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Masonry Arches: Historical Rules and Modern Mechanics

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ABSTRACT: Ancient structures have been designed according to semi-empirical rules based on few principles; nevertheless, their structural performance is usually rather good. In this paper, the ancient rules for masonry bridges and arches, derived from historical manuals, are analyzed on the bases of modern mechanics. Some issues on the material mechanical characteristics and on the structural response of masonry arches allow a better understanding of the response of old arch structures.

1 INTRODUCTION

The birth date of modern Civil Engineering is in the first half of the 19^{th} century, when building technology asked such a detailed knowledge that the formal aspect and the working plan of a structure could no longer be performed by the same designer. Till then, the structural issues were all gathered in some rules of thumb mainly based on a long-lasting experience and were considered of minor importance if not entirely left to the contractor.

Arches and vaults have been used as an alternative to wooden beams and floors, while arch bridges were the structures at the highest technological level of Civil Engineering. The traditional building rules were joined to the growing mechanics and collected in several theoretical-practical treatises, aiming at a rational and widely spread diffusion of the technical knowledge. Being experience different from one country to another, and being mechanics still a young and growing science, the practical rules are different from country to country. The manuals suggest rules that, from the modern point of view, often represent only a first and simplified step towards a rational approach to Structural Engineering; nevertheless, these rules allowed the construction of thousands of bridges (it is estimated that the population of masonry bridges in Europe is around 400.000 individuals), the vast majority of which is still in service.

In this paper, several manuals have been studied collecting all the information for the interpretation of the ancient rules from the point of view of modern mechanics, i.e. looking at: i) geometrical rules, ii) materials, iii) methods for the assessment of the arches, iv) construction phases, mainly the decentering one. Modern mechanics is applied on the bases of recent theoretical and experimental results (Brencich and De Francesco, 2004, Brencich and Gambarotta, 2005) on prototype arches. The response of the structures allows an estimate of the average actual safety factor of ancient structures, showing that masonry arches exhibit much higher limit loads than is usually expected. Discussion is provided on the mechanical principles those rules had been based on.

2 HISTORICAL MANUALS

The technical rules related to brickwork arch barrels derived from some historical manuals are discussed in this section. Due to the period in which the manuals have been written covering something more than a century, and probably gathering the practical knowledge of a wider period, they show the evolution of the technical practice throughout the nineteenth century. The main issue of this section is the analysis of the rules of thumb suggested by the authors aiming at the interpretation in the frame of modern structural mechanics. Some notes on the materials are discussed in the next section.

2.1 Arch thickness

The arch thickness, probably the most important geometric parameter of an arch, is provided by all the authors on the bases of unclear grounds, probably originating from experience, from some mechanical assumption, i.e. the *Mery Method*, and some personal and/or practical optimization of previous formulas, summarized in Figs. 1a and 1b referring to brickwork arches only and to the most important authors and technical codes.

Croizette-Desnoyers in 1885 (Baggi, 1926) worked out an empirical formula on the basis of a statistical survey of the French existing bridges:

