

## Application of Structural Dynamic Methods in Diagnosis of Historic Buildings

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**ABSTRACT:** The article presents an overview of diagnostic researches, conducted for two historic buildings, situated in Gdańsk (Poland) or in Vilnius (Lithuania). The researches are based on vibration measurements made during ambient excitations, which influence the structures every day. As the vibration magnitudes are rather low and the excitation forces are unknown, the advanced methods of signal processing must be implemented. In this work the correlation analysis is applied. The method is fully non-destructive and allows formulating a number of conclusions about technical condition of the structures as well as their interaction with soil. The paper contains a complex description of the methodology: starting from sensors mounting, via signal processing, ending with final concluding.

### 1 INTRODUCTION

The process of spatiotemporal development of the Mediterranean culture has significantly influenced history of two metropolises: 1000-years old Gdańsk (Poland) and 700-years old Vilnius (Lithuania). Both cities are nowadays pride of many historic buildings of unique value.

In 2003, within the confines of Center of Excellence CURE, the programme of the European Union “City of Tomorrow and Cultural Heritage”, the authors formed a partnership with, among others, Vilnius Gediminas Technical University and the Museum of History of Gdańsk. The cooperation provides a great possibility for mutual exchange of experiences and so far resulted in diagnostic researches conducted for selected monuments situated in Vilnius and in Gdańsk. The Center of Excellence CURE is still looking for new partners.

The main methodological assumption in the Centre's CURE activity is studying mathematical models, which reflect behaviour of historical buildings. In the authors' opinion, a rational modelling enables to find a reason of structural damage and to put a problem of prediction of the structural behaviour within time. Two kinds of models are considered: deterministic and stochastic ones. Special attention is paid to non-destructive vibration testing of structures under ambient excitations, analytical and numerical approaches to damage detection and localization, as well as development of models with random parameters.

Two monuments have been selected from the CURE library for discussion in this paper. They are similar constructions but they work in different environment, which involve various problems. The first one, situated in Vilnius, is the Arch-Cathedral Belfry (see Fig.1) and the second is the Tower in the Wisłoujście Fortress in Gdańsk (see Fig.2). In those two case studies the authors wish to present a certain range of problems connected with diagnosis of historic buildings, starting from vibration measurements, interpretation of their results, application of methods of defect identification and ending with a proposal of solution for identification of unknown foundation parameters. The role of vibration measurements as the main source of information about various elements of structural behaviour will be emphasized.



Figure 1 : The Arch-Cathedral Belfry in Vilnius



Figure 2 : The Wisłoujście Fortress in Gdańsk

## 2 DESCRIPTION OF THE SELECTED STRUCTURES

### 2.1 *The Vilnius Arch-Cathedral Belfry*

The historical centre of Vilnius City in Lithuania, is one of the largest in Eastern Europe. It is unique and is inscribed on the UNESCO World Heritage List. The most important and prominent building in there is the Arch-Cathedral Basilica with the Belfry beside. The Belfry has been built over several centuries so it is diverse as regards architecture and structural solutions. The first part was built in the XIV<sup>th</sup> century on foundation and undergrounds dated on the XIII<sup>th</sup> century (Venclova 2002). It was a defensive tower then with almost circular cross section, thick walls and small shooting windows. Three lower floors are built of stones, while fourth, the upper one, is built of masonry. Ceilings are built of wood with one except of a reinforced concrete floor, built at a top of the masonry part as a protection against bell falling down. In 1522 works of the tower's conversion into a belfry started. In succeeding years three stores of octagonal cross sections have been built on the existing tower. They have masonry walls and wooden ceilings. After several reconstructions, since 1893 the Belfry has a present form. The walls thicknesses are:  $3.3 \div 1.8\text{m}$  in the circular part,  $1.5\text{m}$  at the V<sup>th</sup> floor and  $1.3\text{m}$  at the VI<sup>th</sup> and VII<sup>th</sup> floor. The structural height without a covering helmet equals  $41.4\text{m}$  but the very top of the Belfry reaches  $56\text{m}$ . An external diameter at the tower's footing equals approximately  $12.5\text{m}$ .

