A displacement-based approach for the safety assessment of masonry structures

J.A. Ochsendorf
Massachusetts Institute of Technology, Cambridge, MA, USA

ABSTRACT: Historic masonry structures are subjected to severe displacements over their long life. Engineers are frequently asked to determine the safety of masonry structures that have been distorted grossly over the centuries, often due to subsidence or other long-term movements in the foundations. This paper proposes a new approach for assessing the safety of masonry structures based on their capacity for imposed displacement. In considering the applications of displacements to masonry structures, there is a need for new theoretical and analytical approaches. This paper presents a theoretical framework for this approach and gives several examples to illustrate its application. Structural analyses based on imposed displacements illustrate the potential for new safety assessment techniques which are appropriate for historic masonry structures.

1 INTRODUCTION

For historic masonry buildings, the applied loads such as wind or snow loading are often small in relation to the weight of the structure, and the elastic deflections due to these applied loads are very small. But in a masonry vault supported on buttresses, the actual distortion of the structure with respect to its as-built shape can be very large, often exceeding 300 mm in the case of large churches or cathedrals. These distortions tend to increase throughout the life of the structure, usually on account of foundation movements. The collapse condition of such structures may depend on the size of these slowly-increasing displacements, and not on the magnitude of the applied loads. In most cases, the safety of traditional masonry structures depends on their stability, rather than on the strength of the material (Heyman 1995). For all of these reasons, structural engineers often have difficulty assessing the safety of historic masonry buildings.

To determine the safety of a historic building, it is necessary to consider the mode of failure and the conditions that would lead to collapse. The collapse of a masonry structure may be caused by one of three general actions:

1) Applied loading (as in the overloading of a masonry bridge);
2) Applied displacements (as in the differential settlement of foundations); or
3) Applied ground accelerations (as in the case of a strong earthquake).

Engineers have explored the first action in great detail, particularly for masonry arch bridges, and the response of masonry structures to applied loading continues to be the focus of much research (Boothby 2001). However, overloading is generally not a problem for masonry buildings such as vaulted cathedrals. The second action of applied displacements is a very real problem, particularly due to the long-term movements of the deformable foundations of a masonry building. Likewise, ground accelerations as a result of seismic activity are also a significant threat to historic masonry building. For many masonry structures, such as bell towers or Gothic cathedrals, increased displacements can lead to collapse and should be investigated. For these structures, displacement-based approaches may be more valuable than conventional load-based approaches. This paper focuses on the second action, and seeks to determine the influence of applied displacements on the stability of masonry structures. The goal of the paper is to spur further research in the use of imposed displacements as a means of testing the relative safety of historic masonry constructions.

As an example of this approach and the new problems which must be solved, an arch supported on buttresses serves as a case study. Over time, the buttresses may lean outwards and destabilize the structure. As the buttresses lean, the buttress resistance is reduced and the thrust of the arch increases. With sufficient displacement, the thrust of the arch can exceed the capacity of the buttress and collapse will occur. Figure 1 illustrates such a vaulted structure.
with severe deformations over time. Even with the simplified material assumptions of limit analysis, the structural behavior of buttressed arches is complex. Increased deformations can lead to greatly increased internal forces. A first-order structural analysis, based on theories of small deformations, will not reveal the sensitivity to additional movements that can determine the safety of the structure. Huerta and Lopez (1997) analyzed this problem in some detail and inspired the author to pursue the problem more generally. In order to assess the stability of an arch on buttresses, three distinct problems must be tackled:

1) The change in arch thrust as the supports undergo large displacements and the required displacements for the arch to collapse;
2) The resistance of a buttress to horizontal thrust, taking into account the influence of leaning; and
3) The collapse configuration for an arch-buttress system due to large displacements.

The author has analyzed these problems for circular masonry arches supported on rectangular buttresses and has identified the collapse conditions for various geometries (Ochsendorf 2002). Based on these considerations, new methods are proposed for assessing the safety of masonry structures and for determining the influence of future movements on the stability of existing masonry structures. The aim of the paper is to illustrate that large displacements are a significant concern for historic masonry structures and that engineers should determine the magnitude of support displacements that will lead to collapse.

