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Gothic Roof Structures Modelling

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ABSTRACT: The Gothic roof structures – some of them being 600-800 years old, and representing valuable parts of the wooden culture of Romania – have spatial systems consisting (quite often) of thousands of elements, using an empirical-intuitive conception based on the experiences of carpenters accumulated along the centuries. The numerical modelling – expansively accepted as a consequence of the world-wide use of PCs – is the adequate tool for identifying the mechanical-structural role of each element, as well as of the frames within the spatial unit.

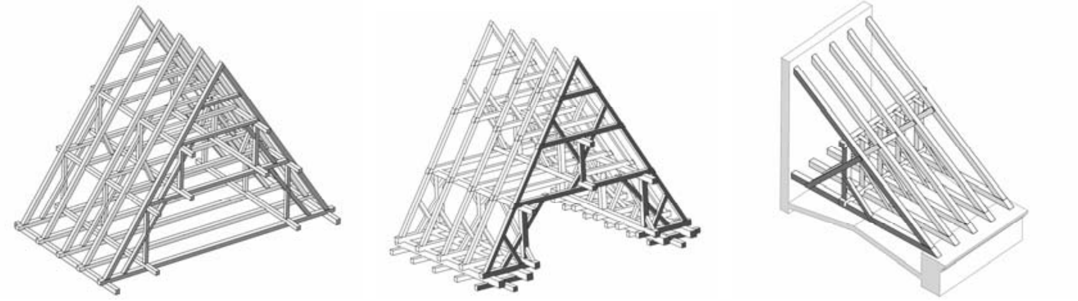
The empirical-intuitive conception needs to be identified – beyond the scientific interest – in order to allow professional interventions aiming the protection of Gothic roof structures. Just in Romania hundreds of Gothic roof structures under use justify the major interest focused on these inestimable heritage values. Within the Built Heritage Research and Design Centre »UTILITAS« in Cluj Napoca, Romania there has been a concern related to the research and restoration of historic roof structures for 15 years. The paper sums up the achievements carried out within the centre.

1 INTRODUCTION – GOTHIC ROOF STRUCTURES ON COMMON RAFTERS AND TIE-BEAMS WITH COLLAR BEAMS

1.1 Gothic roof structures

Load-bearing structural sub-unit placed in order to sustain the roof covering; constitutes a spatial network of lineal load-bearing elements, arranged in cross- and longitudinal bracing frames built as spatial systems proper(with elements placed radial and annular).

The ones arranged in bracing frames (Szabo 2005) were constructed on common rafters and tie-beams with collar beams – Fig. 1a (having a small sub-group on trestles – Fig. 1b), on beams, as well as on common rafters and plates – Fig. 1c.



(a) (b) (c)

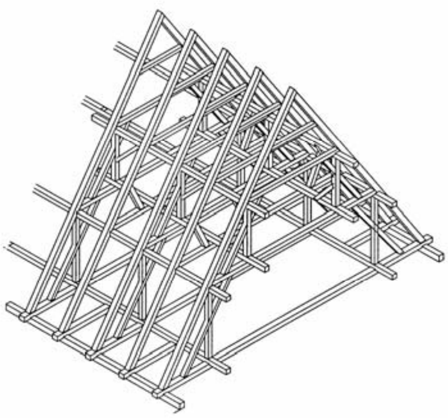
Figure 1 : Gothic roof structures (a) on common rafters and tie-beams with collar beams, (b) on trestles, (c) on common rafters and plates

Historic roof structure, constructed in continental Europe from the 13th century. It's the favourite type of roof of the Gothic and Renaissance architectural styles. In the Eastern part of its area of expansion (in Transylvania – Romania) –, it is also constructed in the first half of the 18th century.

Thousands of Gothic roof structures, hundreds only in Transylvania, have been preserved – unfortunately detailed investigations have not been conducted, the somewhat exact number is not known. From the multitude of the roof categories, the ones on common rafters and tie-beams with collar beams (without trestles) are being discussed, as these are more spectacular and show a greater efficiency (maximum roof spans over 20 m, the maximum volume of the roof being over 6000 m³).

