The influence of bells’ movement on the adjacent masonry vibrations

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ABSTRACT: The gable wall standing over the entrance into a neo-gothic church and fixed on both sides of its base into towers, swayed with well-visible amplitudes out of its plane in the rhythm of the swinging of the bells. The gable is of triangular shape with the height of 8.60 m and base-width 9.0 m, not being supported horizontally by the roof or by any other stiffening structure. The measurements revealed that the bells excite the vibrations of towers and of the basement of the gable with moderate amplitudes, while the cantilever of the gable wall amplifies the motion by the resonance-effect. The impression of the gable motion was made worse due to the beat-effect of superposition of more bells (up to 6, with the masses between 150 and 1000 kg) with slightly different swaying frequencies (0.44 thru 0.52 Hz). The way of reinforcing the wall is now being sought with respect to the cultural heritage conserving aspects.

1 INTRODUCTION

The triangular gable wall in the western front of the cca 100 years old neo-gothic cathedral in Prague-Vysehrad was observed to vibrate out of its plane during ringing. Our Institute was asked to measure these vibrations and to assess their influence on the stability (safety) of the wall. In what follows, the measurements of wall-motion have been described and the results analyzed. Simple shaking-table experiments, modelling the behaviour of the damaged wall, have been added.

2 DESCRIPTION OF THE STRUCTURE AND OF ITS BEHAVIOUR

The cathedral in question was built at the boundary between the 19th-20th centuries from traditional materials (stone, bricks, covered with slate on wooden truss), having about 30 × 56 m in plane; its two towers are about 60 m high (see Figure 1). The church interior was recently refurbished and also its structure seems to be in good order and condition. Its triangular gable wall over the main entrance starts at the height of 19.60 m above the church floor, its width between both church towers is 9.0 m and the height of its rectangular part fixed into the towers is 2.60 m. The height of the free-standing triangular part is 8.70 m and the cross on its top reaches the height 30 m. The gable wall does not lean against the wooden construction of the roof covering the main nave of the church. The masonry of the gable is 0.57 m thick, but the niches containing statues, of total width (from 5 to 1) × 1.20 m, reduce it up to 0.27 m. (the photo annexed).

The towers are of square cross-section 7 × 7 m, the bells are hung in classical wooden stools placed at the level about 29 m. There are two old bells in the southern tower (J1, J2), four recently mounted ones in the northern tower (S1–S4). All bells exert swaying motion driven separately by electric motors. Besides, there is a set of 12 stable bells with clappers controlled by computer in northern tower, used for playing several songs. The characteristics of the bells are given in Tab. 1. The amplitude of all bells’ swaying is about ±45 deg. Both old stools in the southern tower are evidently damaged and need to be repaired.

Standing in the entrance and looking up along the gable, the swaying of its top could be sometimes observed during ringing, similarly also when looking from the window of the tower the movement of the gable edge versus the roof slates. The movement in general was not stationary, its amplitude swayed,
especially when several bells were ringing together. In order to define better the phenomenon, to assess its influence and to decide about adequate measures, the measurements of vibrations of the gable and expert examination was ordered.

3 MEASUREMENT OF THE VIBRATIONS

There were 12 measuring points chosen on the front wall of the church, as shown on Figure 1, viz:

- 5 positions along the height of the southern tower, incl. its top and bell-floor level
- 4 positions on the gable wall, incl. its base and top
- 3 positions on the northern tower.

Our facility was able to pick up and register 6 signals simultaneously. Different combinations were chosen for different excitations, viz:

- each of the bells separately
- all bells together.

The pickups used were of the production of Wilcoxon Research CMSS 916VD: the signals were stored in a standard notebook and analyzed with the ONO SOKKI CF-350Z analyzer. Every registration lasted 1 minute or more, from which the RMS of the displacements and the frequency peaks were evaluated. As the registrations were more or less of the sinusoidal shape, the informative maximum amplitudes were checked as $\sqrt{2} \times$ RMS value. Some results (amplitudes and frequencies, the most important underlined) are given in Tab. 2, some frequency analyses are shown on Figures 2, 3.