$$e = a + b\,(2r)^{1/2} \tag{1}$$

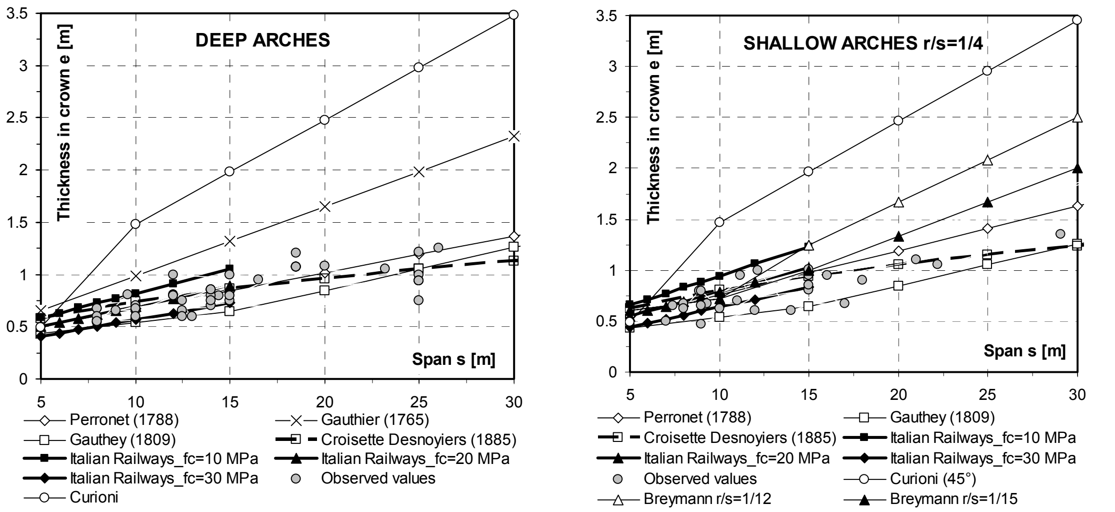
where e stands for the arch thickness (in crown), r for the arch radius while a and b are parameters depending on the rise-to-span r/s ratio: $a\in[0.11,\ 0.15]$m, $b\in[0.13,\ 0.17]\text{m}^{1/2}$ if $r/s\in[0.08,\ 0.5]$.

In 1907 the Italian Railways (*Ferrovie dello Stato*) issued the first National Technical Code, in which the crown thickness of arch bridges e was given as a function of the arch span s:

$$e = a + b\,\mathrm{s} \tag{2}$$

where the constants a and b are similar to those of eq. (1) depending also on the brickwork compressive strength f_b: $a\in[0.25,\ 0.40]$m, $b\in[0.032,\ 0.095]$m if $r/s\in[0.10,\ 0.5]$ and $f_b\in[10,\ 20]$ MPa.

Fig. 1 compare these rules of thumb to the actual values measured in existing Italian railway bridges: i) different authors suggest substantially different rules; ii) the rules of thumb would suggest a dispersion of the actual arches that is not found in existing bridges, probably because the empirical rules gather the knowledge of a large period of time while the Italian bridges have been built in 150 years; iii) the approach suggested by the railway authorities seem to best fit the actual arch geometries; iv) the existing arches considered in this paper refer to arch bridges only, so that different outcomes could be found if data from the vaults of ordinary buildings are considered.



(a) (b)

Figure 1 : Thickness in crown vs. span for a) deep and b) shallow (rise/span = 1/4) arches. N-W Italy Dep.ts

3 MATERIAL DATA AND CONSTITUTIVE MODELS

The first rational approach to arches is the *Mery Method* (1840), still suggested by Breymann for the assessment of masonry arches, that assumes an arch to be safe under the design loads if the axial thrust follows the *middle third* rule: from the point of view of modern mechanics, this means that the cross section should be entirely compressed. Even though this is not a safety assessment, nowadays we know that such a condition is highly conservative for the vast majority of arches. Besides, it does not need a constitutive model for masonry to be defined. For this reason, in past times masonry was characterized only by: i) the compressive strength of the bricks; ii) the mix of the mortar. The main idea was that the higher the brick compressive strength and the better the workmanship, the better the brickwork would result, but no idea of the safety margin was behind such an approach. For these reasons, Rondelet and Curioni provide specific instructions for (lime only) mortar mix only and some suggestion is given for the manufacturing procedure; Breymann focuses on bricks, probably because the mortar production was considered something to be left to bricklayers.

Some concepts of modern mechanics and material science can be recognized, such as: i) if sand is wet, no water should be added; ii) water should be enough for good workability and nothing more; nevertheless, the proportions are quite approximate, mainly for Curioni, and beach sand is admitted, which is no more acceptable due to its high salt content.

