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Dynamic Monitoring and Model Updating of a Masonry Bell Tower in Pisa

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ABSTRACT: Since the methods used to identify natural frequencies of existing structures have different resolution capabilities, depending on the algorithms adopted for the dynamic analysis as well as on the type of the dynamic forces used to excite the structure, an appropriate dynamic characterization procedure should combine several signal processing techniques. In the paper, a suitable natural frequencies identification methodology is illustrated in the framework of the dynamic identification of the bell-tower of St. Niccola's church in Pisa, where classical methods, based on Fourier transform, are combined with more modern methods, based on wavelet transform, integrated by sophisticated finite element investigations as well. The proposed methodology allows to identify the field of application and the soundness of each signal processing technique, achieving an accurate estimation of the main parameters governing the dynamic structural response, especially when dynamic time histories are characterised by limited amplitude and/or are strongly contaminated by noise.

1 INTRODUCTION

Dynamic identification plays a key in structural monitoring and control of historical masonry buildings, especially when safety assessments are framed in a more general policy of preservation of architectural heritage.

Masonry structures are generally very stiff, so their dynamical characterisation is not trivial. In fact, since dynamic time histories induced by periodic or random excitations result generally limited in amplitude and often contaminated by noise, the capability of recognition of natural frequencies depends on the signal processing method adopted as well as on dynamic forces used to excite the structure. It is therefore evident that suitable dynamic characterization methodologies should combine different processing techniques.

Due to their relatively simple static scheme, the dynamic behaviour of towers and bell-towers is largely predictable, at least when damage is not relevant. For this reason, towers and bell-towers are particularly suitable for testing the performances of new modal identification methodologies.

So, the occasion of the dynamic characterization of the bell tower of the in Pisa (Fig. 1), represented the occasion for testing a new methodology of processing dynamic signal, based both on classical methods and on more modern methods.

The present paper illustrates the numerical and experimental studies performed for the dynamic characterization of. the bell-tower of St. Niccola's church in Pisa (Fig. 1), aiming to determine the most significant mechanical parameters and to evaluate the relevant stress patterns, as well as to setup an innovative methodology to identify the mode shapes and the natural frequencies of the bell-tower itself, where the classical signal processing methods, based on Fourier transform, are combined with more modern ones, based on wavelet transform (Addison 2002, Staszewski at al. 1998), integrated by refined finite elements investigations.

This combined methodology, corroborated by experimental results, not only allows to verify the reliability and the field of application of each signal processing technique, but also to attain a good estimation for the main geometric, physics and mechanics parameters of the structure, governing its dynamic behaviour.

Figure 1 : The bell tower of the St. Niccola’s church in Pisa.

2 THE BELL TOWER OF THE ST. NICCOLA’S CHURCH IN PISA

The bell-tower of St. Niccola’s church in Pisa, considered a masterpiece of the Romanesque art, is characterized by a particularly complex architectural organization. In fact, the cross section, circular at the base, becomes octagonal to higher orders and finally hexagonal at level of the bell cell.

The bell tower, standing to the left of the façade of the church, appears quite well preserved and doesn't show evident cracking. It is characterized by an inclination of about 1°13' with respect to the vertical line, caused by sinking of the ground, so that its maximum overhanging, referred to the road level, is about 100 cm (Fig. 2).

The dating and the attribution of the bell tower are not certain and different hypotheses have been proposed. Vasari attributes the construction to Nicola Pisano, but recently several different hypotheses have been proposed, for example by Ragghianti, that attributes it to Diotisalvi (1170) and by Nannicini-Canale and Testi-Cristiani, that dates it between 1230 and 1250, while Frey assumes that it was built in two or three constructive phases, the first one starting around 1173 and the following ones between 1230 and 1250.

Certainly, the bell tower, which was originally isolated, has been included in the adjacent convent of St.Augustin’s Fathers in 1295, during the extension works of the convent.

Inside the bell tower there is a refined staircase in stone, supported by a helical gallery of elegant rampant arches, sustained by columns (see Fig. 3). Above the gallery, articulated in two fornices on each side of the octagon, the masonry is realized with stones perfectly trimmed, flattened and prepared in precise rows without joints. Each side is decorated with blind arches, sustained by angular pentagonal pilasters and containing deep lozenges, sometimes passing through. In the hexagon of the bell cell, built with rectangular cut sandstone, flattened and precisely lined up, a single lancet window for the bell is present in each side. Each lancet window is surmounted by an embedded circular arch and by flat angular pillars ending with three small circular arches.