By investigating the destabilizing effect of imposed displacements on the possible collapse mechanisms, this analysis represents an extension of conventional limit analysis as proposed by Heyman (1995). In considering masonry structures as an assemblage of rigid blocks, this paper makes the well-known assumptions of infinite compressive strength, zero tensile strength, and sufficient friction to prevent sliding. However, the methods proposed in this paper are not restricted to limit analysis and can also be applied in cases where the investigator is concerned with the elasticity of the material. Therefore, displacement analysis can be conducted in the context of limit analysis as well as nonlinear analysis using finite element methods (FEM). For analyses with FEM, the user must apply support displacements, perhaps postulating hinge locations according to the actual state of the structure. For example, a masonry barrel vault with a clear longitudinal crack at the crown of the vault indicates the location of a hinge along the vault. The finite element user can model this condition by releasing the restraints at this location. This is an obvious extension of FEM, which allow for support translations and rotations, and has more physical meaning that applying large concentrated loads. The purpose of this paper is to illustrate that imposing displacements to a historic masonry building has more physical meaning than imposing applied loads. In summary, we are looking for a measure of the stability of the structure by seeking the displacements necessary for collapse to occur. This capacity for displacements is at least as important as the load capacity of a historic building, and in the case of masonry construction, displacements pose a far greater threat than applied loading.

2 ARCH ON SPREADING SUPPORTS

Few researchers have investigated the influence of displacements on the stability of masonry arches. Almost all recent studies on the safety of arches have been concerned with the stability of an arch under applied loads. In the 19th century, Viollet-le-Duc investigated the collapse state of the grossly deformed vault in the church at Vézelay, France, but he did not consider the general problem of arches on spreading supports (Viollet-le-Duc 1854). (Fig. 2) More recently, Smars (2000) has studied the stability of arches and vaults, considering the influence of displacements. Smars identified the domain of statically admissible movements for a chosen mechanism in a semi-circular voussoir arch.

Although these studies have considered the influence of movements, they have not investigated the implications of movements on the horizontal thrust of the arch. For masonry arches, a small increase in the span leads to a deformed geometry, which increases the horizontal thrust. For a given arch, the maximum displacement before collapse, as well as the corresponding horizontal thrust for a given increase in span length, must be determined.

The problem is illustrated in Figure 3 for a circular arch. As one support is displaced horizontally by a distance \( x \), the crown of the arch descends and the
Figure 2. Deformed arch in the church at Vézelay, France due to spreading supports (Viollet-le-Duc 1854).

Figure 3. Circular arch segment with spreading abutments at state of minimum thrust.

horizontal thrust, $H$, changes as a result. It is well-known that a perfectly constructed arch supported on rigid supports can resist a range of thrust values between the maximum thrust and minimum thrust (Heyman 1995). However, most arches exist in a deformed state due to movements of the supports. The arch thrust may cause the abutments to spread apart, increasing the span of the arch. As soon as the abutments spread apart, the arch exists in a state of minimum thrust. The smallest outward movement of the abutments will cause the arch to form three hinges (A, B, and C in Figure 3). At this point the arch is statically determinate and the abutment thrust can then be determined uniquely from the geometry and the location of the hinges. In this case there are intrados hinges at A and C, and an extrados hinge at B. (For small angles of embrace, $\alpha$, the intrados hinges will form at the abutments.) If the span continues to increase, the arch will deform according to rigid-body kinematics. Eventually the arch will collapse due to the excess displacement or in some cases, the thrust of the deformed arch may exceed the capacity of the buttresses supporting the arch.

Solving for the collapse state of a masonry arch on spreading supports is not a trivial problem. Ochsendorf (2002) showed that the location of the intrados hinge C at an angle of $\beta_o$ from the vertical can change as the arch geometry changes. A detailed discussion of this problem is beyond the scope of this paper, which aims to illustrate the need for such displacement-based analysis of masonry structures. While hundreds of papers have addressed the load capacity of masonry arches, surprisingly few have considered the displacement capacity of arches and the influence of displacements on the horizontal thrust of masonry arches.

