of longitudinal translation; longitudinal vibration only appears in torsional modes. This is because this model is very rigid in its longitudinal direction; the walls of the nave, the apse and the façade give great rigidity to the church. On the other hand, the eighth mode of the Puebla model corresponds to longitudinal vibration. The first seven modes are related to transverse, vertical or torsional vibrations.

Some modal shapes can be correlated to observed collapse mechanisms; for example, vertical vibrations of the vaults correspond to the typical damage related to opening and closing of the supports. Most common collapse mechanism of the façade is its detachment from orthogonal walls and its overturning due to the development of a horizontal cylindrical hinge at the basis of the façade, how reflected in the vibration mode shown in Figure 9. Another typical collapse mechanism involves the top of the façade (frontispiece), and it is due to out-of-plane rotation of the wall restrained on three sides (Fig. 10). It is possible to identify the most

![Figure 7a. First mode. Puebla model.](image)

![Figure 7b. Second mode. Puebla model.](image)

![Figure 7c. Third mode. Puebla model.](image)

![Figure 8a. First mode. Oaxaca model.](image)

![Figure 8b. Second mode. Oaxaca model.](image)

![Figure 8c. Third mode. Oaxaca model.](image)
probable collapse mechanism of the façade by observing the modal shapes. The Puebla model has various modes in which the out-of-plane rotation of the whole façade appears. This type of mechanism mostly develops due to inertial forces of the bell towers (Fig. 9), and becomes critical due to the slenderness of the façade.

Overturning of the frontispiece is less critical for Puebla model than Oaxaca model. This second model shows several vibration modes in which this part of the façade is involved. This collapse mechanism is favored by the pushing of the barrel vaults on the façade. Barrel vaults are more flexible than quadripartite vaults, which move as a block with lateral walls and do not thrust on the façade. On the other hand, the top of the barrel vaults has greater displacement than lateral walls due by inertial forces, causing a significant thrust on the façade. In Oaxaca model, bell towers, which are low and stiff, work as buttresses of the façade, and contribute to avoid the overturning of the façade as a whole; then the frontispiece becomes the most vulnerable zone of the façade. On the contrary, the great height of bell towers in Puebla model increase the inertial forces, contributing to the separation of the façade from the main nave.

6  RESPONSE FROM TIME HISTORY ANALYSES

For time history analyses of the finite element models, two typical records of earthquakes generated in the subduction zone were selected. Both seismic events had produced some damage to historical constructions. Figure 11 shows the response spectra of the two seismic records, and Table 3 gives their basic characteristics.

These seismic records were selected for their frequency content, more than for their maximum accelerations. Huatulco record, obtained near to the epicentre, has a response spectrum for horizontal accelerations with peaks for periods from 0.05 to 0.3 s, whereas, Acapulco record, obtained at a moderate distance from

Figure 9. Overturning of the façade. Puebla model (Mode 8).

Figure 10. Overturning of the frontispiece, Oaxaca model (Mode 13).

Figure 11a. Response spectra. North–South.

Figure 11b. Response spectra. Vertical.
the epicentre of a greater earthquake, shows a spectrum with spectral peaks for periods from 0.25 to 0.7 seconds. As already mentioned, Puebla model has its fundamental period at 0.396 s and Oaxaca model at 0.152 s. These periods are expected to increase significantly when severe cracking reduces the lateral stiffness; therefore, the shape of the response spectrum of the Acapulco record is more severe for both structures, because the lengthening of their periods due to progressive cracking during intense ground motion will maintain them in the zone of maximum spectral ordinates, whereas for the Huatulco record, the softening of the structures due to cracking will eventually move them away from the zone of highest spectral ordinates.

Vertical seismic acceleration is an important factor in the seismic vulnerability of this type of structure; particularly because response of masonry vaults and domes is affected by changes in their vertical loads, it is necessary to include the vertical component in the analysis of seismic response. In this regard, Acapulco record has similar accelerations for horizontal and vertical component, while vertical acceleration in Huatulco record is almost 50% that of the horizontal acceleration.

Time history analyses of both models were performed for the simultaneous action of the horizontal component in the transverse direction and the vertical component, for each earthquake. Results allowed determination of base shear coefficients (ratio between maximum shear at the base of the structure and total weight of the structure), as shown in Table 4. Total weight of the Oaxaca model is 19060 kN, while the Puebla model weights 96098 kN, about five times heavier. As can be appreciated, base shear coefficients are very near to the spectral ordinates corresponding to the fundamental period of each model.

Tables 5 and 6 show maximum displacement and acceleration, respectively, at the keystone of the vault, in the centre of the nave for both models. Displacements of Puebla model are greater than for Oaxaca model. Horizontal displacements and accelerations in Oaxaca model are almost three times those in the vertical direction, when Acapulco record is used, while this ratio is ten with Huatulco record. In the Puebla model, the vertical displacements and accelerations are slightly greater than the horizontal ones, when Huatulco record is used. With the other record, displacement ratio is almost two and acceleration ratio is almost one. This shows that vaults of this second model are very sensitive to vertical accelerations, and, therefore, are more vulnerable to ground motions with significant vertical component.