The inner structure is built with sandstone, the pyramidal hexagonal based roof is realized with masonry of full bricks, while the columns are realised with Apuan marble and Elba granite.

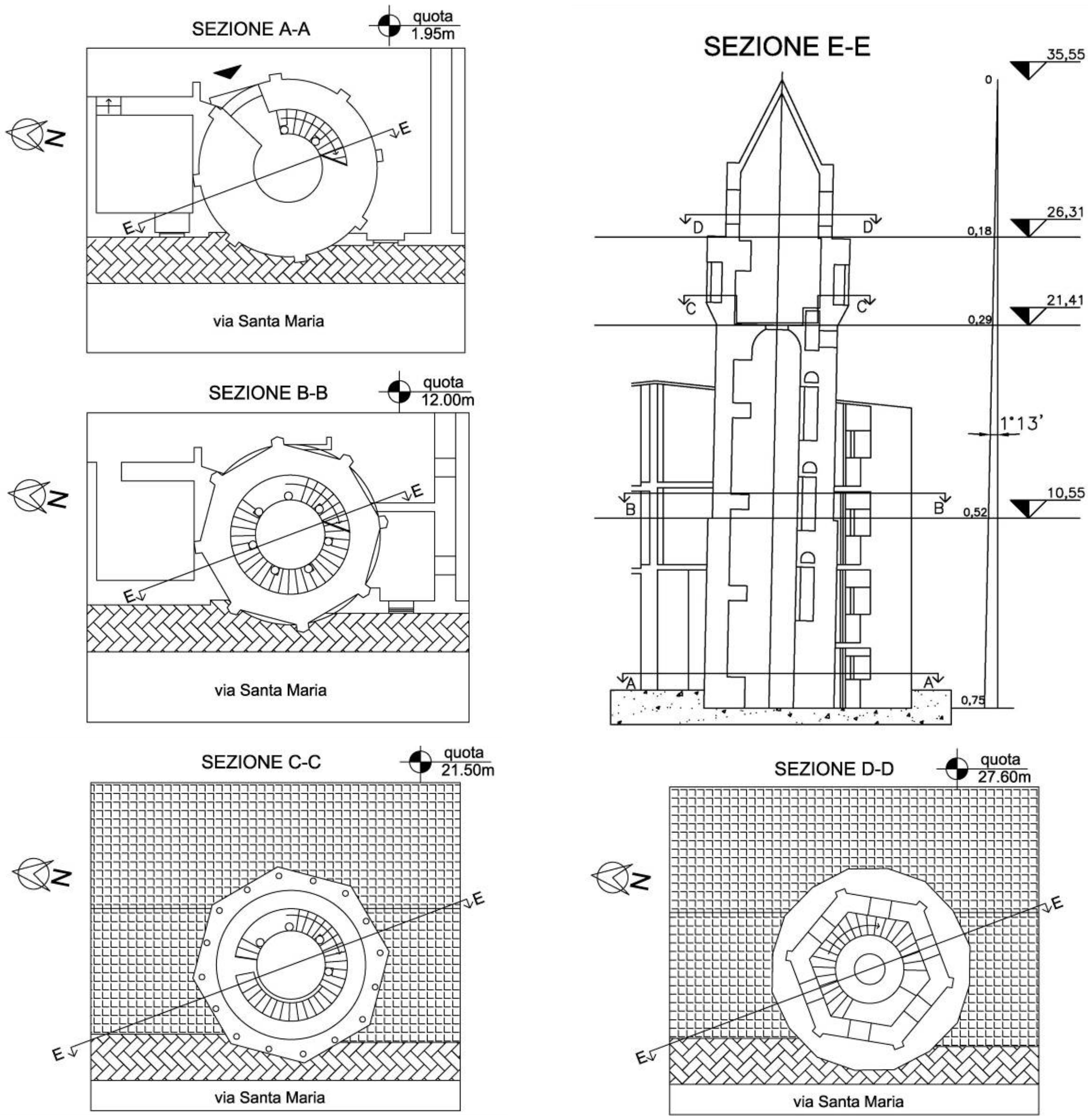


Figure 2 : Bell tower cross section – (a) circular, (b) octagonal, (c) exagonal.