1.2 Gothic roof structures on common rafters and tie-beams with collar beams

Spatial load-bearing sub-unit of great complexity (Fig. 2), able to cover layouts of large sizes (Szabo 2005). The frames placed in two directions allow the taking over and transmission of actions in the following way: the cross frames function at both gravity and non-gravity actions [load-bearing systems with thrusts, made up of elements placed according to triangular outlines (common rafter – tie-beam – common rafter), the gravity actions being divided into slanted compression components – according to the common rafter direction – balanced by the stretching of the tie-beam and by the wall plate reaction], the longitudinal rectangular-formed frames transmit mostly non-gravity actions (usually only in theory II are these employed by gravity actions). The lack of the purlins – as well as of the main discharging elements of the secondary trusses onto the main ones – makes the system more vulnerable: the longitudinal frames (although more efficient in themselves) are not organically connected to the cross frames, and thus the spatiality of the roof suffers damages.



(a)

(b)

Figure 2 : Gothic roof structure on common rafters and tie-beams with collar beams
(a) axonometric projection, (b) common joist of cross and longitudinal frames

2 INVESTIGATION ON ROOF STRUCTURES ON COMMON RAFTERS AND TIE-BEAMS WITH COLLAR BEAMS

2.1 Survey of a load-bearing structure

The survey of a load-bearing structure – usually presented in the technical survey of load-bearing structures – requires the determination of the geometrical dimensions, as well as of the mechanical characteristics of the load-bearing cross sections, elements, sub-units and unit.

(1) The determination of the geometrical dimensions allows a series of particularities worth mentioning: (a) the geometrical dimensions of the structural elements are specified without the dimensions of the architectural details that they support, (b) the geometrical dimensions of the elements and of the sub-units are pointed out by determining their relative positions, specifying at the same time the way they lean and the eccentricities; (c) the local modifications of the dimensions of the structural elements are emphasized; (d) the survey of the load-bearing structure

has to ensure all the necessary data for the determination of the geometrical characteristics of the elements (area, center of gravity, way of resistance, moment of inertia etc.)

(2) The identification of the mechanical characteristics of the historic load-bearing structures (cross sections, structural elements, sub-units and units) implies measurements taken according to criteria belonging to: (a) the materials: own weight, other physical and chemical properties; (b) the resistance of the materials: mechanical characteristics determined through lab experiments on samples; (c) the statics of the constructions: load schemes, supports, joists, grade of fixing; (d) the dynamics of the constructions: grades of freedom, own periods, movements, speeds, accelerations etc.; (e) the technology of the constructions: specific construction technologies, areas constructed with the same or similar technologies, technological aims etc.

2.2 Biological investigations

The biological investigations on the timber used ensure the determination of the biological characteristics, they point out the biological damages and – with the use of the dendrochronological calendar – they allow us to find out the date of extraction of the timber.