To investigate this problem, the author conducted experiments on model arches on spreading supports (Ochsendorf 2002). Figures 4 and 5 illustrate the undeformed arch and the collapse state for a circular arch of 10-degree voussoirs subtending a total angle of embrace of 160 degrees. The thickness of the model arch is 13% of the centerline radius. In this case, the intrados hinge forms initially at $\beta_o = 50^\circ$ from the crown of the arch, though the hinge shifts to $\beta_o = 60^\circ$ at the collapse state. The collapse mechanism is a symmetrical five-hinge mechanism in theory, though only four hinges are required for collapse. The arch collapses when the span is increased by approximately 8% at which point the minimum horizontal thrust has increased by more than 50%. At the collapse state, the crown of the arch has descended by an amount almost equal to the thickness of the arch. Thus, small changes in the support conditions can lead to gross deformations and eventual collapse, with drastic changes in the internal forces in the structure. Such gross deformations are visible in many Romanesque and Gothic vaults throughout Europe.
In the case of historic masonry buildings, the stability of the arch or vault will often depend on the resistance of the arch to support movements and this is an area which requires further research. Such an approach has more physical meaning than applying heavy loads on building vaults. The author is currently investigating the collapse conditions for arches of various geometries, including pointed arches. Such two-dimensional results can then be extended into three dimensions to consider the collapse of domes and vaults.

3 LEANING BUTTRESSES

The deformation of arches is often due to the movement of walls or buttresses which lean over time. Outward leaning is the greatest threat to the stability of a wall or buttress. Most masonry buttresses exist in a state of leaning, which increases throughout the life of the structure. In general, the lean of a buttress may be due to any or all of the following:

- Deformation and subsidence in the foundations during construction;
- Construction defects or small movements between stones;
- Elastic deformation of the masonry and mortar (usually small);
- Additional subsidence due to changes in the soil conditions (caused by consolidation of the soil, enhanced by changes in the water table, adjacent excavations, long term creep of the foundations, etc.);
- Ratcheting movements of the stones due to vibrations of the structure (from earthquakes, bell-ringing, wind loading, etc.); and
- Seasonal effects of temperature and moisture over long periods of time.

As the buttress leans, the eccentric loading on the foundations can cause additional leaning, which will continue to increase during the life of the structure. The lean of the buttress causes an increase in the span of the vault or arch, which will increase the thrust of the arch. In some cases, the increased thrust of the arch may exceed the decreased horizontal thrust capacity of the leaning buttress, leading to collapse as a result of deformations.

A leaning buttress overturns at a lower load than the same buttress in a vertical position, due to the horizontal shift of the centroid of the buttress. Even small amounts of leaning will significantly alter the equilibrium conditions. For buttressed vaults, a small amount of leaning in the buttress, such as 1° from vertical, will alter the line of thrust and increase the applied thrust of the vault (Huerta and López 1997). For example, consider an arch spanning five meters, which is supported at a height of 10 meters. If each buttress leans outward by one degree, then the horizontal movement at the height of the arch will be approximately 0.17 meters on each side. Over the arch span of 5 meters, this constitutes an increase in span of approximately 7%, which is sufficient to cause the collapse of many typical masonry arches.

In addition, recent work by Ochsendorf et al. (2003, 2004) has illustrated the potential for a fracture to form in a masonry buttress at the collapse state, reducing the capacity for horizontal thrust by up to 30%. Experiments on model buttresses have illustrated the existence of this fracture for buttresses under horizontal loads (Ochsendorf 2002). Figure 6 illustrates two model buttress configurations at the collapse state. The buttress on the right is loaded with a vertical surcharge in addition to the horizontal load, while the buttress on the left is subjected only to its own weight and the horizontal force. The fracture is slightly different in each case as a result.

