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# Structural Identification and Seismic Vulnerability of the Tower of Matilde in Italy

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ABSTRACT: The purpose of this paper is to carry out the structural identification and to evaluate the seismic vulnerability of the Tower of Matilde, the bell tower of the Cathedral of Santa Maria and San Genesio in San Miniato, Italy. This bell tower may be susceptible to the seismic actions because of its modifications and current damage. The first part of this paper deals with the dynamic identification of the tower based on acceleration measurement by means of ERA (eigensystem realization algorithm) model and Choi-Williams distribution. The first, second and third natural frequencies of the tower are estimated to be about 2.734 Hz, 3.418 Hz and 6.543 Hz, respectively. The second part covers model updating of the tower based on direct method. The third part discusses the seismic vulnerability of the tower by means of pushover analysis using the tower's updated FE (finite element) model.

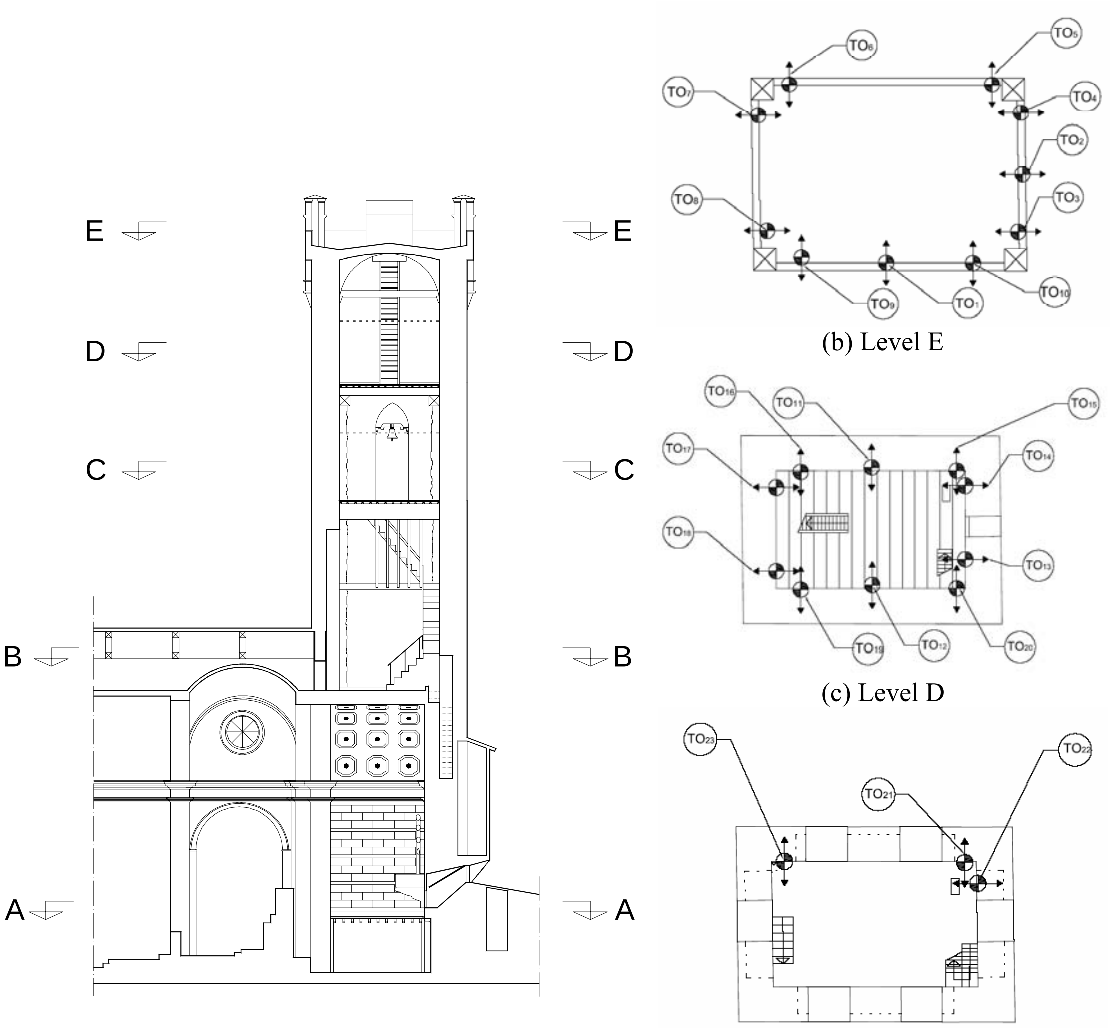
## 1 INTRODUCTION

In the middle of $12^{th}$ century, the tower of Matilde, the bell tower of the Cathedral of Santa Maria and San Genesio in San Miniato, Italy (Fig.1), was initially built as a military fortification incorporated into the city walls. But today the city walls are completely destroyed. Over the following centuries, the tower was under substantial modifications that have considerably changed its original configuration. In fact, it probably had a crenulated crown and three tiers of full-centre arches of varying heights single on the short sides and paired on the long ones. Today, the arches on the two lowest levels have been sealed off with masonry walls. The most structurally significant operations were carried out at the end of the $15^{th}$ century, when the church of Santa Maria was enlarged to include the tower, which was thus transformed into its bell tower, situated above the apse of the nave. This restoration involved demolition of the tower wall on the church side up to the height of the nave itself (Bennati et al. 2005a).

The Tower of Matilde is about 35 m in height, with a rectangular cross section of about 10 m x 7 m (Fig. 2). The cross-sectional dimensions of the tower vary from about 12.5 m x 8.2 m in its lower parts to 10.3 m x 7.3 m in the upper portions up to the crown, which is 11 m x 7.8 m. As shown in Fig. 2a, horizontally, the Tower is divided into four floors: the first, corresponding to the cathedral apse, extends for a height of about 13 m and ends in a barrel vault with 6.5 m radius. The second floor, under the bells, is 9 m in height; it is accessed through a narrow staircase and ends in a masonry floor supported by cracked wooden beams reinforced with riveting. The third floor, about 5.2 m in height and closed off above by wooden planking, is the belfry, where 6 different-sized bells have been set in ogival openings. The fourth floor is about 6.2 m in height; this contains the clock and ends with a vault supporting the flat floor (Bennati et al. 2005b).