The results of the measurements can be summarized as follows:

(a) The biggest response was excited by two old bells in the southern tower due to their biggest mass and to the bad order of their stools. The frequencies of all measured motions correspond with the swaying of the bells, its double and triple; similar results were obtained in (Bennati et al. 2002).

(b) The amplitudes of the tower and the front wall (incl. the base of the gable) are smaller than 0.5 mm and can not in the least endanger the structure. Mostly the 3rd harmonic (1.32–1.50 Hz) prevails in the response of the towers.

Table 2. Response of the gable to the ringing of old bells (southern tower, bells J1, J2) observed frequencies [Hz] and displacements (RMS) [mm].

<table>
<thead>
<tr>
<th>Point no</th>
<th>Position</th>
<th>Bell J1 frequencies</th>
<th>Bell J1 RMS</th>
<th>Bell J2 frequencies</th>
<th>Bell J2 RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Southern tower, top</td>
<td>0.44; 0.88; 1.32</td>
<td>0.154</td>
<td>1.32</td>
<td>0.325</td>
</tr>
<tr>
<td>20</td>
<td>Southern tower above bells</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.108</td>
<td>0.44; 0.88; 1.32</td>
<td>0.218</td>
</tr>
<tr>
<td>2</td>
<td>Southern tower, bells</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.080</td>
<td>0.44; 0.88; 1.32</td>
<td>0.218</td>
</tr>
<tr>
<td>21</td>
<td>Southern tower, gable base level</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.051</td>
<td>0.44; 0.88; 1.32</td>
<td>0.058</td>
</tr>
<tr>
<td>22</td>
<td>Southern tower bottom</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.054</td>
<td>0.48; 0.88; 1.34; 1.78</td>
<td>0.048</td>
</tr>
<tr>
<td>6</td>
<td>Gable top</td>
<td>0.88; 1.76; 2.64</td>
<td>7.163</td>
<td>0.88; 1.33; 1.77; 2.66</td>
<td>4.944</td>
</tr>
<tr>
<td>7</td>
<td>Gable low</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.063</td>
<td>1.32</td>
<td>0.233</td>
</tr>
<tr>
<td>4</td>
<td>Gable base</td>
<td>0.44; 0.88; 1.32; 1.77</td>
<td>0.053</td>
<td>0.88; 1.32</td>
<td>0.078</td>
</tr>
<tr>
<td>11</td>
<td>Northern tower, gable base level</td>
<td>0.44; 0.88; 1.76</td>
<td>0.050</td>
<td>0.50; 0.88; 1.78</td>
<td>0.040</td>
</tr>
</tbody>
</table>
(c) The top of the free standing triangular gable vibrated with visible amplitudes, up to 18 mm, with prevailing frequency corresponding to the 2nd harmonic of the fundamental frequency of southern bells’ swaying (0.88–1.18 Hz).

4 EFFECT OF BELL’S SWAYING ON THE CONSTRUCTION

Neglecting the mass of the clapper, the bell can be considered as a physical pendulum, which is described by its mass \( m \), eccentricity of suspension \( r \) (distance between the centre of gravity and the axis of rotation) and distribution of its mass around the centre of gravity, given e.g. by the radius of inertia with respect to the centroidal axis parallel to the axis of bell’s rotation, \( i \), or by the ratio

\[
\kappa = \frac{i}{r}
\]

This coefficient \( \kappa \) can also be determined from the period of small free vibration (total period, both ways movement), for which it holds \( (g - \text{acceleration of gravity}) \).

\[
T_o = 2\pi \sqrt{\frac{i}{r}}
\]

Figure 2. Frequency response of some points to the ringing of the biggest bell J1.

