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Probability Density Functions for Masonry Material Parameters – A Way to Go?

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ABSTRACT: Nowadays, powerful probabilistic methods are available for the calculation of structural safety values. These permit to calculate the global probability of failure of complex structures, relying on deterministic techniques able to determine the stability state for a prescribed set of parameters. The tendency towards a probabilistic approach strongly depends on the availability of material data to provide the probability density functions for the main material parameters involved. Strength and stiffness of masonry are the material properties used in the design or evaluation of masonry structures. However, in the case of masonry relatively large test samples are needed to retrieve the material properties of the composite material. Dealing with existing (historical) buildings, this adds additional complexity. This paper focuses on the methodology on how to process in establishing the correct probability density function and on estimating the parameters of the proper distribution for the masonry compressive strength. The methodology is illustrated on a case study covering the compressive strength of an historical masonry structure.

1 INTRODUCTION

The concepts of probabilistic design and assessment of existing structures are generally accepted and translated into several design codes (EN 1990, 2002; ISO 2394, 1998; ISO 13822, 2003; JCSS, 2001). Their use strongly depends on the availability of material data to provide probability density functions for the main material parameters involved. With regard to the development of probabilistic material models, different rates of implementation are observed for different materials. Probabilistic design with steel, concrete and wood is ahead of masonry. The first European probabilistic design code includes material models for steel and concrete. Timber and masonry are not yet included (Vrouwenvelder, 2002). It is clear that also for masonry as a basic and universally used construction material, these probabilistic material models should become available.

This paper focuses on the methodology on how to process in establishing the correct probability density function and estimating the parameters of the proper distribution. This is illustrated for the compressive strength of masonry. In that special attention is paid to the numerical relation provided by EC6 (EC6, 1995) in between the compressive strength of the components brick and mortar and the composite masonry.

2 GATHERING DATA FOR THE MASONRY COMPRESSIVE STRENGTH

Depending on the structural model used for the stress distribution in the structure, data for the material parameters are required. Globally, strength and stiffness are the material properties used in the design or evaluation phase. In a partial safety factor method (level I method), use is made of a masonry partial safety factor (γ_m) (EN1990, 2002) to include the spread or physical uncertainty on the material characteristics. In case a probabilistic evaluation method is used, the material properties are defined as random variables. Using masonry, additional uncertainties are traced when compared to steel or wood. Masonry is a composite material. It is built from bricks and mortar following a certain (ir)regular framework. Relatively large test samples are needed to retrieve the material properties of the composite material. Dealing with existing buildings, there are different sources to acquire the material properties needed for a safety assessment (Schaerlaekens et al., 1999). The following are possible sources listed in the order of decreasing reliability but in the order of increasing simplicity for gathering large numbers of data, required for statistical purposes (Schueremans and Van Gemert, 2001):

- Relatively large samples removed from the building itself. Advantages: include effects such as workmanship, actual three-dimensional layout, original materials and environmental influences. Disadvantages: the reliability of these results will depend upon the method employed in the removal of samples and the implementation of the appropriate test method; very time-consuming, expensive procedure and therefore not optimal for statistical processing that requires a relative large number of samples; for historical monuments this type of heavily destructive tests is not acceptable (Charter of Krakow, 2000);
- Masonry samples - small or large wallets - rebuilt with original/new bricks and a similar fresh mortar based on the chemical analysis of the original mortar. Advantages: best alternative as still the masonry properties are directly derived on experimental basis. Disadvantages: the effect of time and original workmanship are lost; due to the large scale, most researches are restricted to 3 full-scale walls;
- Retrieve the material properties of the components brick and mortar and in addition, numerical relationships between component characteristics and composite characteristics. Advantage: tests on bricks and mortar are less time-consuming, less expensive and can be performed in relatively large numbers, which enables the probability distribution function of the material properties to be determined. Disadvantage: because of the numerical model involved, an extra model uncertainty has to be taken into account;
- Literature as an additional source of experimental data. This is gaining field. Many different test setups and results can be retrieved as listed in several reviews. On the other hand, the need for experiments and experimental data grow with the increasing insight in the behavior of masonry and the needs coming from more accurate numerical modeling such as non-linear finite element methods and fracture mechanics. In case a probabilistic evaluation method is used for an assessment of masonry structures, the needs are similar. In addition, the number of test samples should be increased in order to estimate the probability distribution functions that are representative for the uncertainty on the material properties. Moreover, evaluation deals with existing structures, not with design of a new masonry structure, adding a major difficulty in gathering consistent information;
- Detailed databases on which a reliability analysis can be based. These should become available in the future. In case of existing buildings it should be possible to gather data from comparable constructions. The international community is aware of the lack of consistent data on existing masonry structures and the fact that for the moment probabilistic approaches usually fail from lack of sufficient data (Melchers, 1999). To meet this requirement, data are collected on an international basis using a database structure that can be consulted via the internet. These should become a full alternative for in site testing. For the moment, available test data on historical masonry are limited.