The bell system has been changed several times as well. In documents one can find information about an old system of 10 bells and 17 modern ones, placed additionally in the belfry in 1967. In 2002 a new bell system has been put in the building to replace the existing one. It comprises of 6 heavy bells, electrically controlled. They hang on three massive wooden grids, supported on opposite walls. The grids are crossed as shown at Fig.3b. The hanging scheme and the bells' working principle is completely different than before. In addition, lastly a street traffic volume in vicinity of the Belfry has increased because of a new bridge opening nearby. The final new factor, concerning the Belfry, is appearance of seriously looking cracks at the bells level. The cracks are situated in each internal corner of the octagonal cross section, they start app. at level No 3 (see Fig.3a) and end below level No 2. That event has alarmed the authorities and various investigations started. The CURE team was responsible for a dynamic research, in particular for assessing an influence of the traffic and bells ringing on the building structure as well as checking its overall technical condition with emphasis on the cracks influence on the dynamic structural behaviour.

### 2.2 *The Tower in the Wisłoujście Fortresses in Gdańsk*

The Tower, dated from 1482, is the first masonry structure erected in Wisłoujście, situated by the Vistula River mounting in Gdańsk (Zbiński 2000). It had a crucial role in medieval port navigation and protection – it was a lighthouse and a defensive tower then. Within centuries various nations tried to capture Gdańsk as it was an attractive trade city. That constantly pushed

the local authorities to fortify Wisłoujście. That is why the Tower has been changed into a strong fortress surrounded by a moat. Many warfare and fires have destroyed the Wisłoujście Tower several times. The latest disaster took place in 1953, when a hurricane has struck down 70% of the Tower, which was weakened after the Second World War. It has been rebuilt and nowadays diagnostic as well as conservation works last.

The Tower is 22.65 meters high, its external diameter is 7.7 m. The structure has 7 floors with reinforced concrete ceilings. Walls are built of masonry of various ages, because of many restorations. Walls thicknesses are: 1.45m on the ground level, 1.2m on the first floor and 1.1 on remaining ones. The Tower stands by the Leniwka River (actual name), on the layered ground, where damp or wet sands are alternated with aggregate mud. The foundation of the Tower is only known to lie shallow below the ground level, made of boulders.

In this case, similarly like in the previous one, the CURE team has undertaken the task of dynamic identification of the structure (natural frequencies and modes) along with drawing conclusions about its technical condition. An attempt to specify an interaction ground-structure has been also made.

### 3 THEORY OF IDENTIFICATION OF MODAL CHARACTERISTICS

In this paragraph the theory of natural frequencies and mode shapes identification on a base of vibration measurements under environmental excitations is outlined.

The nature of the considered structures allows determining the natural frequencies and modes only on the base of their ambient vibrations, caused by environmental excitation, which are: wind, traffic, water, seismic waves etc. As vibrations of the structures under such influences do not have significant magnitudes, it is difficult to distinguish structural characteristics among measurement noises. This problem is solved by applying advanced methods of signal processing, which in this work, are based on correlation analysis.

From the measurements one obtains accelerations' (displacements'/velocities') time histories  $x(t)$  of selected measure points. Determination of frequencies involved in the signals is possible after transforming them into a frequency domain. The desired transformation function should distinguish between frequencies of structural vibration, frequencies of excitation forces or noises. Such a tool, which emphasizes natural frequencies from measured signal of ambient vibrations, is the spectral density function (auto-spectrum)  $G_{xx}(f)$ . By the definition it is equal to the Fourier transform of a correlation function  $R_{xx}(\tau)$  (Szabatin 2003):

$$G_{xx}(f) = 2 \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau, \quad f \geq 0. \quad (1)$$

Auto-spectrum is a real function. Analogically one can calculate a cross-spectrum  $G_{xy}(f)$  of two signals  $x(t)$  i  $y(t)$ , which is a complex function. Practical method for calculating auto- and cross-spectrum describe following formulae (Bendat et al. 1980):

$$G_{xx}(f) = 2 \lim_{T \rightarrow \infty} \frac{1}{T} E \left[ |X_k(f, T)|^2 \right], \quad G_{xy}(f) = 2 \lim_{T \rightarrow \infty} \frac{1}{T} E [X_k^*(f, T) Y_k(f, T)], \quad (2)$$

where  $T$  means the measurement time, \* denotes complex conjugate,  $X_k(f, T)$ ,  $Y_k(f, T)$ -coefficients of Fourier transform calculated for succeeding signals  $x_k(t)$  and  $y_k(t)$ , measured in given measure point,  $E$  – operation of an expected value.