The maximum displacements are rather small, especially for the Oaxaca model, because they correspond to high frequency movements. However, these displacements cause damages on this type of structures, as it was observed after these and other similar seismic events. At their undamaged state, both churches have great lateral stiffness, and only when significant cracking occurs, displacements could reach values that endanger the overall safety of the temples.

Stress distribution in critical parts of the model can help finding probable cracking patterns. For example, Figure 12 shows the principal stresses in the vault of the Oaxaca model under the Huatulco record. In this case, the zone near to the keystone at intrados is the most probable zone of flexural cracking. In fact, the tensile stress at the keystone is 0.6 MPa, while tensile strength of masonry is 0.3 MPa.

Figure 13 shows the maximum acceleration and displacements along the height, obtained from records at the basement, roof and top levels of bell tower for both models, for the Huatulco record. In the Puebla model,
maximum acceleration at the top is 20 times greater than at the basement, while this ratio for the roof level is about 3. For the Oaxaca model, these ratios are 4 and 2 times, respectively, and distribution along the height is quasi-linear, indicating that the behaviour of the tower is not an appendix-like vibration; whereas for the Puebla model the large amplification from the top to the roof levels reveals an appendix-like vibration, due to the high flexibility of the upper part of the tower.

7 CRACKED MODEL

First five periods of cracked and uncracked models are compared in Table 7. Only a slight increment of periods is produced by the type of cracking introduced in the model, that does not affect the main lateral force resisting elements, walls and buttresses; cracking mainly affects the vertical movement of the vault and modifies the modes that are correlated to vertical vibrations, as can be seen by comparing differences in the second and the third modes. In the uncracked model the second mode corresponds to a lateral vibration in which the apse moves in one direction, while the facade with bell towers moves in opposite way (Fig. 8b); the third mode corresponds to a vertical vibration of the vault (Fig. 8c). On the other hand, the second mode of the
cracked model corresponds to a vertical vibration of the vaults (Fig. 14a), while the third mode is similar to the second mode of the uncracked model (Fig. 14b). In other words, the cracks imposed to the vault made it more flexible for the vertical vibration; therefore, the third mode of the uncracked model became the second mode of the cracked model.

Table 8 shows the three modes that undergo significant change of period due to cracking. In all cases the modes are related to the vertical vibration of the vaults. The increment in the period is smaller as the order to the mode increases, because the vertical vibration of modes up to the twelfth are related to torsional shapes that are not greatly influenced by the vertical motion.

Table 9 shows the maximum acceleration at keystone of the central bay of the nave due to both records. Lateral acceleration is only 10% greater in the cracked model than in the uncracked model for Acapulco record, while it is the almost the same for Huatulco record. On the other hand, vertical acceleration is practically the same for Acapulco record. In the case of Huatulco record, vertical acceleration is three and half times greater.

Part of the increase in the response of the cracked model is due to the particular spectral shape. Figure 15 shows the acceleration response spectra of both records. It can be seen that the increase in the period of
Table 10. Stresses at keystone.

<table>
<thead>
<tr>
<th></th>
<th>Axial (MPa)</th>
<th>Shear (MPa)</th>
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</thead>
<tbody>
<tr>
<td><strong>Extrados</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncracked</td>
<td>-1.14</td>
<td>-0.23</td>
</tr>
<tr>
<td>Cracked</td>
<td>-1.26</td>
<td>-0.34</td>
</tr>
<tr>
<td><strong>Intrados</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncracked</td>
<td>0.63</td>
<td>-0.22</td>
</tr>
<tr>
<td>Cracked</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Tensile stresses are positive.

the third mode causes the structure go out of the peak of the vertical spectrum of Acapulco record, this is the reason because vertical acceleration remains almost the same despite of the reduced stiffness of the model. In the case of the Huatulco record, the lengthening of the modes does not change the spectral acceleration. Horizontal accelerations are not incremented because the modes fall in a spectral plateau.

Stresses at keystone due to Acapulco record are shown in Table 10. In the uncracked model tensile stresses generated at the intrados by flexural moment are greater than tensile strength of the masonry (0.3 MPa). When cracks are introduced, compression stresses at the extrados increase, because neutral axis moves upwards. Shear stresses increase too, since the crack is unable to transfer shear stresses.

It can be concluded that the longitudinal cracking frequently found in vaults of churches of this kind does not significantly affect their seismic response, because it has little effect in reducing their lateral stiffness. It can be assumed that only important flexural cracking of the longitudinal walls and their separation from the façade and from the transept could soften the structure and give rise to the large lateral displacements needed to produce the collapse of the vaults.

8 CONCLUSIONS

Elastic-linear models are able to give preliminary information about the seismic behaviour of masonry churches and their seismic vulnerability. The main information that they could provide is: (a) vibration modes, (b) weak zones of the structures, (c) elements with undesirable behaviour, etc.

It is clear that it is necessary to make inelastic analyses to get full information about seismic vulnerability. However, the complexity of these analyses and the lack of reliable information about some of the parameters often limit the possibility of their practical application for complex structures. An approach that offers some advantages is to use information obtained from elastic analyses as guideline for constructing models of critical parts of the structures which could be studied separately with more complex analyses.

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