3 MASONRY COMPRESSIVE STRENGTH – NUMERICAL RELATION BY EC6

Different numerical models have been proposed and refined to estimate the compressive strength of masonry, based on the compressive strength of bricks and mortar. Former research (Schueremans, 2001b) revealed that the formula proposed in the European Standard EC6 (EC6, 1995) is one of the most reliable and practically suitable numerical relationships. The formula is the result of a statistical regression, based on a large number of test results. In that, it is usable over a wide range of input values, which is particularly interesting in case of extension to random variables. The characteristic compressive strength (f'_k) of masonry according to EC6, based on the average compressive strength of bricks (f_b) and mortar (f_m) yields:

$$f_k^{'} = K\left(\delta . f_b\right)^{0.65}\left(f_m\right)^{0.25} \tag{1}$$

The fact that this formula is a product of two factors makes it interesting for stochastic extension. Assumed that f_b and f_m are lognormally distributed, which seem to fit the wide range of experiments performed, the resulting random variable will again be lognormal distributed because of the multiplication model (Van Dyck and Beirlant, 2001). In literature, two elements in this formula are subject of discussion (Santos, 1995; Krischig, 1999):

- Because of the curve fitting used to estimate the parameters, a dimensional problem occurs. In case the compressive strength of bricks and mortar are expressed as [N/mm^2] and the factor K is a non-dimensional constant, the resulting dimension is no longer the dimension of a stress value [N/mm^2]. In Eurocode 6 this problem is solved by assigning a dimension [N/mm$^{0.1}$] to the constant K. Other authors propose to increase the exponent of the mortar strength to 0.35. This has the additional advantage that the (too) limited influence of the mortar strength, certainly in case of lime mortars, is increased a bit;
- Starting from mean values of bricks (f_b) and mortar (f_m) a resulting characteristic strength of masonry is calculated, which is at least an unfortunate choice. This has its consequences for the stochastic extension of the formula and the calculation of mean value and spread.

Despite these inconveniences, the original formula is maintained since the parameters are optimized based on a large number of test samples. This is required when it will be used for a wide range of variables as in case of a stochastic extension.

For bricks, the normalized compressive strength is used, introducing a normalizing factor $*$. This normalizing factor $*$ is a function of the brick's geometry. Assuming a lognormal distribution function for the strength of mortar and brick, the numerical model can be translated into:

$$\ln\left(f_k\right) = \ln(K) + 0.65 \times \ln(\delta) + 0.65 \times \ln\left(f_b\right) + 0.25 \times \ln\left(f_m\right) \tag{2}$$

Mean value and spread can be calculated (Van Dyck and Beirlant, 2001) using:

$$\begin{aligned} \mu_{\ln f_k} &= \ln(0.60) + 0.65\ln(\delta) + 0.65\mu_{\ln f_b} + 0.25\mu_{\ln f_m} \\ \sigma^2_{\ln f_k} &= \left(0.65\sigma_{\ln f_b}\right)^2 + \left(0.25\sigma_{\ln f_m}\right)^2 \end{aligned} \tag{3}$$

From these values, the mean value and the spread of the resulting lognormal distribution for the compressive strength are calculated, using:

$$\mu_{\ln f'} = \mu_{\ln f_k} - 1.645\sigma_{\ln f_k} \tag{4}$$

$$\sigma^2_{\ln f_k} = \sigma^2_{\ln f'} \tag{5}$$

With the inverse relations, mentioned on the right hand side of Eq. 6, the mean value and spread of the resulting lognormal distribution can be calculated:

$$\begin{cases} \sigma^2_{\ln f'} = \ln\left(1 + \left(\dfrac{\sigma_{f'}}{\mu_{f'}}\right)^2\right) \\ \mu_{\ln f'} = \ln\left(\mu_{f'}\right) - \dfrac{1}{2}\sigma^2_{\ln f'} \end{cases} \Leftrightarrow \begin{cases} \mu_{f'} = \exp\left(\mu_{\ln f'} + \dfrac{1}{2}\sigma^2_{\ln f'}\right) \\ \sigma^2_{f'} = \mu^2_{f'}\left(\exp\left(\sigma^2_{\ln f'}\right) - 1\right) \end{cases} \tag{6}$$

4 NUMERICAL ANALYSIS AND EXPERIMETAL VERIFICATION

The formula is used to determine the probability distribution function of the masonry compressive strength for which both the strength of the components brick and mortar and the composite masonry are verified experimentally.