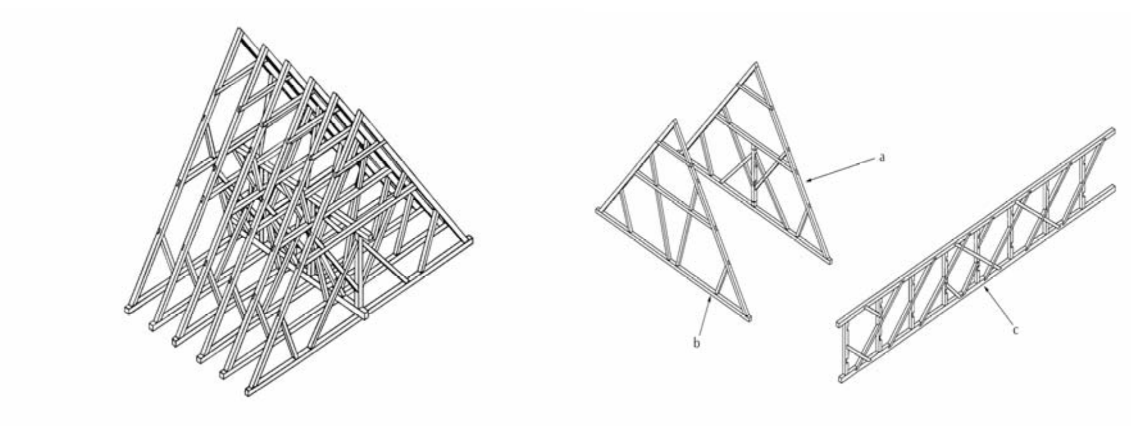
2.3 Mechanical investigations

The mechanical investigations on roof structures imply the dismantling of the load-bearing structural sub-unit into frames and elements, determining their efforts and deformations (structural analysis), as well as the re-building of the elements into frames and spatial sub-unit, identifying the role of the constituent parts in the mechanical behavior of the load-bearing structure (structural synthesis).

2.4 Analysis of the Gothic roof structures on common rafters and tie-beams with collar beams

The analysis – an investigation method for load-bearing structures – implies the dismantling of the historic roof structures into constituent sub-units, which in their turn are dismantled into groups of elements (devices) or elements, according to each case; the load-bearing structural elements resulting from this separation are investigated with the identification of the conceptual, material, technological, geometrical and mechanical characteristics developed in space and time.

The spatial system of the Gothic roof structures on common rafters and tie-beams with collar beams can be grouped into three categories of frames: main cross frames (trusses), secondary cross frames and longitudinal bracing frames (Fig. 3).



(a) (b)

Figure 3 : Frames in Gothic roof structures on common rafters and tie-beams with collar beams, (a) unit, (b) frames

The secondary trusses are built of a tie-beam, an upper collar, collar beams (on one or two levels), (pairs of) common rafters, angle braces, (occasionally) counterbraces and passing braces; the main trusses are built of: a tie-beam, an upper collar, collar beams (on one or two levels), (pairs of) common rafters, angle braces, passing braces and (occasionally) counterbraces, as well as king/ queen posts; the longitudinal bracing frames have (pairs of) plates, king/ queen posts, passing braces and (occasionally) counterbraces;

The material mainly used is hardwood (oak, evergreen oak etc. – having a durability of 500-1000 years inside the building, in a dry environment), fit for constructing thin elements; the construction technology implies processing by trimming and the forming of joists achieved by the typical dovetail half-lap joints, reinforced by cylindrical fibrous oak pegs; the pegs are capable of taking over the shearing strain, preventing the elements from detaching from each other at the joists; the dovetail lap joints contribute to taking over axial and bending stresses, achieving more strongly fixed joints (Szabo 2005).

The stress of the elements is mainly bending stress with axial strain – the weight of the axial strain being influenced by: (i) the role and position of the element, (ii) the rigidity relations to the axial strain and to the bending stress of the elements, (iii) the load method, (iv) the grade of the fixing of the joints. The results of a qualitative structural analysis are presented in Table 1.

Table 1 : The results of a qualitative structural analysis of a Gothic roof structure