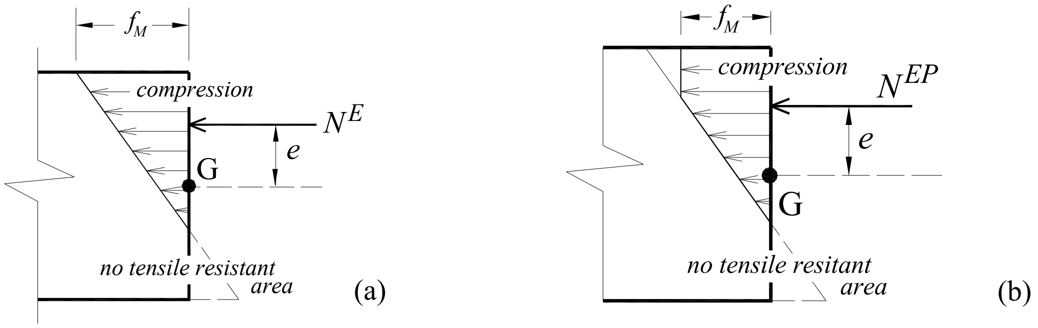


Figure 2 : (a) NTR – Perf. Brittle and (b) NTR-El. Perf. Plastic with limited Available Ductility models for masonry.

The first constitutive model for masonry, in the modern sense, has been formulated by Castigliano (1879) in the frame of a Static approach to arches. Due to the poor knowledge of the mechanical response of brickwork, masonry was assumed a perfectly No-Tensile-Resistant (NTR) material in which the compressive strength is reached just at the end of the elastic limit, i.e. a perfectly brittle material characterized by two mechanical parameters: the compressive strength and the elastic modulus, Fig. 2a.

The Kinematic approach to arches (Heyman 1982) looks for the work done by external and internal forces once a collapse mechanism is activated, assuming that plastic hinges may develop in the arch in a number large enough to transform it into a mechanism which, in turn, is possible if masonry exhibits unbounded inelastic strains. Therefore, this approach asks a NTR-Elastic Perfectly Plastic constitutive steel-like model for masonry; practically, this requirement can be slightly released asking inelastic strains to be 3 to 5 times the elastic limit (Crisfield 1985, Crisfield and Packham 1985).

A unifying approach is that of a NTR-Elastic Perfectly Plastic constitutive model with limited inelastic strains allowed, Fig. 2b; in this case a third mechanical parameter is defined: the available ductility δ_{av}, i.e. the ratio between the ultimate strain ε_{ul} and the value at the elastic limit ε_{el} ($\delta_{av}=\varepsilon_{ul}/\varepsilon_{el}$, Brencich and Gambarotta, 2005). While the Static approach assumes a perfectly brittle material (δ_{av}=1), the Kinematic Approach implies the exact opposite, i.e. an available ductility not less than 3-to-5 ($\delta_{av}\geq 3$).

Figs. 3a, and 3b show the stress-strain response of concentrically loaded solid clay brickwork for a modern (Brencich et al., 2006) and an ancient solid clay brickwork taken from the pier of a railway bridge (approx. 1880), respectively. The materials of Fig. 3a are characterized in table 1, while the materials of the existing brickwork could not be identified. The mechanical response of both the masonry types is linear elastic in the first part of the diagram, than inelastic strains are activated close and after the peak load is reached. On these bases, a NTR-Perfectly Plastic model turns out to be a reasonable approximation of the actual behavior provided that the available ductility is assumed in the range [1.2, 1.5] for cement-lime mortar brickwork and reaches the upper bound of 2.0 for historic masonry, Fig. 3b.

Table 1: Materials for the tests of Fig. 3 and main results of the tests.

	Brick		Mortar 1		Mortar 2	
f_t – av. [MPa]	4.7	c.o.v.: 10%	3.4	c.o.v.: 15%	2.7	c.o.v.: 12%
f_c – av. [MPa]	19.7	c.o.v.: 17%	13.1	c.o.v.: 18%	10.0	c.o.v.: 16%
E – av. [MPa]	1530	c.o.v.: 30%	1546	c.o.v.: 16%	1365	c.o.v.: 22%

MASONRY average values							
	Mortar 1	f_c [MPa]	12.77	E [MPa]	2150	δ_{av}	1.37
	Mortar 2		12.81		2355		1.33