As the buttress leans outward, the centroid of the buttress shifts. Typical values of lean for historic buildings may be on the order of one or two degrees, though in extraordinary cases, such as the tower of Pisa, the lean from vertical can approach five degrees or more. Regardless, for values of lean less than five degrees it is appropriate to approximate the movement of the centroid using the small angle assumption that it is a linear decrease in buttress capacity for small angles. As the centroid of the buttress shifts horizontally, the load capacity of the buttress is reduced linearly. The exact reduction in capacity depends on the geometry of the buttress.

Figure 6. Tests on model rectangular buttresses showing the collapse state of a buttress due to horizontal thrust at mid-height.
4 ARCH ON LEANING BUTTRESSES

The results for an arch on spreading supports can be combined with the analysis of the leaning buttress to gain a greater appreciation for the stability of an arch on leaning buttresses. This problem has been encountered by several researchers in recent years, such as Huerta & Lopez (1997) and Deshpande (2001), and is illustrated in Figure 7. For such a deformed structure, it is crucial to understand the source of the displacements and to determine the capacity of the structure for increased displacement.

The author’s PhD dissertation examined this problem in detail and concluded that there are two dominant modes of collapse due to the outward leaning of the buttress (Ochsendorf 2002). In some cases, the arch may collapse before the buttress capacity is reached, in a mode defined as “strong-buttress” failure. For strong-buttress failure, the support displacements are too large for the arch to stand and the arch collapses, even though the thrust of the arch has not exceeded the capacity of the leaning buttress. Alternatively, failure may occur by a “weak-buttress” mode, in which the horizontal thrust capacity of the buttress is exceeded. In this case, one buttress gives way due to the increased thrust of the arch. As the buttress leans outward, the arch collapses. Though the buttress capacity is exceeded, the arch will collapse and the buttress will remain standing. Once the arch collapses, it will no longer exert a horizontal overturning force and the center of mass of the buttress will remain within the thickness of the buttress. Thus, even in the case of “weak-buttress” failure, the buttress will not collapse. For all cases, the displacement-based approach provides insight into the possible collapse modes for a vaulted masonry structure.

5 DISCUSSION

The use of a displacement-based approach for the assessment of vaulted masonry structures provides many advantages. The arguments in favor of this approach can be summarized as follows:

1) Small movements in the geometry of masonry structures can greatly alter the equilibrium configuration and can lead to collapse.

2) In order to assess the safety of historic monuments, engineers should be most concerned with the collapse state, rather than the current state or previous state. By determining the magnitude of displacements required to cause collapse, engineers can assess the relative stability of a structure.

3) In the case of historic buildings, collapse is more likely to occur from excessive support displacements than from excessive loading.

4) This method can provide a measure of comparison which is useful for assessing a large group of structures. For example, 30 masonry churches in a particular region can be evaluated quickly to see which require more detailed analysis.

5) A displacement-based approach has physical meaning since the foundations of historic monuments are often insufficient, causing walls and buttresses to lean outwards.

6) Such an approach can be implemented using various structural analysis methods. It can be carried out as an extension of conventional limit analysis by considering the blocks as rigid, or it can be applied to elastic models by introducing support displacements to a non-linear finite element model.

7) There is scope for much additional work in this area, in particular for the consideration of three-dimensional collapse mechanisms in masonry towers and vaults as a result of support displacements.

6 CONCLUSIONS

This paper has proposed a general analytical approach for masonry structures based on the application of
support displacements. The use of such a method has exposed a number of previously unsolved problems in masonry structure, such as the collapse state of an arch on spreading supports and the thrust capacity of a leaning buttress. Together these problems define the collapse conditions for an arch supported on leaning buttresses. Solutions to these problems are presented elsewhere and the current paper argues only for the wider consideration of displacements on the stability and safety of masonry structures. In summary, displacement-based approaches can be implemented with numerous analytical techniques and can provide a powerful method for assessing the stability and safety of historic structures.

ACKNOWLEDGEMENTS

The author would like to acknowledge a debt of gratitude to Professors Jacques Heyman, Christopher Calladine, and Santiago Huerta for their helpful discussions on this topic and other problems in masonry structures.

REFERENCES