The environmental excitations are random forces. Hence, vibrations of building structures subjected to such forces are also random. A main advantage of implementing spectral density functions to the analysis is changing its random nature to a deterministic domain, which allows for using statistical tools in analyzing the measured signals. Additionally, due to the expected value operation, included in equations (2), short term harmonic components are removed from the resulting spectrum while the dominant ones are amplified. Calculating the auto- and cross-spectra is a first step for filtering out some parts of noises and excitation forces from the measured signals. Further steps, leading to natural frequencies and modes of the building structure specification, need a calculation of the coherence function  $\gamma_{xy}^2(f)$ . This function provides information about linear dependence between auto-spectra of two signals for each

frequency  $f$ . The value  $\gamma_{xy}^2(f_i) = 1$ , for a given frequency  $f_i$  signifies a perfect linear relation between signals  $x(t)$  and  $y(t)$ , while  $\gamma_{xy}^2(f_i) = 0$  denotes no such relation between them. If the two signals are measured in the same structure in two different points, the linear dependence between them for some  $f_i$  means a common mode for that frequency. Practically, if for several pairs of signals measured in various points of some structure one obtains several results of  $\gamma_{xy}^2(f_i) \approx 1$ , that means that  $f_i$  is a natural frequency of the structure (called also resonant frequency  $f_r$ ). The coherence function is calculated according to the formula:

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)}, \quad 0 \leq \gamma_{xy}^2(f) \leq 1. \quad (3)$$

When the resonant frequencies are already specified, one can calculate displacements of each measure point, being a coordinate of a mode shape coupled with each  $f_r$ . If the measurements are conducted in several series, then few sensors (dependently on type of the modes, which are to be identified) must be placed constantly in the same locations in order to assure data for scaling the signals. They are called reference points. The coordinates of modes in this case are calculated according to the formula:

$$\phi_p(f_r) = \sqrt{\frac{G_{x_p x_p}(f_r)}{G_{x_{ref} x_{ref}}(f_r)}}, \quad (4)$$

where  $G_{x_p x_p}(f_r)$  and  $G_{x_{ref} x_{ref}}(f_r)$  denote values of the auto-spectra calculated for signals measured in some point  $p$  and in the reference point respectively for the frequency  $f_r$ .

As the auto-spectra are real functions, one can not specify a direction of any displacement building the mode shapes. Such directions can be determined after analysis of the cross-spectra, by checking a phase shift between subsequent pairs of signals. The following formula is valid:

$$\theta_{x_p x_{ref}}(f_r) = \tan^{-1} \left[ \frac{Q_{x_p x_{ref}}(f_r)}{C_{x_p x_{ref}}(f_r)} \right], \quad (5)$$

where  $Q_{x_p x_{ref}}(f_r)$  means an imaginary part of  $G_{x_p x_{ref}}(f_r)$  and  $C_{x_p x_{ref}}(f_r)$  - its real part. The phase shifts equal to  $0^\circ$  or  $180^\circ$  allow specifying directions of displacements of measure points in relation to each other.

It should be finally noted, that this method is applicable for structures with low damping ratio  $\xi < 0.05$  (Bendat et al. 1980).

## 4 ANALYSIS OF THE TWO HISTORIC BUILDINGS

### 4.1 *Vibration measurements*

All vibration measurements have been conducted by usage of the system PULSE 3560c and one-dimensional piezoelectric accelerometers. For both building structures, considered in this article, the main aim was identifying their natural frequencies and mode shapes. As it is shown further, much information can be read from them. For this purpose the ambient vibrations have been measured and correlation analysis conducted, according to the theory presented in the previous paragraph. Additionally, measurements while bells' ringing have been made for the Vilnius' Belfry (to assess their influence on the structure) and while truck driving through a threshold lying on a street nearby (after such impulses the logarithmic decrement of damping was estimated). There was an attempt to increase magnitudes of vibrations of the Wisłoujście Tower also by a truck driving nearby (but solid buildings around the Tower have smother the waves) or by striking a wall at the Tower's top by a group of people – the effect was non-visible. The results presented in this paper are derived from the analysis of signals measured in several points, marked at Fig.3 and Fig.4. For the Vilnius Belfry 4 sensors have been placed at 6 structural levels, which gave 24 measure points. For the Wisłoujście Tower 9 levels have been distinguished, in which 4 sensors have been placed – that gave 36 measure points. The sensors have been fixed to the walls by usage of magnets and glue.