3 DYNAMIC TESTING

The experimental research work consisted in a wide series of dynamic tests on the bell tower, using three different exciting forces: a random one, environmental-type, produced by the normal vehicular traffic, a sinusoidal one, produced by the motion of the main bell, and an impulsive one, caused by the impact of the wheels of a heavy calibrated lorry driving on a 10 cm height timber step, placed on the roadway surface, normally to St. Maria street axis (Fig. 4). It is important to stress that the interaction between the bell-tower and the walls of the adjacent convent, modified the stiffness and the dynamic behaviour of the bell-tower, so that the test the operational management of the tests resulted more complex and the results more difficult to understand than expected, for the isolated structure case.

A total of 41 dynamic tests was performed: 20 using environmental excitation, 10 using sinusoidal excitation and 11 using impulsive load. Under sinusoidal and impulsive excitation structural displacements and accelerations were recorded at different heights using seismometers and accelerometers, while under environmental excitation only seismometers were employed.

Figure 3 : Internal helical staircase; view from atop.

Figure 4 : Impulsive excitation – calibrated lorry

4 SIGNAL PROCESSING TECHNIQUES AND WAVELET TRANSFORM

The dynamic time histories recorded during experimental tests can be analyzed with different techniques. Classical techniques in structural dynamic are mostly based on Fourier transform, nevertheless, in the last years, some interesting alternative techniques, borrowed from other sectors of the engineering, have been suggested, like the ones based on wavelet transform, considered in the present work.

Wavelet transforms are systematically used in various scientific fields, like analysis of climatic data, analysis of financial indexes, cardiac monitoring, identification of statistic fluctuations in turbulent motions, characterization of fracture surfaces, compression of images and so on.

The wavelet analysis uses compact (*let*) oscillatory functions (*wave*), deriving their name from the form of a fundamental function Ψ, the mother wavelet, used to build such functions (Fig. 5).

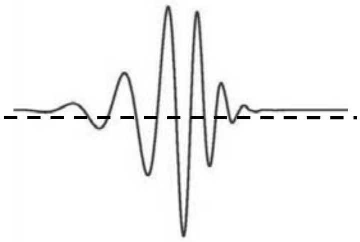


Figure 5 : Mother wavelet

The wavelets are basis functions, of a real or complex variable, produced by translation and expansion of the mother wavelet, Ψ, which is a regular function, defined on a limited interval and equal to zero outside the interval, characterised by null average and finite energy [Addison 2002]. These mathematical properties make the wavelets particularly effective in the analysis of aperiodic, intermittent, transitory or noisy signals.

In analogy to Fourier transform, they exist continuous (CWT) and discreet (DWT) wavelet transform and the relative inverse transform. Moreover, unlike Fourier transforms,, that associates to a temporal impulse an infinite spectrum in the frequency domain, wavelet transform associates to a function defined in a short time interval, a function defined in an analogously short time interval in the transformed domain.

The continuous wavelet transform of a function $f(t)$ is defined as

$$T(a,b) = w(a)\int_{-\infty}^{+\infty} f(t)\psi^{*}\left(\frac{t-b}{a}\right)dt \tag{1}$$

where a is a scale parameter, b represents the translation on the time axis, $w(a)$ a normalization function that assures that the wavelets have the same energy for every value of the scale parameter and the star means conjugate complex. The scale parameter has a meaning analogous to the cartographic scale: high values of the parameter give global information on the signal, associated with low frequencies, while small values give local information on the signal, associated with the high frequencies.

5 ANALYSIS OF THE EXPERIMENTAL TEST RESULTS

Because of the stiffness of the structure, the signals recorded during the experimental campaign are characterized by small amplitudes and by a high noise to signal ratio, as underlined in figure 6, where the time is expressed in second and the displacement in mm.

Dynamic time history have been analysed using three different signal processing techniques, based on the Fourier transform (case *a*), on the Fourier transform with triangular window (case *b*), and by the wavelet transform, (case *c*). For example, power spectra relative to some significant dynamic time histories, obtained with the different methods, are compared in Figs. 7, 8 and 9, referring, respectively, to environmental excitation, to sinusoidal excitation and to impulsive one.

