Ultimate behavior of masonry arches reinforced with FRP at the intrados: comparison between analytical and numerical models

U. Ianniruberto  
*Department of Civil Engineering, University of Rome “Tor Vergata”, Rome, Italy*

Z. Rinaldi  
*Department of Mechanics, Structures and Environment, University of Cassino, Cassino (FR), Italy*

**ABSTRACT:** The application of FRP materials at the extrados or intrados of masonry arches sharply modifies the bearing capacity and the failure mode of the structures. In this case the classical limit analysis theory, suitable for the collapse behavior of un-reinforced masonry structure, cannot be applied due to the FRP presence, which prevents the formation of a mechanism. The evaluation of the ultimate behavior of reinforced arches with a suitable analytical model has been already proposed by the authors. Aim of the paper is a validation of this model by means of a numerical procedure developed with a FEM program, able to account for the presence of the composite material bonded at the surface. In particular in this paper the analyses will be devoted to the collapse response of masonry arches reinforced with FRP at the intrados and subjected to vertical loads. On the basis of the comparison with numerical outcomes the effectiveness of the analytical methodology in predicting the ultimate load and the failure mode is pointed out.

1 INTRODUCTION

The adoption of composite material in the framework of strengthening and rehabilitation of masonry structures has encountered a great favor in both scientific and technical fields. This aspect is witnessed by the available studies on this topic and by actual application on real structures.

The presented study is devoted to the analysis of masonry arches as this typology is widely spread, particularly in the Italian building heritage. The adoption of FPR sheets on the surface of these structures proves to be the most competitive solution for increasing the strength, compared to more traditional techniques, due to the ease of application.

The increasing interest in this technique and particularly the frequent interventions carried out with it highlight the need for the engineer to design the strengthening intervention with suitable models, able to catch the ultimate bearing capacity of the reinforced structure and its failure mode. With particular reference to the arches, if the collapse load of the unreinforced system can be easily evaluated by using the limit analysis theorems (Kooharian, A. 1953, Heyman, 1966 and 1982), different procedures are needed when FRP sheets are adopted. The application of the composite material at the entire innermost or outmost surface of arches, in fact, prevents the opening of “classical” hinges at the un-reinforced side of the structure. Therefore the hinge pattern usually does not allow the onset of a kinematically admissible mechanism and the ultimate strength of the structure is governed by local failures due to masonry crushing, masonry slip at a mortar bed joint or FRP debonding.

Como et al. (2001) proposed some simplified models to evaluate the internal forces and then the ultimate strength of arches completely reinforced with FRP at the intrados or at the extrados, and subjected to a vertical force at the crown. Furthermore, in order to evaluate the effectiveness of the reinforcement, a parametric enquiry is carried out in (Ianniruberto et al., 2004) on reinforced arches, by varying the position of the load application point.

Aim of the present paper is a comparison between analytical and numerical response of a reference arch reinforced with FRP at the intrados and subjected to vertical loads.

The effectiveness of the analytical model in predicting the ultimate behavior of this particular scheme, both in terms of collapse load value and collapse mode will be pointed. Furthermore the sensitivity of the reinforced arch to the vertical load position will be highlighted.
2 ANALYTICAL MODEL

The adoption of the FRP technique in strengthening a masonry arch at the extrados or at the intrados, besides varying the ultimate capacity of the structures, modifies its collapse mode (see for instance Briccoli Bati, & Rovero, 1999, Como et al., 2001 and Valluzzi et al., 2001).

The formation of a mechanism is, in fact, prevented by the composite material, as the classical hinges can form only at the reinforced surface.

In Figure 1 the hinge arrangement at collapse of an un-reinforced arch is compared with the one reinforced at the intrados, in the hypothesis, often satisfied, that the FRP sheets be not bonded below the springing of the structure. In this case the hinges filled in black can open, while the formation of the blank one is prevented by the FRP material.