Element (no.)	Plan system	Role and position of element	Stiffness ratio	Loads: Symmetrical	Loads: Non-symmetrical
Tie beam (1)	SPT_{PR}/SPT_{SEC}	Balancing the pulling	S_N	T_m	T_m / C_M
Rafter (2)	SPT_{PR}/SPT_{SEC}	Taking over the dead loads from the roofing	S_M	C_M	T_m / C_M
Upper collar (3)	SPT_{PR}/SPT_{SEC}	Fixing the joints	S_M	C_m	T_m / C_M
Collar (4)	SPT_{PR}/SPT_{SEC}	Reducing the span of rafter	S_N	C_m	T_m / C_M
Brace (5)	SPT_{PR}/SPT_{SEC}	Discharging the suspending system	S_M	C_m	T_m / C_M
Brace (6)	SPL	Taking over the longitudinal sliding	S_M	C_m	T_m / C_M
Angle brace (7)	SPT_{PR}/SPT_{SEC}	Fixing the joints	S_M	C_m	T_m / C_M
Upper beam (8)	SPL	Longitudinal stiffening	S_M	–	T_m / C_M
Longitudinal beam (9)	SPL	Longitudinal stiffening	S_M	–	T_m / C_M
King, queen post	SPT_{PR} + SPL	Suspension of tie-beam, collar	S_M	I_m	T_m / C_M

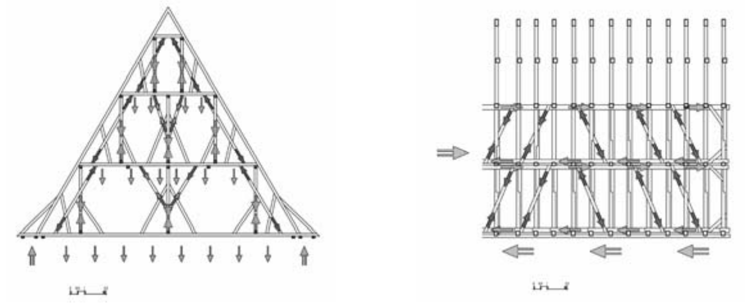
Note:
TPS_M / TPS_S – Main transverse plan system/secondary one
LPS – Longitudinal plan system
C_M/T_M – Compression/tension with large eccentricity
C_m/T_m – Compression/tension with small eccentricity
S_N / S_M– Element with axial stiffness / Element with bending stiffness

2.5 Synthesis of the Gothic roof structures on common rafters and tie-beams with collar beams

The synthesis – an investigation method for load-bearing structures – determines the role of the elements, groups of elements and frames in the building and behaviour of the load-bearing sub-unit (Fig. 4). The elements of the roof structures are built of cross and longitudinal frames.

The trusses – cross frames with a triangular form – rest through the wall-plates on the supporting sub-unit, having the main role of leading the symmetrical and non-symmetrical gravity actions, as well as the cross non-gravity actions to the supports. The significant pitch of the common rafters (between 50° and 75°) eliminates the majority of the loads resulting from the snow. The distribution of the main and secondary trusses varies, characteristic being the situation when the row of trusses starts with and ends in a main truss (M), and every second truss is secondary (S), thus: M–S–M; a more rare type is the one having only main trusses: M–M, or the one in which between two successive main trusses there are two secondary ones: M–S–S–M; the connection between cross and longitudinal frames is achieved by king/queen posts belonging to both structures.

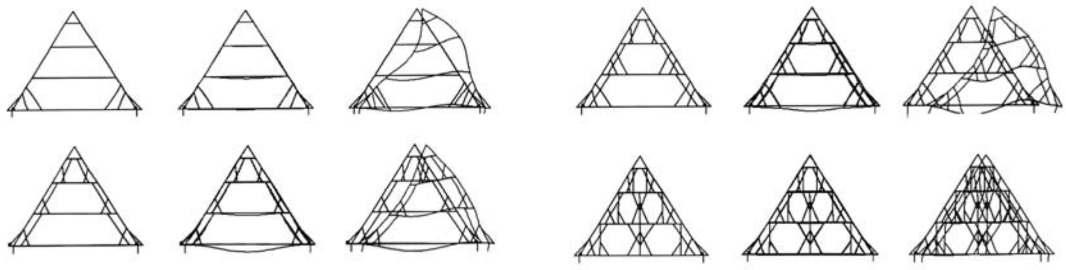
The quality of main truss manifests itself mainly at non-gravity actions, there being a difference in rigidity between the two cross fames, especially at this type of actions. The self-bearing secondary trusses are of a rigidity close to that of the main trusses in the case of symmetrical gravity actions.