The critical examination of the spectra shows clearly that:

- the Fourier analysis of weak and/or noise contaminated signals allows leads to satisfactory results only in case of sinusoidal forcing (Fig. 8a), whereas the results obtained by the wavelet transform result much less satisfactory (Fig. 8c);
- when environmental or impulsive excitation are considered, the capacity of identification of the Fourier transform is considerably reduced, as highlighted by Figs. 7a and 9a;
- the efficiency of the Fourier transform is not significantly improved using windowing techniques, as it is evident comparing cases *a* and *b* in Figs. 7, 8 and 9;
- wavelet transform is particularly effective to identify natural frequencies, when environmental or impulsive excitations are considered (Figs. 7c and 9c).

Spectral analysis also shows that, in case of impulsive excitation, dynamic time histories are clearly interpretable and give satisfactory both in terms of displacement and in terms of acceleration, while, in case of sinusoidal excitation, best results are obtained in terms of displacement.

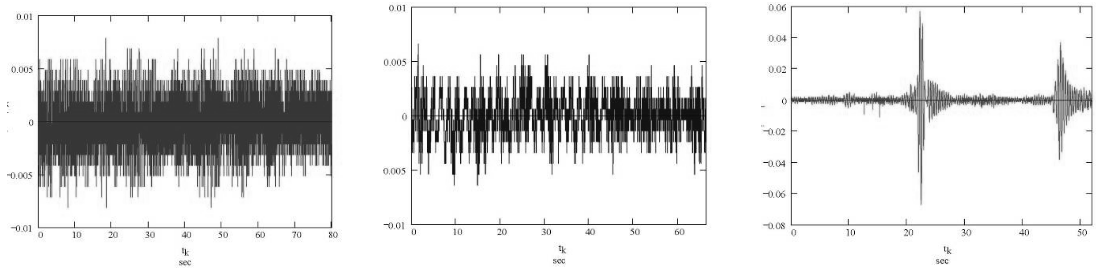
Special comparison is obviously immaterial in case of environmental excitation, where only seismometers were used during the tests.

Values of natural frequencies identified analysing all the recorded time histories are summarised in Table 1. Values, derived through a cross check of results given by different signal processing methods, are divided in Table 1 on the basis of the nature of the forcing function, so stressing the capacity of the forcing function itself to excite the natural modes of vibration.

For each type of excitation and each mode shape, the value of the natural frequency has been evaluated averaging values identified in different trials. Besides, in order to reduce errors due to the noise or to the testing technique, only natural frequencies recognized in more than 70% of the total number of trials, have been considered.

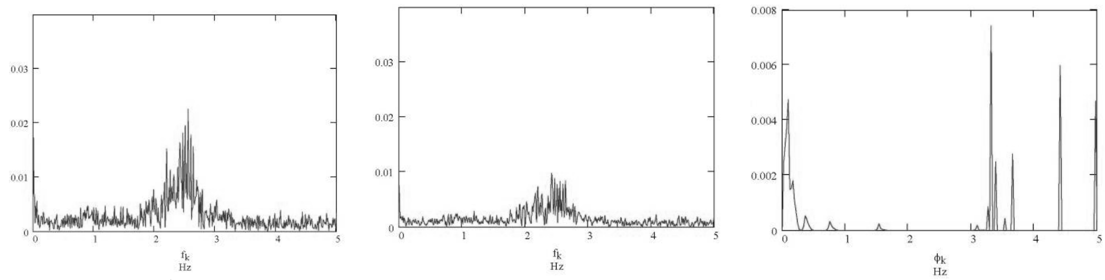
Experimental results show that the first four detected mode shape are primarily flexural: the second one is a bending mode belonging to a plane parallel to the axis of St. Maria’s street, while the other three are bending modes belonging to a plan perpendicular to the street axis. The mode shapes identified in the two perpendicular planes are quite different, because of the interaction between the bell-tower and the wall of the adjacent convent.

Analysis of experimental results underlines that, as reasonably expected, the environmental excitation doesn't have enough intensity to excite the first mode shape, confirming also that technique based on wavelet transform are more effective than those based on Fourier transform to identify frequencies of higher modes.