At each cross section the thrust line can be external to the arch thickness only at the opposite side of the reinforcement but, because of the previous assumption, at the springing sections the thrust line is forced to lie within the masonry thickness, as occurs in the un-strengthened scheme.

The collapse of the reinforced structure is then governed, not by a mechanism formation, but by a local failure at some critical sections due to masonry crushing or debonding phenomenon at the masonry- FRP interface or slip at a mortar bed orthogonal to the arch axis. This last failure mode has not been considered in this paper.

The evaluation of the ultimate behavior of the reinforced arches, then, requires a correct definition of the material properties and of the failure criteria. In this paper the masonry is simulated as suggested by the EC6 with a maximum strength equal to 10 MPa. The influence of the shear stresses on masonry strength is neglected.

The FRP sheet is considered linear elastic in tension and unable to sustain any compressive stress. The main geometrical and mechanical properties of the adopted composite reinforcement are reported in the Table 1.

In this paper the collapse behavior of a reference arch (Fig. 1), reinforced at the intrados and subjected to vertical load with different application points along the structure, is evaluated. The mean radius of the arch $r$ is equal to 1.5 m, its thickness $t$ and its width $b$ are set equal to 0.35 m, and the abutment angle $\theta = 10^\circ$.

The extension to different geometries and load conditions and then the assessment of the obtained results is now in progress.

2.1 Masonry arches strengthened with FRP at the intrados

The analytical evaluation of the collapse behaviour is carried out on the reference masonry arch reinforced with FRP at the intrados and subjected to a vertical load. Different application points along the arch span are considered.

In the considered case the hinge formation is prevented at the extrados except for the sections at the abutments (Fig. 2).

The position of the hinges is not known a-priori, because it is a function of both the load value and application point. Indeed, besides the right abutment hinge, the two hinges at the intrados must be located in the sections where the shear is equal to zero, in order to obtain an admissible thrust line (Como et al., 2000). This condition, if we adopt the hypothesis of negligible self-weight, acceptable for reinforced arches, can be satisfied by the geometrical construction of Figure 2 (Ianniruberto et al., 2004).

In particular it can be noted that the hinge $C_2$ is fixed for each load position, while the hinge $C_3$ moves downwards as the force moves from the keystone to the abutment. In particular when the hinge $C_3$ reaches the abutment its position remain fixed, even if the distance of the load application point from the crown increases. For this reason it exists a limit position for the load ($x = 864$ mm, in the examined case) and beyond it the hinge is always placed at the abutment.

<table>
<thead>
<tr>
<th>Table 1. FRP properties.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Tensile Young modulus</td>
</tr>
<tr>
<td>Tensile strength</td>
</tr>
<tr>
<td>Ultimate strain</td>
</tr>
</tbody>
</table>
As a consequence of the hinge pattern, the reinforced arch, close to the collapse, behaves as a statically determined structure. This allows the evaluation of the internal loads and then, by imposing the flexural failure criterion in the critical section, the definition of the ultimate load.

The debonding of the sheets is taken into account by reducing the ultimate strain in the fiber $\varepsilon_{u,\text{FRP}}$, according to the approach suggested in the FIB Bulletin n.14, (2001) for FRP reinforced concrete structure.

Due to the lack of experimental and theoretical results two values of the limit strain in the FRP are considered: $\varepsilon_{\text{lim}} = 0.4 \varepsilon_{u,\text{FRP}}$ and $\varepsilon_{\text{lim}} = 0.2 \varepsilon_{u,\text{FRP}}$. For both these conditions the ultimate load is plotted versus its application points in Figure 3.

As expected the failure is always governed by the FRP debonding and then the reduction of the limit strain from 0.4 to 0.2 $\varepsilon_{u,\text{FRP}}$ leads to a shifting of the curve towards lower values of the ultimate capacity.