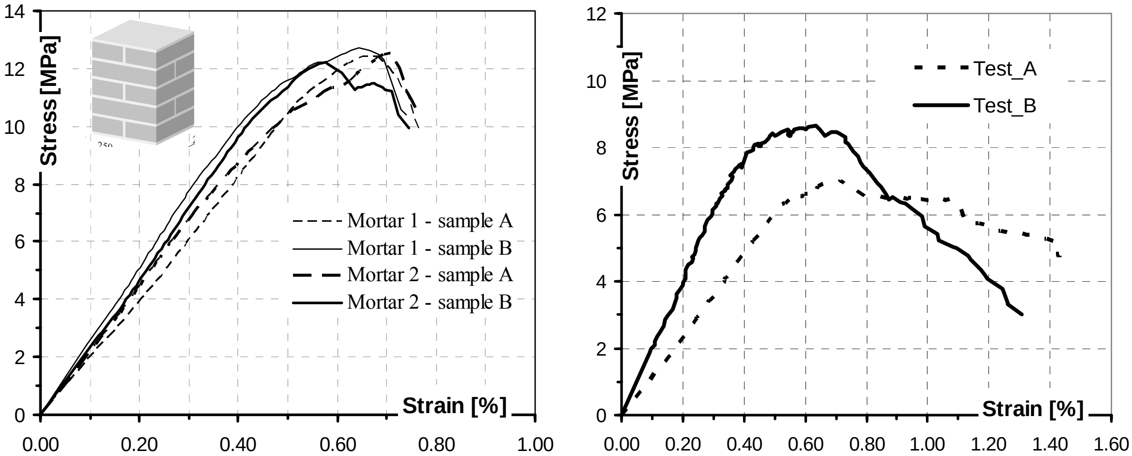


Figure 3 : Stress-strain response under concentric loading for solid clay brickwork: a) 60x120x240 mm bricks, 10mm thick mortar joints – 5 courses; b) 75x140x260 mm bricks, 20mm thick mortar joints – 3 courses.