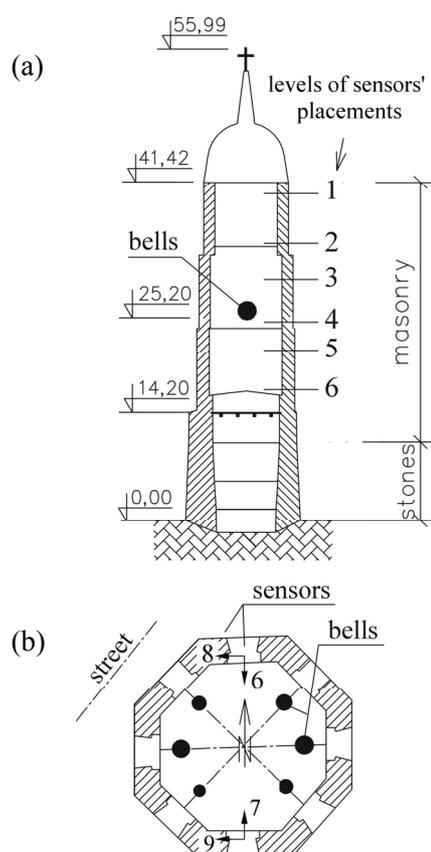


Figure 3 : (a) Scheme of the Vilnius Belfry with levels where the sensors were placed; (b) cross section of level No 4 and typical orientation of the sensors

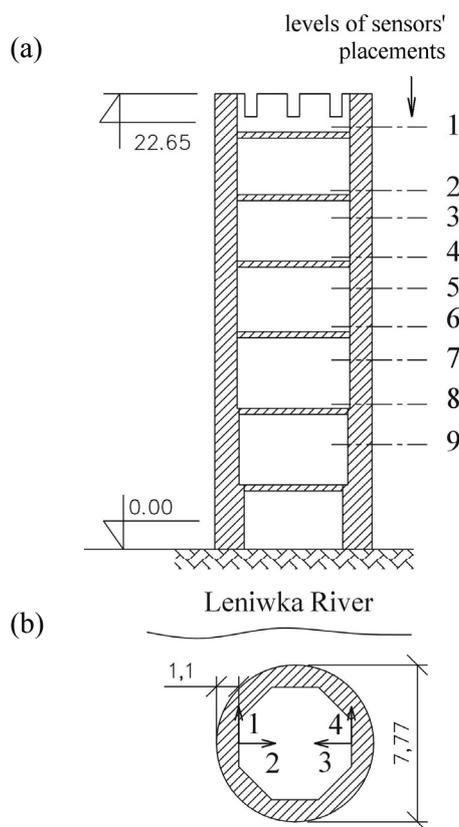


Figure 4: (a) Scheme of the Wisłoujście Tower with nine measurement levels; (b) typical cross section and orientation of sensors

#### 4.2 The Finite Element Method models

Numerical models of the two structures have been built before the vibration measurements. The aim was to get known provisionally dynamic behaviour of the structures, which was helpful in calibrating the measurement equipment and deciding about locations of sensors. The models have been built in the commercial program SOFiSTiK by the usage of the Finite Element Method. Solid elements have been used. The model of the Belfry has a fixed support, which, as appeared later, was a good solution, because the measured and calculated natural frequencies were similar. The Belfry has massive undergrounds and is situated in the main square of Vilnius, where soil certainly is well consolidated. On the other hand, the natural frequencies calculated in the model of the Wisłoujście Tower with a fixed support, were different than the measured ones. The structure stands on a weak ground, by a river. Its foundation is shallow. It certainly has a flexible support. After measurements, the Wisłoujście Tower's model support condition has been calibrated to obtain similar frequencies (measured and calculated).

#### 4.3 Analysis of the Vilnius Arch-Cathedral Belfry

To assess the bells' ringing influence on the building structure the acceleration time histories measured during ambient vibrations and during bells' moving have been compared for different measure points. It appeared that the bells' swinging amplifies the 'ambient' signals about 10 times. A suspicion of a resonance appeared. Calculation of auto- and cross-spectra allowed specifying natural frequencies of the Belfry on the base of the 'ambient' signals. First three of them equal:  $f_{1e} = 1.31\text{Hz}$ ,  $f_{2e} = 1.45\text{Hz}$ ,  $f_{4e} = 3.95\text{Hz}$ . Fig. 5 presents the time history of acceleration of the measure point No 6 (see Fig.3b) under ambient excitations and further while bells' swinging. Auto-spectra of the two signals are given as well. From the time histories one

can read app. 30 times amplification of the ‘ambient’ signal after switching on the bells. In the auto-spectra one can see, that the first two natural frequencies are exactly those, which are amplified. That confirms that the bells’ ringing may be dangerous for the building structure.