Masonry towers including bell towers are thought to be susceptible to seismic actions because of their modifications and current damage such as the presence of wide-spread cracking. Recent papers related to the Tower of Matilde show that its dynamic action is influenced by the bell's action and the potentiality of vulnerability to seismic actions. The purpose of this paper is to carry out the structural identification and to evaluate the seismic vulnerability of the Tower of Matilde. The first part of this paper deals with the dynamic identification of the tower by means of ERA (eigensystem realization algorithm) model and Choi-Williams distribution. The second part covers model updating of the tower based on direct method. The third part discusses the seismic vulnerability of the tower by means of Pushover analysis using the tower's updated FE (finite element) model.

Figure 1 : Tower of Matilde in Italy




(b) Level E

(c) Level D

(a) Section (d) Level C

Figure 2 : Section and plan of Tower of Matilde and measuring points

## 2 DYNAMIC IDENTIFICATION

### 2.1 ERA technique

Using the ERA (eigensystem realization algorithm) technique (Jung and Pappa 1985) the modal parameters such as natural frequencies, mode shapes and damping factors of a dynamic system based on the description of its behaviour in the space of the phases can be determined.

The general principle is to tie the theoretical response to the measured response in the points of acquisition. Treating the discreet process, the relationship is:

$$\{x(k)\}=[R]\{u(k)\}=[R][A]^{k-1}\{B\} \tag{1}$$

where $[A]$ and $\{B\}$ are the model parameter matrices and vector, and $[R]$ is a transformation matrix. Considering the excitement of all the points of input we can write:

$$[X(k)]=[R][A]^{k-1}\{B\} \tag{2}$$

The matrices $\left[X(k)\right]$, called Markov parameters (response functions to the impulse), are used for the construction of the matrices of generalized Hankel:

$$[H(k-1)]=\begin{bmatrix} [X(k)] & [X(k+1)] & \cdots & [X(k+j)] \\ [X(k+1)] & [X(k+2)] & \cdots & [X(k+j+1)] \\ \vdots & \vdots & \ddots & \vdots \\ [X(k+i)] & [X(k+i+1)] & \cdots & [X(k+i+j)] \end{bmatrix} \tag{3}$$

Through the use of the Singular Value Decomposition it is possible to determine the minimum order of the system and to trace back again the modal parameters of the model such as natural frequencies, mode shapes and damping factors.