(a) (b)

Figure 4 : Gothic roof structure on common rafters and tie-beams with collar beams, (a) Main transverse frame (b) Longitudinal frame

The homogenization of the deformations at different actions is only partially carried out through the longitudinal frames (Fig. 5).



(a) (b)

Figure 5 : Deformations in the Gothic roof structures on common rafters and tie-beams with collar beams, at (a) symmetrical, (b) non-symmetrical actions

The longitudinal frames rest on main trusses through the elements common with these trusses (king/queen posts), being usually connected through the notching of the plates and to the secondary trusses. Their main role is to rigidify the spatial sub-unit of the roof structure in the longitudinal direction; the passing braces that join the plates – intersecting the king/ queen posts – are in this sense of a major importance. Through the rigidity in the direction, which is perpendicular to the plane, in which they are developed, they contribute to the homogenization of the transversal deformations of the main and secondary trusses. They are well-delimited, being placed according to the roof span, in the vertical symmetry axis, and/ or – symmetrical about this axis – in pairs of planes, which are always vertical, on one or more levels.

3 MECHANICAL MODELLING OF THE ROOF STRUCTURES ON COMMON RAFTERS AND TIE-BEAMS WITH COLLAR BEAMS

3.1 The necessity of the mechanical modelling

The Gothic roof structures on common rafters and tie-beams with collar beams – from the 13th century on – are constructed according to an empirical-intuitive concept, much earlier than the formulation of the engineered conformation theories regarding the load-bearing structures. The computer-assisted mechanical modelling is more justified as determining the efforts and deformations in the constituent elements of these roofs was not possible before the spread of the automatic programs.

The modelling principles are exemplified through the restored (Kirizsan 2005) load-bearing structures or those under restoration (the Evangelical Church Huet square, Sibiu – Romania).

The mechanical modelling of the spatial unit on computer – from which the correct initial mechanical concept derives, that meets the present performance requirements – is recommended to be continued through the restoration of the historic roof structure, intervening only in the damaged areas.

3.2 The mechanical modelling process

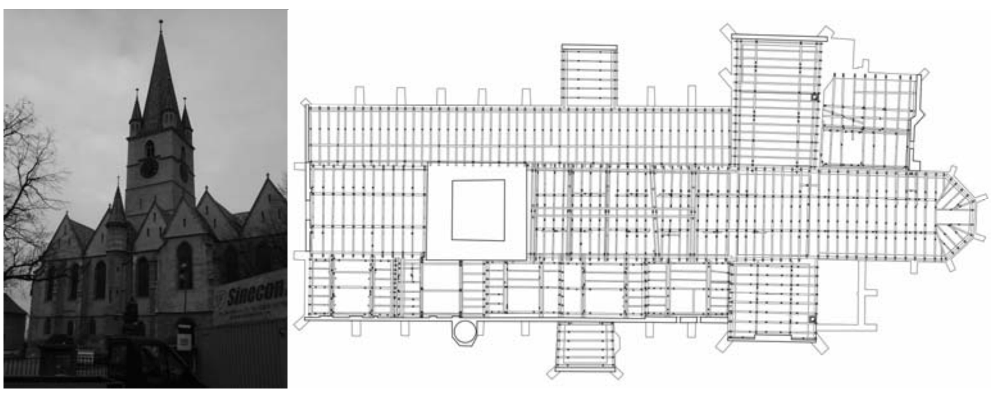
The geometrical model allows cross frames (main and secondary trusses), as well as longitudinal ones. The trusses – of a triangular outline – are edged with common rafters and tie-beams assembled between them. The rigidity of the outlines is differently provided through angle braces, upper collars and collar beams (components adherent to all the trusses), as well as through passing braces and hanging trusses (characteristic only of the main trusses), there being possibly such secondary trusses, which have passing braces. The relative displacements of the elements and frames have to be noticed, as the estimation of the possibilities of returning to the initial geometry is necessary, modelling the existing situation and the one after the foreseen interventions.