Figure 3. Frequency response of some points to simultaneous ringing with all bells.
in which the reduced pendulum length is,

\[ l_{red} = r + \frac{j^2}{r} = r \left(1 + \kappa^2\right). \]

Then

\[ T_0 = 2\pi \sqrt{\frac{r}{g}} \frac{\sqrt{1+\kappa^2}}{\left(1+\kappa^2\right)}, \quad \text{or} \quad \kappa = \sqrt{\frac{T_0^2}{4\pi^2}} \frac{g}{r} - 1. \]  

(2)

A physical pendulum, swaying with the angle-amplitude \( \pm \varphi_0 \), acts in its general angle-elongation \( \varphi \) on the supporting point by a force with horizontal and vertical components (Dašek V. 1955).

\[ H = \frac{m}{1+\kappa^2} \left[2\cos\varphi_0 - 3\cos\varphi\right] \sin\varphi \]

\[ V = \frac{m}{1+\kappa^2} \left[3\cos^2\varphi - 2\cos\varphi_0 \cdot \cos\varphi + \kappa^2\right] \]  

(3)

Supposing the bell's motion as harmonic, viz

\[ \varphi(t) = \varphi_0 \sin \left(\frac{2\pi}{T_0} t\right) \]  

(4)

the horizontal reaction can be expressed as

\[ H(t) = \frac{m}{1+\kappa^2} \gamma(t), \]  

(5)

in which the dimensionless time-function \( \gamma(t) \) is defined by both previous expressions (3) and (4). Its time-course, for the values corresponding to the bells J1, J2, i.e. for

\[ T_o = \frac{1}{0.44} = 2.273 \text{ s}; \]

\[ \varphi_0 = 45^0 = 0.785 \text{ rad} \]

is given on the Figure 4. Maximum of the vertical reaction is evidently in the moment when the bell passes through its equilibrium position \( \varphi = 0 \), maximum horizontal force corresponds to the angle \( \varphi_m \) given by zero value of the derivative of (3), namely

\[ \frac{d\gamma(t)}{d\varphi} = 3\sin^2\varphi + 2\cos\varphi_0 \cdot \cos\varphi - 3\cos^2\varphi = 0 \Rightarrow \]

\[ \varphi_m = \arccos \left[ \frac{1}{6} \left(\cos\varphi_0 + \sqrt{\cos^2\varphi_0 + 18}\right) \right] \]  

(6)

Here \( \varphi_m = 33.41^\circ \). In our case, during one cycle \( (t = 0 \text{ thru } 2.273 \text{ s}) \), this maximum (absolute value) occurs four times, namely, according to (4), for

\[ t = \frac{T_0}{2\pi} \arcsin \varphi_0 \Rightarrow \frac{2.273}{2\pi} \arcsin \frac{33.41}{45} = 0.303 \text{ s}, \text{ and then } 0.834, 1.439, 1.970 \text{ s} \]

The function \( \gamma(t) \) can be developed into a Fourier series

\[ \gamma(t) = (2\cos\varphi_0 - 3\cos\varphi) \cdot \sin\varphi \equiv \]

\[ \pm b_1 \cdot \sin \left(\frac{2\pi}{T_0} t\right) + b_3 \cdot \sin \left(6 \pi \frac{t}{T_0}\right) \]  

(7)

in which (in our case)

\[ b_1 = \frac{2}{T_0} \int_0^{T_0/4} \gamma(t) \cdot \sin \left(\frac{2\pi}{T_0} t\right) \, dt = 0.673; \]

\[ b_3 = \frac{2}{T_0} \int_0^{T_0/4} \gamma(t) \cdot \sin \left(6 \pi \frac{t}{T_0}\right) \, dt = 0.179 \]

As it can be seen, the excitation effect of a swaying bell has two harmonic components, the first and third. The higher ones are unimportant (e.g. \( b_3 = 0.0065 < 4\% b_1 \)). Both these frequencies, the fundamental one and its triple, can be identified on the measured responses, but this analysis does not say anything about the double, which is also very important in the response. It probably comes from the fact that in this case the bells do not sway as a pure pendulum, the driving force of which depends on the elongation, but they are also driven by electric motors, which exert the force depending on velocity. The exact analysis of this phenomenon has not been done till now, but it seems evident that the phase of such an excitation is shifted by \( \pi/2 \) and evidently adds cosine-members into the excitation (7). The double frequency was also observed in (Bennati et al. 2002), where the bells were driven by motors, while in (Fischer O. & Pirner M. 2002), where the bell was moved by human force, only the first and 3rd frequencies were found.