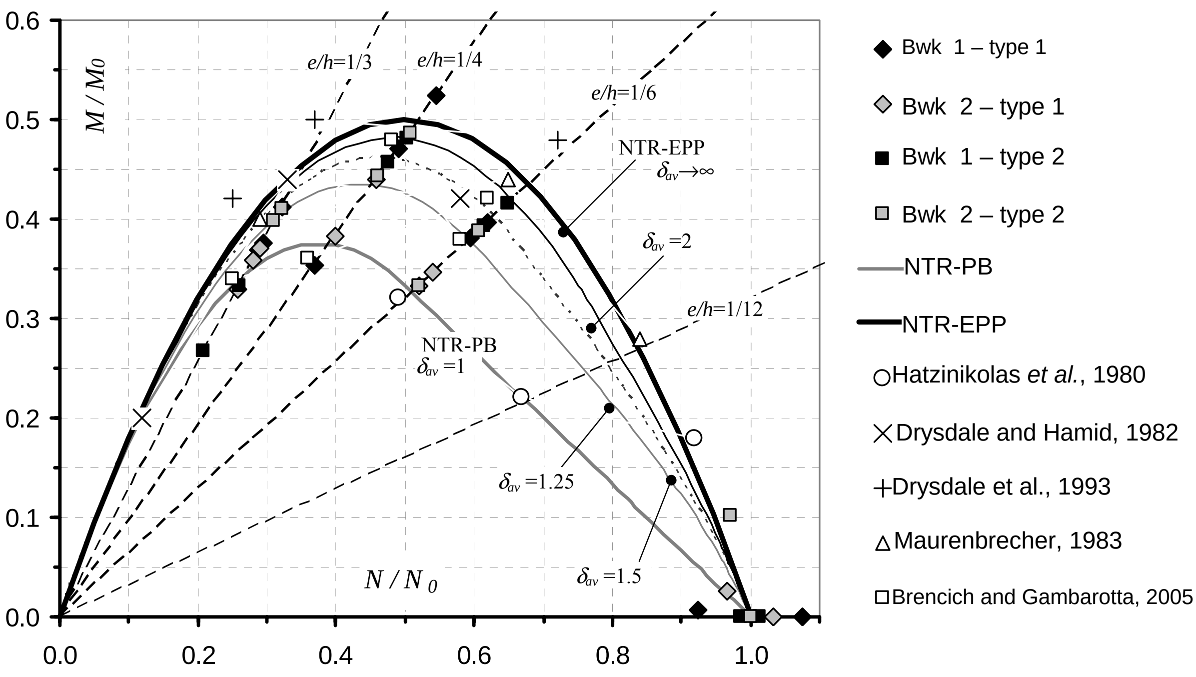


Figure 4 : Limit domains of the NTR–models: exp. data, homogeneous NTR-EPP models with different ductility.

Even though the inelastic phase would be better described by more detailed models then Perfect Plasticity, more detail is not needed for the safety assessment of masonry arches, as showed in the following. The implications of these circumstances are discussed in section 5.

The same conclusions are derived from a large series of eccentric loading tests (Brencich et al., 2006) summarized in Fig. 4 where the limit domains for eccentrically loaded masonry are represented in the N/N_0-M/M_0, plane (N_0=ultimate load for concentric loading, $M_0 = N_0 \cdot h/4$, h being the brick length). The outer parabola is obtained for a NTR-Perfectly Plastic material with unbounded ductility, while the inner domain is that of the classical NTR-Perfectly Brittle model of

Fig. 2a; the other domains, internal grey lines, are obtained setting the available ductility to the measured values of 1.25, 1.5 and 2; the points show the results of tests: the assumption of a NTR-Elastic Perf. Pl. model with Limited Allowed Ductility fits quite well the experimental data; since the widening of the inner domain is due to inelastic strains, modern mechanics needs to assume the inner limit as the reference domain for safety assessments of arches. More detailed discussion on this issue can be found in (Brencich et al., 2006).

4 THE LOAD BEARING STRUCTURE

The standard approach to the assessment of a masonry bridge assumes the arch barrel as the main load bearing structure, the geometry of the arch being defined by a set of data deduced from its external aspect. Some observations on collapsed bridges suggest that the actual load bearing structure, i.e. the "structural arch", is different from what appears from the outside, i.e. the "geometric arch". The position of the actual skewbacks can be roughly identified (Pauser, 2004) connecting the lower point of the backfill to the centre of the arch; Fig. 5 (*Verde viaduct*, North-Western Italy, in service 1st category bridge) shows deep arches spanning 18.5m for which the position of the actual skewbacks is rather different from the apparent external one. In fact, this viaduct presents also internal spandrels (Brencich and Colla, 2002), which suggests that the actual structural skewbacks might be at something more than 40°, at 44.4°, from the top of the pier, dotted lies in Fig. 5.

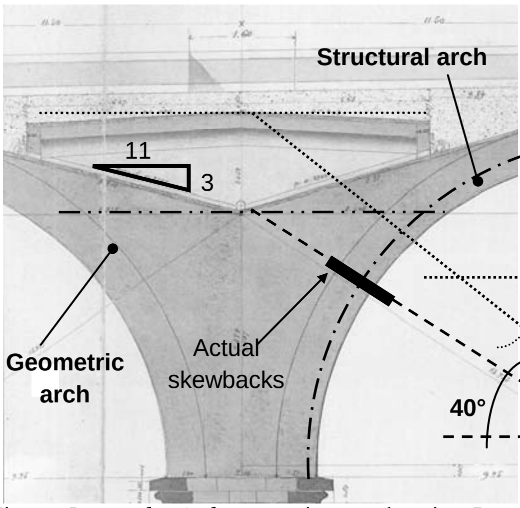


Figure 5 : *Verde viaduct*: section on the pier. Internal spandrels over the backfill.