Three natural mode shapes have been determined from the measurements.  $f_{1e}$  and  $f_{2e}$  correspond with two first bending mode shapes in two perpendicular directions (that is because of a circular symmetry of the Belfry) and  $f_{4e}$  corresponds with a second bending mode shape in the same direction of displacements as  $f_{2e}$ . The modes have numbers 1, 2 and 4 because from the numerical model it is known that the third one is a torsional mode. Fig. 6 presents schemes of the determined modes. They concern a northern wall of the Belfry. The sensors 1÷6 are placed in a vertical line crossing each level 1÷6, shown at Fig.3a. Those with a symbol R are oriented perpendicularly to the wall and those with symbol C – tangentially. For a comparison, two theoretical modes, calculated in the numerical model, are also run.

From the modes analysis one can find out if the observed cracks are so serious that change the structural dynamic behaviour. The test of modal curvatures has been done for this purpose (Pandey et al. 1991, Tomaszewska 2004.). Theoretical modes have been constructed from displacements of nodes situated alike measure points so the modes have similar form as the experimental ones. Their frequencies are equal to:  $f_{1t} = 1.24\text{Hz}$ ,  $f_{2t} = 1.26\text{Hz}$ ,  $f_{4t} = 3.96\text{Hz}$ . The curvatures of each mode shape have been calculated according to the Central Difference Method. The Belfry represents its actual condition while the model – ideal one (no cracks or other defects). A so called defect index  $I$  and standardised index  $Z$  are calculated for each measure point  $p$  according to the formulae:

$$I_p = |C_p^e - C_p^t|, \quad Z_p = \frac{I_p - m}{\sigma}, \quad (6)$$

where  $C_p^e$  and  $C_p^t$  denote the curvatures calculated for a point  $p$  from the experimental and theoretical data respectively,  $m$  – a mean value of curvatures of a given mode,  $\sigma$  – standard deviation of the curvatures. The biggest positive value of  $Z$  in a given mode shape gives information about the localisation of the defect in the structure (if there is any).

From the first and second mode, the directions with the largest displacement at level No 1 have been chosen as representative. Hence, the first mode represents a circular direction of displacement, while the second – the radial one. Table 1 presents the obtained values of damage indices. An analysis of the values of indices, calculated for each structural level, lead to a conclusion that levels No 2 and 3 have stiffness different than assumed in the model. That certainly indicates that the cracks observed in that surrounding are serious, constructive cracks, which influence the dynamic behaviour of the structure.

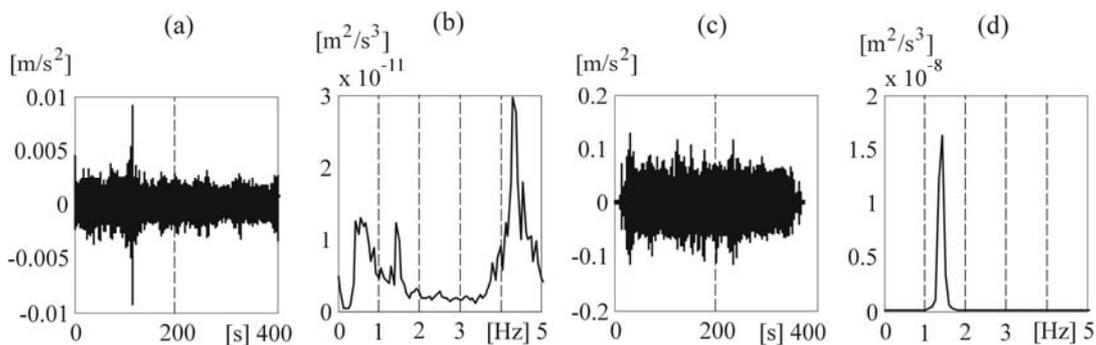


Figure 5 : Accelerations time histories and auto-spectra of the measure point No 6 at level No 4 (see Fig.3); (a) and (b) for ambient excitations; (c) and (d) for ambient excitations + bells’ ringing