The critical section, where the failure occurs, is located at the load application point, for every load position. The maximum bearing capacity is reached when the vertical load is at the crown (about 225 kN and 120 kN for $\varepsilon_{\text{lim}} = 0.4 \varepsilon_{u,\text{FRP}}$ and $\varepsilon_{\text{lim}} = 0.2 \varepsilon_{u,\text{FRP}}$, respectively), while the minimum ultimate load is attained when the force is located at the haunch (about 90 kN and 50 kN for $\varepsilon_{\text{lim}} = 0.4 \varepsilon_{u,\text{FRP}}$ and $\varepsilon_{\text{lim}} = 0.2 \varepsilon_{u,\text{FRP}}$, respectively).

### 3 NUMERICAL MODEL

The collapse behaviour of the reference arch is evaluated also by means of a F.E.M. program – Fiber implemented by (Petrangeli, 1999).

The structural cross-sections are modelled with fibre elements, in the classical hypothesis of plane sections. Due to this assumption the FRP debonding needs to be simulated, once again, by fictitiously reducing the ultimate strain of the composite material. Improvement of the model through the introduction of an interface element, able to simulate the phenomenon and also to validate the adopted simplified procedure is now in progress. The main purpose of this study will be the assessment of the reduction factor to be applied at the ultimate FRP strain, in order to account for the debonding, particularly in the case of masonry-curved surfaces.

The fiber number in which each section is divided, equal to 30, and the number of elements constituting the arch, equal to 26, are defined on the basis of preliminary sensitivity analyses.

The masonry is simulated as a no-tension material, with the non-linear compression behaviour reported in Figure 4, limited to an ultimate strain equal to 0.35%, in agreement with the constitutive law assumed in the analytical model. The FRP sheets are considered unable to sustain any compressive stress and are characterised by an elastic behaviour. The material properties agree with the ones adopted for the analytical model (Tab. 1).

The analyses have been performed both in displacement and force control, by increasing the external action up to the structure failure.

A preliminary study is carried out with reference to the un-reinforced scheme, in order to evaluate the effectiveness of the program in simulating the arch behaviour and in particular its collapse mode, governed by a mechanism formation.

The comparison between analytical and numerical results is reported in Figure 5, where the collapse load ($F_u$) is plotted versus the distance of the point load from the keystone. The numerical results, as can be noted, are quite perfectly coincident with the ones obtained through the application of the classical limit analysis approach.

Furthermore the numerical thrust lines at collapse are plotted for three positions of load and are in perfect agreement with the ones required by the static and kinematics theorems, as can be observed in Figure 5 where the points in which the thrust line touches the surface of the arch are indicated with the classical symbol of hinge.
As known, the maximum bearing capacity of the un-reinforced arch, \((F_u = 27 \text{ kN})\), is obtained when the load is applied at the crown, whilst the minimum strength is related to the force at the haunches \((F_u = 8 \text{ kN})\).

**3.1 Masonry arches strengthened with FRP at the intrados**

The behaviour of the arch reinforced with FRP at the intrados is numerically evaluated by means of a pushover analysis. The external load is increased up to the collapse value, defined as the reaching of the ultimate strain in one of the materials. The self-weight is accounted for in the analyses.

The obtained results confirm that no mechanism occurs in the arch for each load position. A large amount of outcomes are available, and in particular it is possible to follow the whole stress–strain pattern for each section of the arch. Anyway in order to validate the proposed model, related to the collapse stage of a reinforced arch, we will focus our attention on the ultimate behaviour of the structures.

Different load positions are considered varying from the crown to the abutment and some of the obtained results are summarised in Figure 7 where the ultimate load is plotted versus the load application point.

Once again two values of the limit strain in the FRP have been considered equal to \(0.4 \varepsilon_{\text{uFRP}}\) and \(0.2 \varepsilon_{\text{uFRP}}\). It is worth noting that as the FRP debonding governs the collapse for every load position, in agreement with the proposed analytical model, the value to be assigned to \(\varepsilon_{\text{lim}}\) sharply affects the ultimate strength. As a consequence, and particularly for these typologies, the definition of a correct, suitable value of the ultimate reduced strain in the composite sheets is a very important question to be solved.