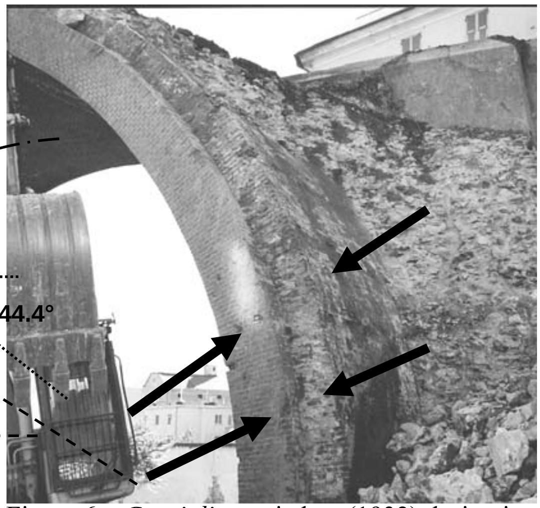


Figure 6 : *Cornigliano* viaduct (1932) during its demolition (June 2001): the barrel thickness is 65% higher than the external apparent value

Whatever the rule for identifying the structural arch, it should be noted that the structural ach is different from the apparent external arch barrel: i) the shallow arches coincide with the actual structural arches; ii) deep arches are, in fact, shallow arches with rise/span ratio in the range [0.20, 0.27]. These conclusions may be applied to any other type of arch (i.e. multi-centre and elliptic arches, etc.): the load bearing structure may be well approximated by a cylindrical arch with span and rise defined according to the previous considerations. The average backfill is 70-80% of the geometric rise, which leads, according to the standard values of the arch thickness and of the pier width, to a $r/s = 0.25$.

Masonry bridges usually exhibit limit loads that are much higher than the calculated values, which explains their good performances in spite of increasing loads and increasing speed. This overstrength might be partially justified on the bases of the previous considerations: i) shallow arches, r/s=0.25, present a collapse load that is twice, on the average, the value for deep arches, r/s =0.35, (Brencich and De Francesco, 2004); ii) the actual rise/span ratio of the structural arches is approx. 0.25 on the average due to the backfill extension.

Some kind of awareness of the fact that the structural arch does not coincide with the geometric one can be found in Curioni, who identifies the arch section sustaining the maximum axial thrust at 30° (referred to the horizontal line) for deep arches, at 50° for shallow arches, and at the geometric skewback if the arch opening angle is 120°. The first two indications mean, for a modern mechanics approach, that the load bearing structure is a shallow arch with rise-to-span ratio equal to 0.29 and 0.18 respectively.

A second basic parameter for the assessment of an arch bridge is the barrel thickness. Often the external arch thickness is different from the internal actual value, as shown in Fig. 6. Statistical data show that most of the arches present a constant thickness equal to $1/20^{th}$–$1/13^{th}$ of the span; when the apparent (external) thickness is outside these values, a higher (or lower) arch thickness has to be suspected and further tests need to be carried out.

5 SOME RELEVANT MECHANICAL PARAMETERS OF THE ARCH

Two sample prototype arches are considered in this section, a shallow (rise/span=0.25) and a deep (rise/span=0.35) one, Fig. 7, aiming at discussing some issues on the mechanical response of arches (Brencich and De Francesco, 2004). The main geometric parameters, arch thickness (75cm $=1/20^{th}$ of the span) and span (l=15m), fill depth in crown (75 cm=arch thickness), are average values of the Italian railway bridge stock as recorded by the statistical data and coherent with the historical manuals, table 2.