The load modelling implies distributed loads coming from the weight of the load-bearing (for timber it is 600 daN/m^3) and non-bearing (roof covering and others) structures, as well as climatic loads: wind and snow. The loads originating from its use have to be taken into consideration as well, if it's the case.

The material model – imposing a relation between effort-deformation – is mainly characterized by the quality of the materials of the load-bearing structure, being also influenced by the history of the loads, the external conditions (temperature, humidity), all developed in space and time. The wooden material is considered to be linear elastic and isotropic, having the elastic modulus equal to 100000 GPa. The structural model greatly depends on the aim of the modelling, as well as if the static and dynamic verification and the verification of the resistance and stability of the roof are required. The seismic damages or the ones deriving from the traffic vibrations being insignificant, the dynamic verifications are not a priority. The quality of the joints between the elements and frames is of great importance, these being usually considered to be articulations or semi-articulations. At present – in the case of the roof structures on common rafters and tie-beams with collar beams – the supports are provided for the cross frames at the extremities.

3.3 The mechanical model and automatic calculation at the roof structure of the Evangelical church in Sibiu

The presented mechanical model is a sub-unit of the roof structure of the Evangelical church in Huet Square, Sibiu – Romania (Fig. 6). Constructed at the beginning of the 16th century, the roof of the aisle consists of five volumes each having 7 trusses placed perpendicularly to the longitudinal axis of the church (volumes marked with E. – F. – H. – K.).

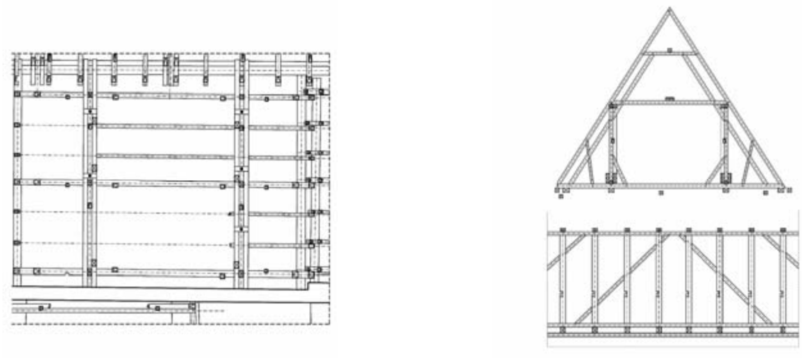


(a) (b)

Figure 6 : The roof structure of the Evangelical church in Sibiu (a) southern façade, (b) plan

Each volume is built of three main trusses placed first, forth and seventh, as well as four secondary trusses (second, third, fifth and sixth) and two longitudinal frames. The secondary trusses are built of a tie-beam, two common rafters (forming angles of approximately 60° with the tie-beam), an upper collar, collar beam and two angle braces (ensuring the connection between the tie-beam and the common rafters). The main trusses have got in addition two king/ queen posts, two passing braces – placed between the upper collar, tie-beam and two angle braces that join the tie-beam with the king/ queen posts. The longitudinal frames have lower and upper plates, as well as passing braces that intersect the king/ queen posts, elements belonging to both the main trusses and the longitudinal frames. The five volumes rest steadily laterally, in the direction of the valleys, these being elastic supports provided by beams.

The lack of a rigid support in the direction of the valleys that are perpendicular to the southern façade (especially during the winter with huge amounts of snow) has caused great deformations – maybe even the local destruction of load-bearing elements – making probably major interventions necessary right after its initial execution (Fig. 7).