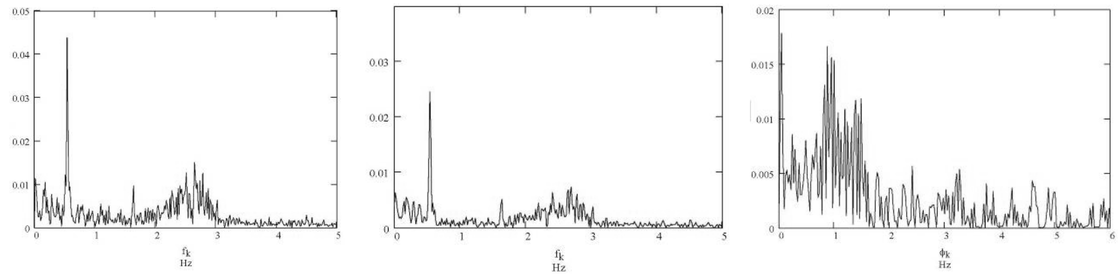
Environmental excitation *Sinusoidal excitation* *Impulsive excitation*

Figure 6 : Seismometer signals acquired during the tests



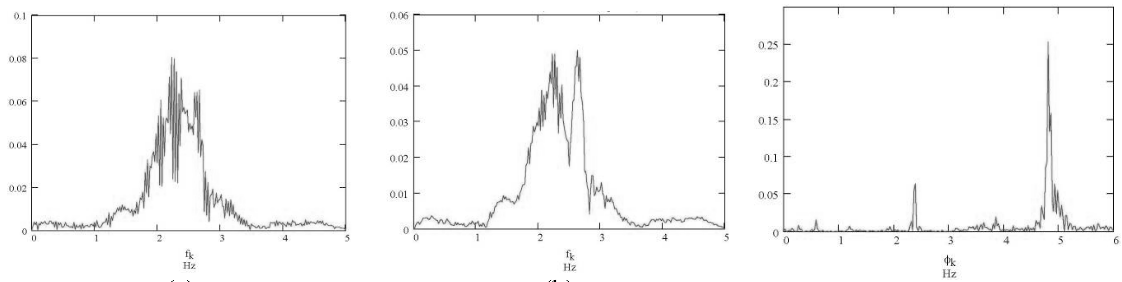
(a) (b) (c)

Figure 7 : Power spectra - environmental signals: (a) FFT, (b) FFT triangular window, (c) wavelet transform.



(a) (b) (c)

Figure 8 : Power spectra - sinusoidal signals: (a) FFT, (b) FFT triangular window, (c) wavelet transform.



(a) (b) (c)

Figure 9 : Power spectra - impulsive signals: (a) FFT, (b) FFT triangular window, (c) wavelet transform.

Table 1 : Experimental eigenfrequencies.

	Measured eigenfrequencies Hz			Type of mode shape
Mode n.	environmental excitation	sinusoidal excitation	Impulsive excitation	
1	/	0.545	0.561	*bending in plane perpendicular to the street axis*
2	/	1.633	1.834	*bending in plane parallel to the street axis*
3	3.198	/	/	*bending in plane perpendicular to the street axis*
4	3.338	/	3.32	*bending in plane perpendicular to the street axis*

6 NUMERICAL ANALYSIS AND COMPARISON OF THE RESULTS

The static and dynamic behaviour of the bell tower has been studied numerically using the FE programme COSMOS/M. The finite element model, consisting of about 160000 8-nodes solid elements, reproduces very precisely all the structural details which are significant for the description of the real behaviour of the bell tower, such as the openings and the helical staircase, and the interactions with the soil and the walls of the adjacent convent. The finite element mesh is illustrated in Fig. 10, where it is compared with the actual vertical section of the bell tower, while the main mechanical parameters considered in the analysis are summarized in Table 2.