Figure 4. Time function \( \gamma(t) \) describing the time-course of the horizontal reaction of the bell.
5 EVALUATION OF THE RESULTS OBTAINED

The only important vibrations found on the construction are those of the free standing triangular gable over the church entrance. This evidently behaves as a cantilever and – due to the resonance effect – amplifies the negligible excitation of the base into about centuple response of the top. Nevertheless, such a trivial explanation has some shortcomings, viz.

- according to even an approximate calculus, the natural frequency of the triangular gable wall about 1 Hz is unrealistically low
- the resonant amplification by centuple would require unrealistically low damping of 0.5%
- the observed resonance frequency band 0.88–1.18 Hz ($f_0 \pm 15\%$) does not match with this low damping.

All these discrepancies seem to reveal that the gable wall does not behave like a linear elastic structural element, i.e. that its masonry has been damaged in some way, most probably by the ageing process and weathering of the mortar in the joints between stones. In order to elucidate the influence of this phenomenon, a simple experiment on the shaking table was arranged.

6 SHAKING TABLE EXPERIMENT OF A BRICK-COLUMN

A column of 590/140 mm comprising 22 layers from 2 (or 1 + 2 halves) pieces of perforated bricks (290/140/65 mm, 3.57 kg) was “built” on shaking table 1.25 x 1.55 m exerting unidirectional horizontal excitation, controlled by electro-hydraulic dynamic system MTS 204.51. In the first case, the bricks were left bare (total height 1.43 m), in the second the joins were filled by mortar, but the horizontal ones were separated by thin plastic sheets (height 1.56 m) – see Figure 5. Two transducers were fixed horizontally in the heights of 475 and 1384 mm (525 and 1554 mm). The columns were tested for free vibrations after a release from static deflection and for harmonic excitation with step-wise increasing and decreasing frequencies starting with small amplitudes and repeated with larger ones. The response is of course strongly non-linear: (i) the “resonance curves” are different for increasing and decreasing frequencies, (ii) the natural frequencies increase with smaller amplitudes, and (iii) the damping is smaller for small amplitudes. One set of measured values is given in Tab. 3 as example, some resonance curves are shown on Figures 6 and 7.

Table 3. Natural frequencies $f_0$ [Hz] and logarithmic decrement $\delta$ of free vibrations of the column made without mortar and with separated horizontal layers.

<table>
<thead>
<tr>
<th></th>
<th>Bare bricks</th>
<th>Separated layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>1.96–2.53</td>
<td>2.53–3.57</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.138–0.078</td>
<td>0.322–0.431</td>
</tr>
</tbody>
</table>

Figure 6. Stationary response of the bare brick column (ratio of the RMS of the motion of the top and of the table) to harmonic excitation with increasing (■) and decreasing (○) frequencies. a – excitation 0.2 mm, b – 0.30 mm.
The experiment has confirmed that the above given discrepancies can be explained by nonlinear behaviour of masonry, the joints of which are opening to a certain extend, thus the wall vacillates, together with bending.

7 CONCLUSIONS

The presented analyses and measurements have revealed that

- the swaying bells excite the structure by considerable horizontal forces with frequencies of one, two and three multiple of the bells' motion. This excitation can be amplified by resonance effect of an arbitrary structural element, which is tuned to one of these frequencies.
- the gable wall of the church under consideration is most probably damaged due to the ageing process of its masonry, and has to be retrofitted either by restoration the mortar joints or by reinforcing the gable, e.g. by an additional supporting system inside the church garret.

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REFERENCES