Table 1 : Values of damage index  $Z$  calculated for levels 2-6 of the Belfry from the data of three modes

	level on the structure				
	2	3	4	5	6
I <sup>st</sup> mode	-0.6783	<b>1.1392</b>	0.1378	0.7116	-1.3103
II <sup>nd</sup> mode	0.7056	<b>0.7912</b>	0.6924	-1.1129	-1.0762
IV <sup>th</sup> mode	<b>1.0074</b>	0.7589	-1.1739	0.3579	-0.9503

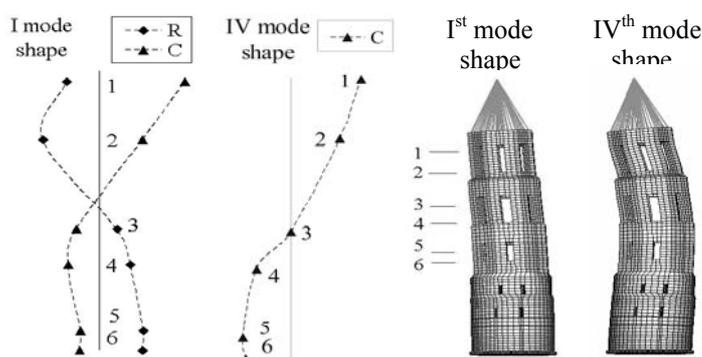


Figure 6 : Mode shapes of the Belfry; left hand side: experimental; right hand side: theoretical ones. Symbols in legends denotes: R – displacement in radial direction, C – displ. in tangential direction

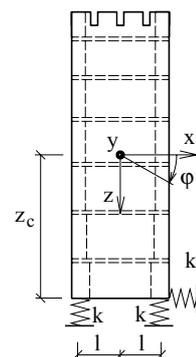


Figure 7 : Mathematical model of free vibrations of stiff structure on elastic supports

#### 4.4 Analysis of the Tower in the Wisłoujście Fortress in Gdańsk

The first problem, which appeared in case of the Wisłoujście Fortress, was a big difference between measured and calculated frequencies. The measured first frequency was equal to 1.44 Hz, while the calculated in the numerical model: 2.6 Hz. That forced the authors to look closer at the model. It has been built with the real structural shape preservation and deduced material properties. The analysis led to a conclusion that supporting conditions assumed in the model must be incorrect. The weak ground, described in the paragraph 2.2, and the shallow foundation make the supporting rather flexible. In the model the structure was assumed as fixed in the ground. The authors decided to identify the ground stiffness parameters basing on the measured natural frequencies. A simple mathematical model of a stiff solid on a elastic supports is assumed (see Fig.7). The equation of motion of such model is:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_y \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{x} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} 2k & 0 & 0 \\ 0 & k_x & -k_x z_c \\ 0 & -k_x z_c & 2kl^2 + k_x z_c^2 \end{bmatrix} \begin{Bmatrix} z \\ x \\ \varphi \end{Bmatrix} = \mathbf{0}, \quad (7)$$

Where  $m$  denotes the structure mass,  $J_y$  is the mass moment of inertia about the  $y$  axis and the other symbols are marked at Fig.7. The unknowns are  $k$  and  $k_x$ , but from the three equations in the system (7) one can identify one more value. The sensitivity analysis, conducted for the natural frequencies of the system (7) proved, that  $z_c$  is of the biggest importance among other parameters as regards calculation of the frequencies (Szymczak and Tomaszewska 2005). Hence, the three values will be identified:  $k$ ,  $k_x$  and  $z_c$ . By assuming harmonic solutions, the mathematical operations lead to equations:

$$k = \frac{m\lambda_z}{2}, \quad k_x = \frac{m\lambda_1\lambda_2 J_y}{2kl^2}, \quad z_c = \sqrt{\frac{1}{k_x}((\lambda_1 + \lambda_2)J_y - 2kl^2) - \frac{J_y}{m}}, \quad (8)$$