### 2.2 TFIE with Choi-Williams

The Choi-Williams distribution (Ming et al. 1999) is one type of TF (time-frequency) distribution. A generalized TF distribution is defined as:

$$GTFD(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f\left(x+\frac{\tau}{2}\right) = f^{*}\left(x-\frac{\tau}{2}\right)\Phi(u,\tau)e^{j(ux-\tau\omega-ut)}dxd\tau du \tag{4}$$

where $f(t)$ is a signal analyzed and $\Phi(u,\tau)$ is a transform kernel. Taking different types of kernel, different types of TF distribution can be obtained. For Choi-Williams distribution, its kernel is provided as:

$$\Phi(u,\tau) = e^{-\frac{u^2\tau^2}{\sigma}} \quad (\sigma > 0) \tag{5}$$

where $\sigma$ is a scaling factor to control its attenuation rate. Choi-Williams distribution sacrifices the non-negativity and does not completely meet the support properties in time and frequency but provides a high resolution in time and frequency.

### 2.3 Acceleration measurement

For the purpose of obtaining the data concerning dynamic structural properties of the tower, acceleration by ambient vibration is measured at different levels in the tower (Fig. 2). The total number of sensors is 23 and each sensor measures acceleration in one direction. The acceleration measurements are carried out for 35 times and are recorded to a sampling frequency of 1.6 kHz.

### 2.4 Result of dynamic identification

The experimental dynamic parameters such as fundamental frequencies, mode shapes and damping factors are identified by the analysis of the acceleration time-history by means of ERA technique and TFIE with Choi-Williams distribution (Fig. 3).

Table 1 shows the results of dynamic identification, the first, second and third natural frequencies of the tower, by a comparison of the ERA and Choi-Williams.

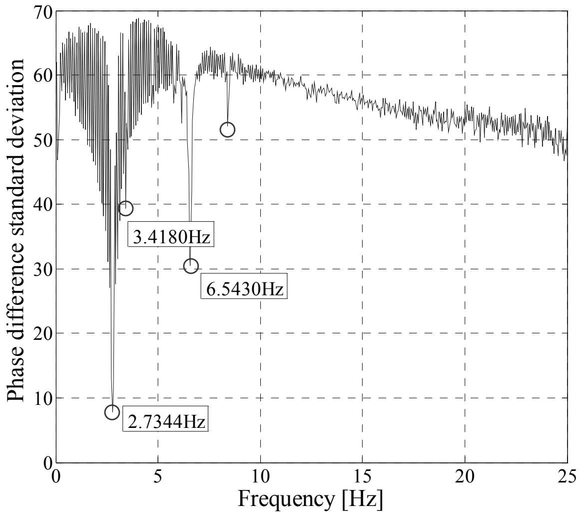
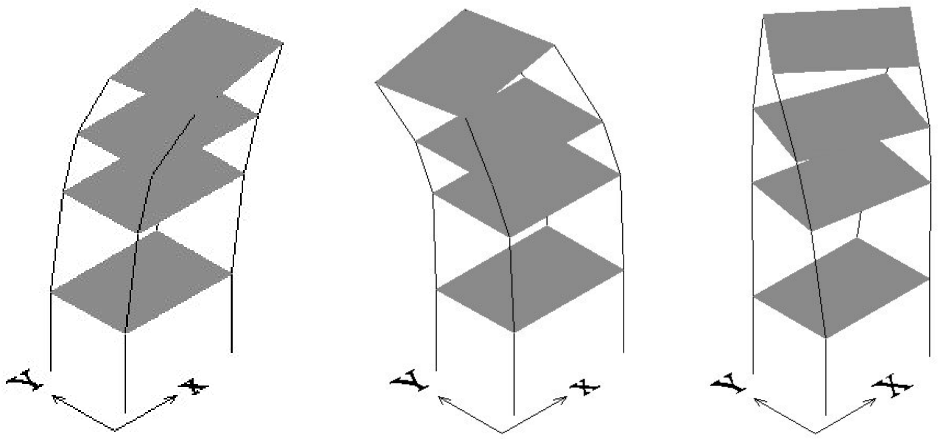