The load applied at the crown provides the maximum bearing capacity. When the force moves towards the springing the ultimate load presents a non-linear behaviour with a minimum related to the force at the haunches.

In order to localize the hinge position, and to make possible a direct comparison with the analytical model, for each section the eccentricity of the internal load,
defining the thrust line, is evaluated, at collapse stage. In Figure 8 some of the obtained curves are plotted and the hinge position is pointed out with the typical symbol.

In the first scheme the external force is located at the keystone and a symmetrical pattern of hinges can be highlighted. Due to the presence of the FRP sheets at the intrados the thrust line can lie outside the arch at the extrados side, as occurs in this case in the cross-sections close to the load application point.

When the force moves towards the abutments the arch becomes a statically determined three-pin arch. In particular the hinges 2 and 3 at the intrados, in Figure 8, moves towards the springing and the keystone respectively, while the hinge at the right abutment remains fixed.

The critical section, affected by the FRP debonding, and then governing the failure of the arch, is located under the force, in agreement with the analytical model.

The discontinuity of the thrust line is related to the discontinuity of the axial load induced by the concentrated force.

4 VALIDATION OF THE ANALYTICAL MODEL

The comparison between the results shown in the previous pages allows validating the analytical model as a powerful tool for the analysis of the collapse behaviour of masonry arches reinforced with FRP at the intrados, subjected to vertical load.

On the basis of the obtained responses the proposed analytical model appears particularly effective in predicting the ultimate behaviour of the analysed reinforced structures. The numerical and analytical collapse behaviours, in fact, are in a perfect agreement, both in terms of hinge location and values of the ultimate load.

The comparison between the two procedures is summarised in Figure 9 where the ultimate numerical and analytical loads are plotted versus their application point for \( \varepsilon_{\text{lim}} \) equal to 0.4 and 0.2 \( \varepsilon_{\text{uFPRP}} \).

The thick curves, related to the proposed model are quite coincident with the thin ones obtained as a result of the Fiber program.

Finally the comparison between the ultimate behaviour of reinforced and un-reinforced schemes, obtained with numerical and analytical results are reported in Figure 10.

5 CONCLUSIONS

The ultimate behaviour of masonry arches, reinforced at intrados with FRP sheets has been analysed in this paper with a proposed analytical model compared to a numerical one. The reference scheme is subjected to a vertical load whose position varies along the span of the arch.

While the un-reinforced scheme fails at the onset of a collapse mechanism, different failure modes occur for the reinforced arches and failure criteria, based on masonry crushing or FRP debonding, are considered. In particular this last phenomenon is accounted for by fictitiously reducing the ultimate strain in FRP sheets.

Due to the uncertainties related to the value to be assigned, the analyses are performed by limiting the FRP strain at two arbitrary values 0.4 and 0.2 \( \varepsilon_{\text{uFPRP}} \). The main assumptions and the results obtained with the proposed analytical model are validated by a numerical procedure based on a FEM program.

The structure is discretized in elements characterised by fiber cross-sections and push over analyses are developed by increasing the vertical load for different application points. In this case the evolution of
the strain pattern and in particular of the thrust line can be followed for each load step.

The comparison among the collapse behaviour of the reinforced arches, evaluated with the two models, gives very satisfactory results, both in terms of failure mode and collapse load values.

The position of the hinges assumed in the proposed analytical model is well confirmed by the numerical results together with the adopted hypothesis of a collapse behaviour evaluated on a statically determined structure.

Finally the paper shows the reliability of the analytical model that proves to be a very simple and powerful tool for the evaluation of the collapse behaviour of masonry arches entirely reinforced at the extrados and loaded by vertical point loads.

REFERENCES


Eurocode 6 – Design of masonry structures, ENV 1996.