(a) (b)

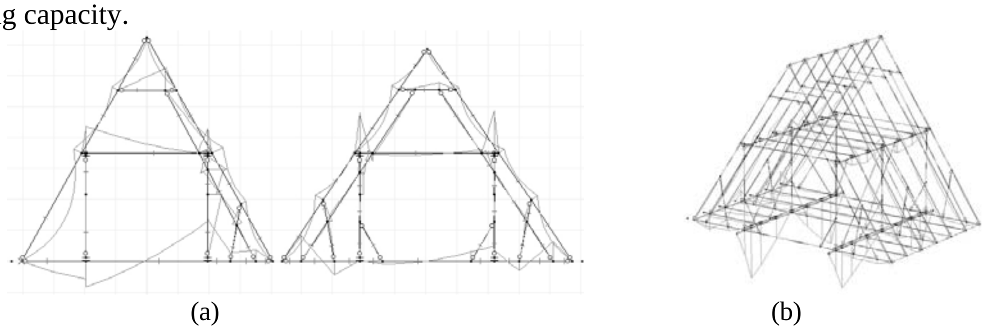
Figure 7 : The roof of the southern aisle of the Lutheran church in Sibiu (a) plan, (b) frames

The geometrical model allows five volumes each with 7 cross frames (3 main trusses and 4 secondary ones) and 2 longitudinal frames. The load model implies distributed loads deriving from the weight of the load-bearing and non-bearing structures (roof covering and access scaffolding), as well as climatic loads: wind and snow. The material model is characterized by pinewood, which is considered to be linear elastic and isotropic, with the elastic modulus 100000 GPa. The structural model implies the verification of the resistance and of the stability of the roof structure. The joints are with mortice and tenon or lapped, being considered articulations or semi-articulations. The support is provided through the cross frames, at the extremities.

3.4 The results of the modelling at the roof structure of the Lutheran church in Sibiu

The results are interpreted both in the present phase and after the interventions. The conception proves to be viable, in contrast with the supports in the direction of the valleys. The results point out the insufficient load-bearing capacity of the supporting elements, as well as the correct conformation of the sub-units, if proper supporting conditions are provided (Fig. 8).

The efforts in the bars are within the limits of the load-bearing capacity and the deformations are compatible with the rigidity requirements of the roof elements. The joists are carried out correctly, the reserves deriving from the partial fixings being left to the increase of the load-bearing capacity.



(a) (b)

Figure 8 : Efforts in the roof elements of the southern aisle of the Lutheran church in Sibiu (a) frames (b) 3D

3.5 Intervention propositions following the modelling at the roof structure of the Lutheran church in Sibiu

For the restoration of the five volumes of the Gothic roof we propose the fitting of a pair of metallic beams in the direction of the intermediary valleys, which are rigid enough to take over the reactions of the trusses. After providing adequate supporting conditions we can pass over to the dismantling of the earlier consolidations, the hanging trusses placed next to the longitudinal frames adjacent to the valleys.

4 CONCLUSIONS

4.1 Conclusions regarding the behaviour of Gothic roof structures on common rafters and tie-beams with collar beams

The Gothic roof structures on common rafters and tie-beams with collar beams belong to the spatial sub-units arranged in frames, preserved in our time in an impressive number. Contrary to the ones on trestles or with common rafters and plates, these roofs proved to be viable, if their maintenance was acceptable, especially those, where through the reduced volume the longitudinal frames provided sufficient spatial rigidity.

4.2 Conclusions regarding the investigations through mechanical modelling

The strains and deformations indicate a correctly constructed roof structure; this is less true in the case of the supporting elements in the direction of the valleys, which need consolidation.

4.3 Conclusions regarding the possible interventions following the modelling

In connection with the results of the modelling, one must determine the proportion of the intervention works: conservation, restoration, consolidation and reconstruction. As the modelling results in a correct initial mechanical concept (the historic roof structure meets the present performance requirements), the indicated intervention is conservation or restoration, through which the initial mechanical concept is preserved and which allows the return of the load-bearing structural sub-unit to the original concept, using materials with features similar to the original ones, and using traditional technologies similar to the originally used ones.

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