Figure 3 : Natural frequencies identified by Choi-Williams

Table 1: Identified modal frequencies

| Mode | ERA | Choi-Williams |
|---|---|---|
| 1st | 2.688 Hz | 2.734 Hz |
| 2nd | 3.411 Hz | 3.418 Hz |
| 3rd | 6.327 Hz | 6.543 Hz |

The first mode shape was identified windowing it around the peak of about 2.73 Hz (Fig. 3). The relevant mode shape is of flexional type in the Y direction shown in Fig. 4a; moreover from the eigenvector analysis we can infer that translational behaviour of the last level of the tower shows itself with a rotational component along X direction. The same phenomenon does not appear in the levels below. The second mode shape was identified around the peak of about 3.42 Hz. This mode shape represents the translational mode that is verified in the X direction (Fig. 4b); the stiffness in the X direction is larger than that in the Y direction. Windowing the signals around the peak between 6.2 Hz and 6.6 Hz we obtain the third mode shape that represents the rotational mode (Fig. 4c). These three mode shapes are shown in Fig. 4.




(a) Mode 1 (2.73 Hz) (b) Mode 2 (3.42 Hz) (c) Mode 3 (6.54 Hz)

Figure 4 : Identification of the mode shapes

## 3 MODEL UPDATING

### *3.1 Analytical model*

With the aim of analyzing the seismic response of the tower with great accuracy, a numerical model updating is applied. FE (finite element) model of the tower is composed of truss elements and gives rise to the same mode shapes that we can observe in the experimental data (Fig. 4). For the floor, roof slabs and structural walls, an equivalent replacement is applied to replace them with the equivalent truss elements. In other words, the areas of the equivalent truss elements are determined by an equivalent replacement method. Young’s modulus, Poisson’s ratio and the specific gravity used in FEA (finite element analysis) are 2.45 kN/mm$^2$, 0.15 and

18 kN/m$^3$, respectively (Bennati et al. 2005b). The total number of nodes and elements are 26 and 78, respectively.

### *3.2 Result of model updating*

In order to minimize the differences between the experimental measurement and a theoretical dynamic response of a model, model updating based on direct method is applied (Friswell and Mottershead 1995). As shown in section 2.4, three principal modes are identified by the experimental measurements and dynamic identification (Table 1). Three fundamental frequencies identified by Choi-Williams, that is 2.734 Hz, 3.418 Hz and 6.543 Hz, are used for FE model, while mode shapes are not considered. The numerical model updating is carried out by using only stiffness corrections, see Fig. 5.

Fig. 6 shows the development of frequency errors in the iterative computation. As the initial FE model is excessively idealized, the errors are significant up to 18.6 % between the measured and the estimated frequencies obtained by the FE model. After updating, the differences between the experimental and the analytical frequencies are almost zero.

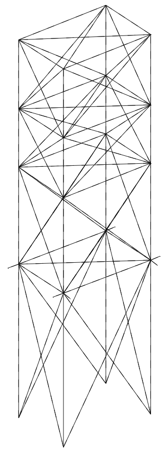


Figure 5 : FE model

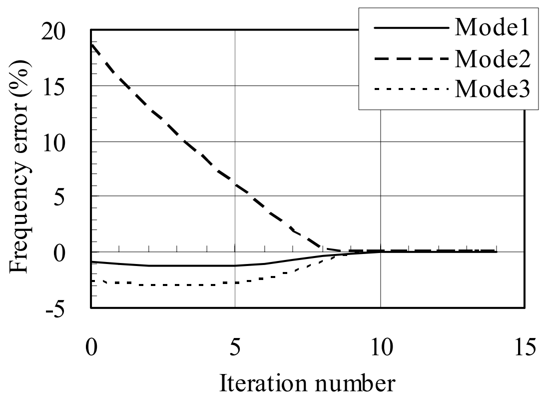
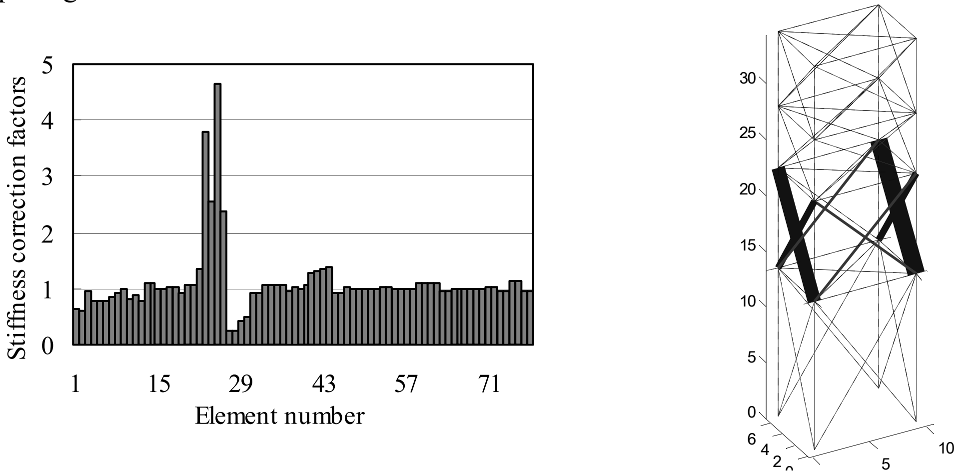