where  $\lambda_1, \lambda_2$  denote squares of the first and the second bending radial natural frequencies in the plane  $xz$  and  $\lambda_z$  is the square of the longitudinal radial natural frequency. Each value of  $\lambda$  comes from the measurements, other values are calculated according to the structural geometry and assumed material properties. The data are:  $\lambda_1 = (2 \pi 1.45)^2 = 81,9$ ;  $\lambda_2 = (2 \pi 6.56)^2 = 1699,09$ ;  $\lambda_z = (2 \pi 4.5)^2 = 799,19$  [rad<sup>2</sup>/s<sup>2</sup>];  $m=9475 \cdot 10^2$  [kg];  $J_y=4340 \cdot 10^4$  [kg·m<sup>2</sup>];  $l=3,16$  [m]. The calculation results are:  $k = 1196.99$  MN/m;  $k_x = 239.37$  MN/m;  $z_c = 13,3$  m. A transposition of the ground stiffness parameters to the numerical model allowed obtaining similar natural frequencies to the measured ones. The first three measured natural frequencies are equal to:  $f_{1e} = 1.42$ Hz,  $f_{2e} = 1.45$ Hz,  $f_{3e} = 4.46$ Hz, while the calculated ones:  $f_{1t} = 1.45$ Hz,  $f_{2t} = 1.47$ Hz,  $f_{3t} = 4.48$ Hz. The measured signals allowed identifying mode shapes connected to those three frequencies. Each mode has 9 coordinates, as only displacements of measure points oriented in parallel to the dominant direction of displacement in a given mode could be measured. The first two modes are the first bending ones in two perpendicular directions, the third frequency is a torsional one.

The analysis of the defect index did not detect any defected section. That proves that the structure is compact, without dangerous cracks. One can also assume that the new part of the Tower, covering four upper floors, rebuilt after the hurricane which has damaged that part, is well connected to the old part and the building material has similar properties as the older one.

Finally one has to stress that the Tower is bended from the vertical direction by  $2^\circ$  so in the future one has to take care of the Tower's stability.

## 5 FINAL CONCLUSIONS

The article proves an efficacy of ambient vibration measurements in diagnosis of historic buildings. The measurements are made during natural everyday conditions of an environment surrounding the structure of interest. The process is fast (takes one day) and is fully non-destructive. On the base of such measurements many conclusions can be formed. In the paper questions of structural integrity and safety are posed.

As far as the Vilnius Belfry is concerned it appeared that the recently observed increased traffic volume does not cause dangerous magnitudes of the structure's vibrations, which unfortunately can not be stated in the case of bells' swinging. The bells cause approximately 30 times amplification of the Belfry's ambient vibration magnitudes. That is assumed as the dangerous situation, which probably caused the recent cracks propagation. The research of the structural integrity (modal curvature approach) showed that the level, where the cracks are situated (top of the sixth floor) is seriously weakened. Further works are planned, which are aimed at checking internal forces of the structure and the material strength conditions. In the case of the Wisłoujście Tower it appeared, that the structural integrity is not unbalanced and the newly rebuilt part (after the hurricane) well cooperates with the old masonry. Another interesting fact came out. It appeared that the Tower's footing is not stiff. The Tower behaves like a stiff structure on elastic supports. Parameters of the support have been specified on a base of equations of motion of a structural model and calculated geometrical characteristics of the Tower. In the future the authors plan to introduce optimization methods based on measured natural frequencies and mode shapes so that the geometrical data needed in identification process could also be identified in parallel.

## ACKNOWLEDGMENTS

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## REFERENCES

- Bendat, J.S. and Piersol, A.G. 1980. *Engineering applications of correlation and spectral analysis*. New York: John Wiley & Sons.
- Szabatin, J. 2003. *Fundamentals of theory of signals*. Warszawa: Wydawnictwa Komunikacji i Łączności, (in Polish).
- Venclova, T. 2002. *Vilnius*. Vilnius: R. Paknio Leidykla.
- Zbierski, A. 2000. The Maritime Fortress of Wisłoujście As Seen in the Complex Research. *Wisłoujście Fortress – History, Present and Future*. Gdańsk: Museum of History of Gdańsk, p. 29-50.
- Pandey, A.K., Biswas, M. and Samman, M.M. 1991. Damage detection from changes in curvature mode shapes. *Journal of Sound and Vibration* 145 (2), p.321-332.
- Tomaszewska, A. 2004. Vibration – based methods applicable in diagnosis of civil engineering structures. *Acoustical and Vibratory Surveillance Methods and Diagnostic Techniques; Proc. Intern. conference, Senlis, 11-13 October 2004*, on CD.
- Szymczak, C. and Tomaszewska, A. 2005. Example of practical use of vibration measurements results – study of settlement of the Wisłoujście Tower. In T. Godycki-Ćwirko and M. Niedostatkiwicz (eds), *City of Tomorrow and Cultural Heritage – Pomerania Outlook; Proc. intern. workshop, Gdańsk, 08-09 December 2005*, p. 67-70. Gdańsk: CURE.