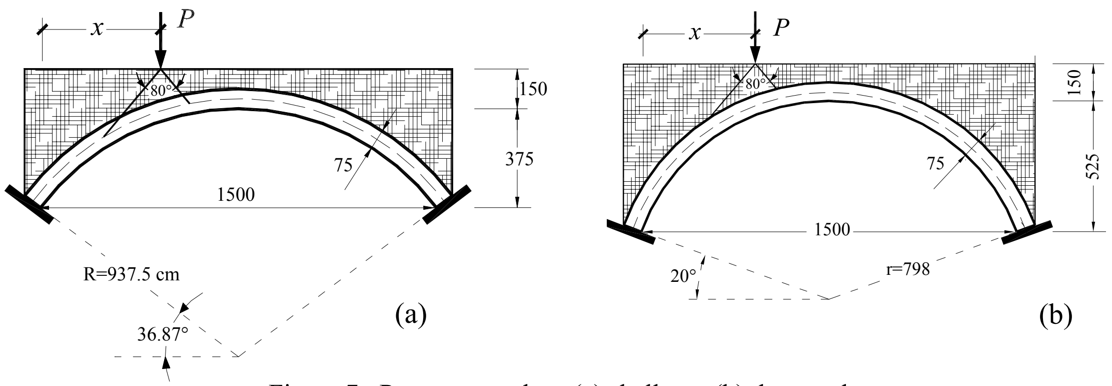


Figure 7 : Prototype arches: (a) shallow ; (b) deep arch.

Table 2 : Main parameters of the sample arches

GEOMETRY				**SHALLOW ARCH**			
Span s	1500 cm	Width	100 cm	Rise r	375 cm	r/s	1/4 = 0.25
Ring th.ss d	75 cm	d/s	1/20	**DEEP ARCH**			
Fill (crown) f	75 cm	f/d	1/1	Rise r	525 cm	r/s	7/20 = 0.35
DENSITY AND LOADS				**MASONRY**	available ductility: δ_{av}=1 / 1.4 / ∞		
Arch density	22 kN/m^3	Load diffusion	40°+40°	Young's modulus	15000 MPa		
Fill density	24.1 kN/m^3	Loaded length	Knife-type	Compressive strength	5 / ∞ MPa		

Figure 8 shows the load-displacement response of the two arches for a concentrated load at $1/3^{rd}$ of the span. The constitutive models of section 3 have been considered: i) NTR model without limit to compressive stresses (NTR-elastic, bold lines); ii) NTR Elastic Perfectly Plastic model (NTR-EPP) with a 5MPa compressive strength and different values for the available ductility, table 3. Circles mark the load at which the compressive strength is attained, squares the load at which the available ductility (δ_{av}=1.4) is reached.

The asymptote of the diagrams corresponds to the load carrying capacity estimated by the Mechanism Method setting no limit to the compressive stresses. If the material is given a finite compressive strength, NTR-EPP models of table 3, the asymptote slightly changes if inelastic strains remain uncontrolled (NTR-EPP_1); a limit to inelastic strains (NTR-EPP_2, δ_{av}=1.4 and NTR-EPP_3, δ_{av}=1) makes the ultimate load to be strongly reduced. For both the arches, the classical NTR elastic model accounts for an ultimate load that is approx. twice the value estimated by a NTR-Perfectly Brittle model, showing a relevant effect of the available ductility. Table 3 shows that the constitutive model (strength and ductility) and the arch geometry (r/s ratio) affect the estimated load carrying capacity.

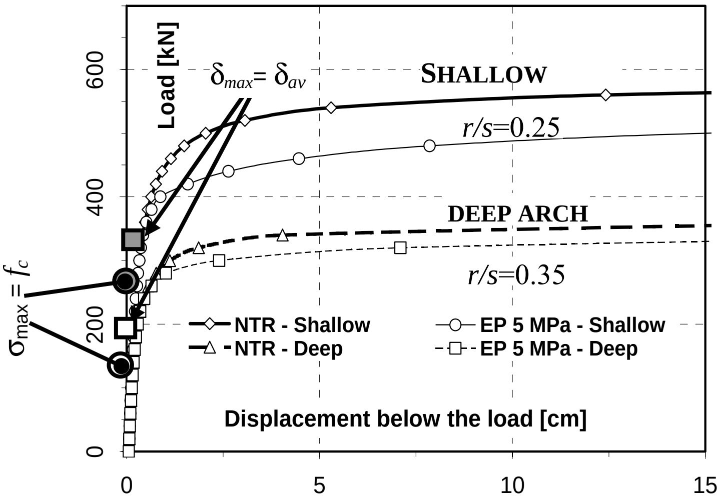


Figure 8 : Load-displacement curves of the prototype arches.