Figure 6 : Development of frequency error in iterative computation

With the model updating according to the experimental measurements, it is possible to determine the variations in the stiffness to the structural elements, as illustrated graphically in Fig. 7. This figure shows the stiffness correction coefficient, the value less than 1.0 represents the stiffness reduction, while the value greater than 1.0 represents its increase. The stiffness of the elements of the walls on the second floor is increased to simulate the effect of the Cathedral. On the other hand, the stiffness of the elements of their orthogonal walls is reduced due to the openings.




(a) Stiffness correction coefficients (b) Stiffness correction coefficients

Figure 7 : Stiffness correction coefficients

## 4 SEISMIC PEFORMANCE OF THE TOWER OF MATILDE

### *4.1 Pushover analysis*

The numerical model, updated on the basis of the results of the dynamic test, the dynamic identification and the numerical model updating, is used in this section to estimate the seismic response of the Tower of Matilde. In the European code, that is Eurocode n.8: "Design of structures for earthquake resistance", non-linear static (Pushover) analysis is available for this purpose.

Pushover analysis is a non-linear static analysis under constant gravity loads and monotonically increasing horizontal loads (Aoki and Sabia 2004). Fig. 8 shows the stress-strain relationship of masonry characterizing the element. The masonry is assumed to crush when the strain $\varepsilon$ reaches the ultimate strain $\varepsilon_{CR}$, and the analysis is performed under a condition that the stiffness after this strain has to be zero. The crack of masonry is assumed to occur when the tensile stress exceeds the tensile strength shown in Fig. 8. After cracking, for the sake of the expediency to achieve numerical efficiency, a small amount of tension stiffening is assumed. Material constants used in the analysis are the same as those of model updating and the areas are modified based on the updating results. The tensile strength $f_t = f_{t2} = 0.375$ N/mm$^2$, its strain $\varepsilon_t = f_t/E$, ultimate strain $\varepsilon_{t2} = 0.003$, compressive strength $f_c = 7.5$ N/mm$^2$, its strain = -0.0075, ultimate compressive strength $f_{CR}$= 1.5 N/mm$^2$, its strain $\varepsilon_{CR}$ = -0.03 are used in the analysis (Fig. 8). The geometrical nonlinearity is not considered.