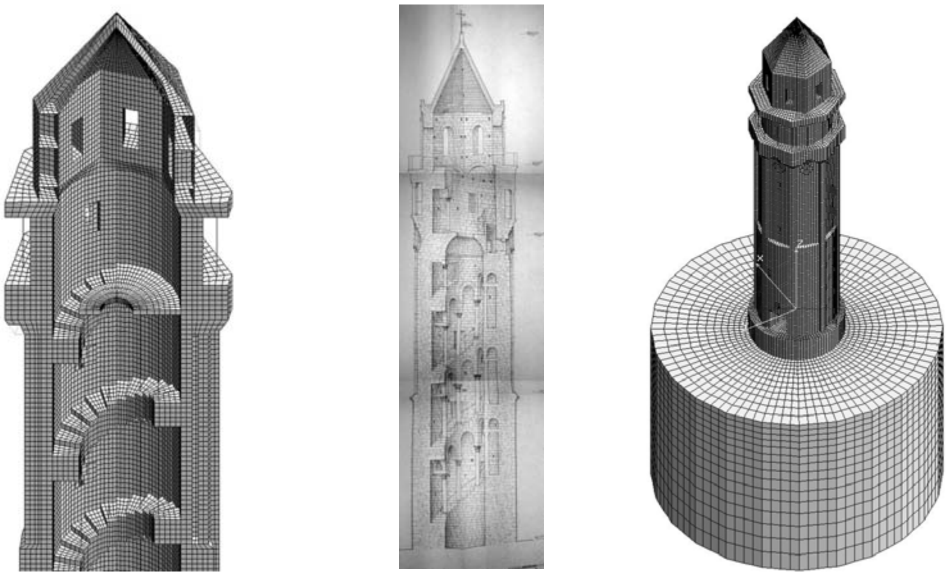


Figure 10 : Finite element model and vertical section of the bell tower

Table 2 : Main mechanical parameters considered in the FE analysis.

Material	Elastic modulus [MPa]	Density [kg/m^3]	Structural elements
Concrete	25000	2400	*Foundation*
Concrete	30000	2400	*Floor at the colonnade level*
Stone	50000	2700	*External faces of stone masonry, bell cell and staircase*
Infilling	5000	1400	*Infilling of the stone masonry*
Brick masonry	10000	1800	*Pyramidal roof*

Experimental and numerical results in terms of natural frequencies are compared in Table 3 for each type of excitation, together with the relative error index E_ω, defined as

$$E_{\omega_i} = \frac{\left| \omega_{num_i} - \omega_{sp_i} \right|}{\omega_{num_i}} \cdot 100 \tag{2}$$

Table 3 : Comparison between measured and calculated natural frequencies

	Experimental eigenfrequencies Hz			Numerical Hz			
Mode	Environmental	Sinusoidal	Impulsive		$E_{\omega,env}$ (%)	$E_{\omega,sin}$ (%)	$E_{\omega,imp}$
1	/	0.545	0.561	0.544		0.18	3.13
2	/	1.633	1.834	1.903		14.19	3.63
3	3.198	/	/	3.067	4.27		
4	3.338	/	3.32	3.141	4.17		4.74

7 CONCLUSIONS AND FUTURE DEVELOPMENTS

As the ability of different signal processing techniques to recognise natural frequencies depends on the type of forcing function used to excite the structure, an effective dynamic identification methodology is proposed, combining methods based on the Fourier transform with more modern ones, based on wavelet transform, integrated by refined finite element analysis.

The good agreement between theoretical and experimental data, resulting in values of error index E_ω lesser than 5% for almost all the detected modes, confirms the correctness of the proposed methodology, stressing also that cross check of the results is quite unavoidable, especially when geometrical, physical and mechanical parameters governing the structural responses need to be calibrated.

Fourier transform is very effective technique in dynamic structural identification, provided that the structural response is clear or periodic exciting functions are used. On the contrary, when the recorded signals are weak or contaminated by noise, the ability of modal identification of the wavelet transform give much more satisfactory results than the Fourier transform.

Although sometimes used for dynamic identification of r.c. bridges (Fasana et al. 1999, Lardies et al. 2004), wavelet transform represents, in consideration of its high sensibility in the low frequency range, of its effectiveness in demodulation, of its accuracy in manipulation of transitory and aperiodic signals, a very powerful tool also in the field of the dynamic identification of stiff structures, like historical masonry constructions. The following phases of the research will be finalized to improve the proposed method, in order to make it more sure and reliable, in view of its application field to more complicated structures, what, for instance, masonry bridges.

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