Table 3 : Load carrying capacity of the arches of Fig. 7. NTR Materials

	Material Model	*NTR-elastic*	*NTR-EPP_1*	*NTR-EPP_2*	*NTR-EPP_3*
	Compressive strength [MPa]	$f_c \to \infty$	f_c=5	f_c=5	f_c=5
	Available Ductility	/	$\delta_{av} \to \infty$	δ_{av}=1.4	δ_{av}=1
Ultimate Load [kN]	**Deep Arch** - r/s=0.35	363	347	217	177
	Shallow Arch - r/s=0.25	578	540	325	268

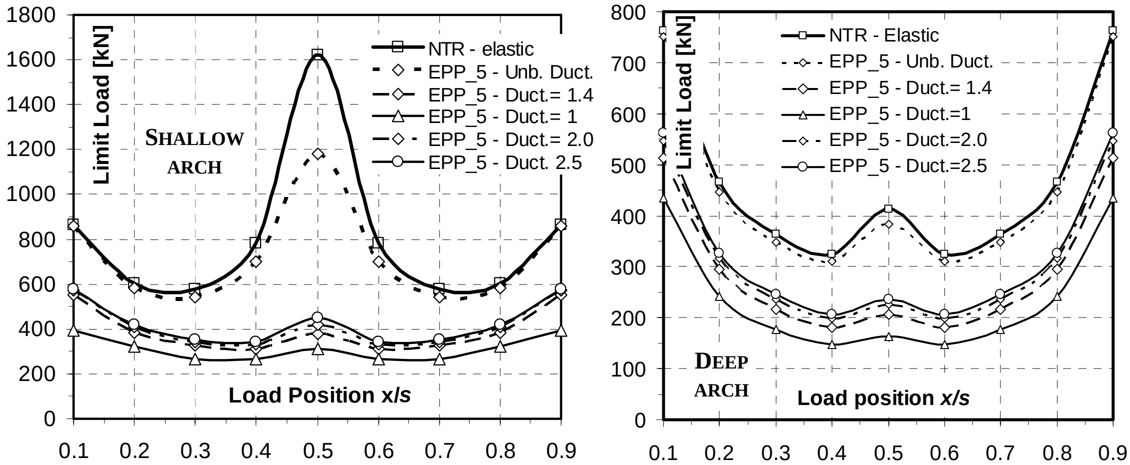


Figure 9 : Limit load vs. load position for the prototype a) shallow; b) deep arches.

Fig. 9 show the ultimate load for the arches of Fig. 8 loaded by a concentrated knife-type force on the fill surface; the position of the load is identified by the distance x of the load from the skewbacks; the load is distributed on the arch by the fill. The upper diagrams (NTR-elastic and NTR-EPP models) show a strong dependence of the ultimate load on its position, with the minimum load being some 2.5 times lower than the maximum value. Nevertheless, the most important effect is related to the constitutive model assumed for masonry: setting a compressive strength and a limit to inelastic strains produces a decrease of the limit load making the load position a parameter of minor importance. With the aim of assessing an arch, no inelastic strain should be considered; under these hypothesis, the limit load seems to be something less than half the ultimate load estimated assuming a NTR-elastic model, i.e. by means of the Mechanism Method. The effect of the arch shape, (r/s ratio) is clearly evident: slight changes (0.35 to 0.25) makes the limit loads to be almost doubled.

6 DISCUSSION AND CONCLUSION

The historical rules for masonry arches, originate from both experience and from the first approaches to modern mechanics. For this reason, some aspect of the building procedure, such as decentering is not defined and some relevant geometric parameter, such as the arch thickness in crown, is given with different formulas, that may lead sometimes to significant differences from one manual to the other. Nevertheless, arch bridges are found to follow the most widespread rules of thumb of ancient authors. Some concepts of modern mechanics were only suspected and not well understood, such as the actual span length, while mechanical characterization was almost unknown. Nevertheless, the arches built according to ancient rules present large safety margins, explaining why thousands of in service masonry bridges still show good performances.

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