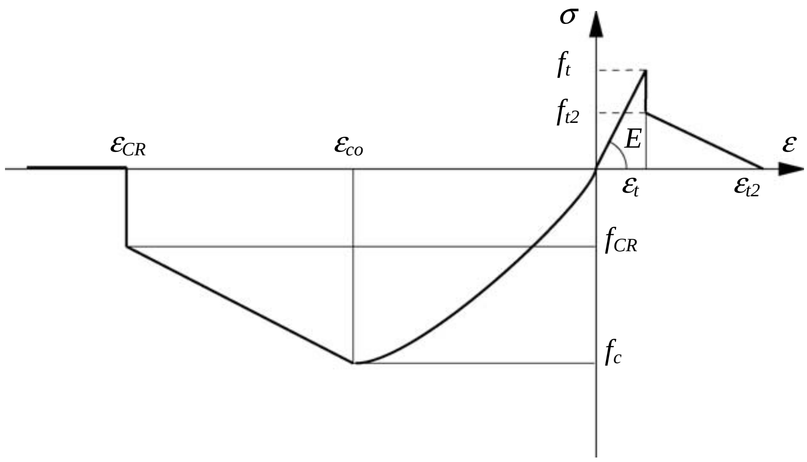



Figure 8 : Stress-strain relationship for masonry constitutive model

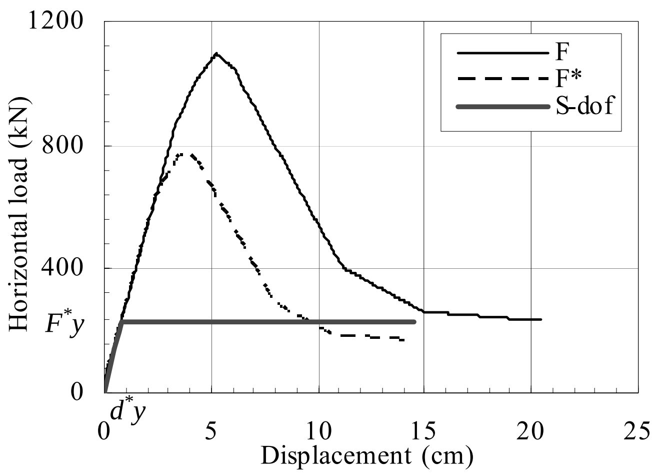



Figure 9 : Relationship between horizontal load and displacement determined by Pushover analysis

### *4.2 Results of static non-linear (pushover) analysis*

Non-linear (pushover) analysis is carried out on the Tower of Matilde applying a “modal” pattern type of horizontal forces (proportional to 1st mode shape) linearity increasing until their collapse. As a result of non-linear response, relationship between horizontal load and displacement of the tower is shown in Fig. 9. The characteristics of the equivalent bi-linear SDOF (single degree of freedom) systems are summarized in Table2, where Γ.is the transformation factor, $F^*_y$ is the yield force, $k^*$, $d^*_y$, and $T^*$ are the initial stiffness, yield displacement and period of the idealized SDOF system, respectively.

Table 2 : Properties of equivalent SDOF system

| Γ | $F^*_y$ (kN) | $k^*$ (kN/m) | $d^*_y$ (cm) | $T^*$ (s) |
|---|---|---|---|---|
| 1.402 | 229 | 27946 | 0.820 | 0.662 |

The evaluated target displacements considering the ground type (Table 4) and the design ground acceleration *a/g* are listed in Table 3. From Table 3 the target displacement of the Tower of Matilde is 252mm if the value of *a/g* equals 0.70 and the ground type is C. It is larger than the limit value of 204mm determined by pushover analysis (Fig. 9). According to the static non-linear (Pushover) analysis, the Tower of Matilde seems to be vulnerable to earthquakes with around 0.6 g.

Table 3 : Target displacements of MDOF system (mm)

| *a/g* | Ground type | | |
|---|---|---|---|
| | A | B, C, E | D |
| 0.15 | 34 | 54 | 93 |
| 0.25 | 57 | 90 | 155 |
| 0.35 | 80 | 125 | 217 |
| 0.50 | 115 | 180 | 311 |
| 0.70 | 161 | $\boxed{252}$ | 435 |
| 0.85 | 195 | 306 | 528 |
| 1.00 | 230 | 360 | 622 |

Table 4 : Ground types

| Ground type | Description |
|---|---|
| A | Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface |
| B | Deposits of very dense sand, gravel, or very stiff clay, at least several tens of m in thickness, characterised by a gradual increase of mechanical properties with depth |
| C | Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of m |
| D | Deposits of loose-to-medium cohesionless soil, or of predominantly soft-to-firm cohesive soil |
| E | A soil profile consisting of a surface alluvium layer with $v$s values of type C or D and thickness varying between about 5 m and 20 m |

## 5 CONCLUDING REMARKS

The following conclusion remarks were obtained:

1) From the results of dynamic tests, the fundamental frequencies of the Tower of Matilde are estimated to be about 2.734 Hz, 3.418 Hz and 6.543 Hz, respectively. These results are obtained by means of Choi-Williams distribution.
2) According to the static non-linear (Pushover) analysis, the Tower of Matilde seems to be vulnerable to earthquakes with around 0.60 g.

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