4.1 Compressive strength of brick

The used brick is a hand made facing brick module 50, l×w×h = 188×88×48 mm, type Kempenbrand. Compressive tests (v = 1 mm/min) are performed both on cores ϕ50mm with a height of 44 mm (h^2/Surface = 1) and couplets with a height of 120 mm. The results of the compressive strength are summarized in Table 1.

Table 1 : statistical summary of compressive strength of bricks and mortar

Compressive strength f_c [MPa]	Cores ϕ40 h= 44mm	Couplets h=120mm	Mortar prisms 40x40x160mm
Average [MPa]	6.34	5.16	8.31
Spread [MPa]	2.37	1.56	1.80
Cov [%]	37.4	30	21.6
Number of samples	51	50	108
Probability density funciton	LN	LN	LN/ (truncated normal)

To check whether or not there is a significant difference in the population's mean (μ) compressive strength between the obtained results, the technique of hypotheses tests (Van Dyck and Beirlant, 1995) is used. A significance level of 95% is preset. The null hypothesis H_0 and the alternative upper tail hypothesis H_1 read: H_0: $\mu_1=\mu_2$, H_1:$\mu_1>\mu_2$. The hypothesis tests indicate indeed that there is a significant difference in average compressive strength in between cores ϕ50 and couplets. The difference however is merely caused by the size effect of the samples. Eurocode 6 provides a normalizing factor to correct for the different sizes of test samples (EC6, 1995): $f_b = \delta \times f_{b,m} = 0.82 \times 6.34 = 5.20$ N/mm^2 for cores ϕ50 and $f_b = \delta \times f_{b,m} = 1.11 \times 5.16 = 5.73$ N/mm^2 for couplets with height 120 mm. Using the same hypothesis test, the difference between the normalized mean values is no longer significant.

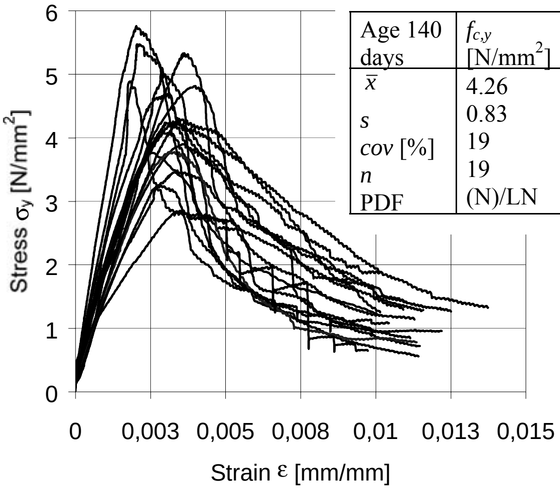
4.2 Compressive strength of mortar

The mortar is a hybrid mortar type M_5 (EC6, 1995). Following composition was used: 1 volume part of cement CEM I 42.5, 1 volume part of hydraulic lime and 6 volume parts of Zutendaal sand. Water is added to obtain uniform workability between 1.9 and 2.1 (NBN B14-207, 1983). The resulting compressive strength on standard prisms is added in Table 1, last column.

4.3 Compressive strength of composite masonry

The first series of samples are small masonry pillars. They contain 6 layers, Fig. 1. The resulting compressive strength f_c, is listed in Fig. 1. The mean compressive strength is lower than the values found on the bricks and mortar samples.

The second type of test samples that has been used to derive the masonry material properties are 5 cores drilled from wallets, with diameter of *150 mm* and a height of *300 mm*. The resulting compressive strength f_c, is listed in Fig. 2. Overall, the mean values are very similar to the value of the masonry pillars, Fig. 1. However, the number of test samples is too small to retrieve a reliable distribution type.



Age 140 days	$f_{c,y}$ [N/mm^2]	E_y [N/mm^2]	$G_{f,c}$ [Nmm/mm^2]
$\bar{x}$	4.26	1673	1.71
s	0.83	368	0.56
cov [%]	19	22	33
n	19	18	18
PDF	(N)/LN	N	N

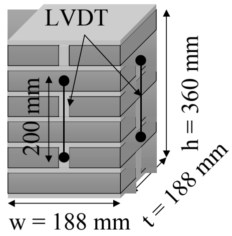
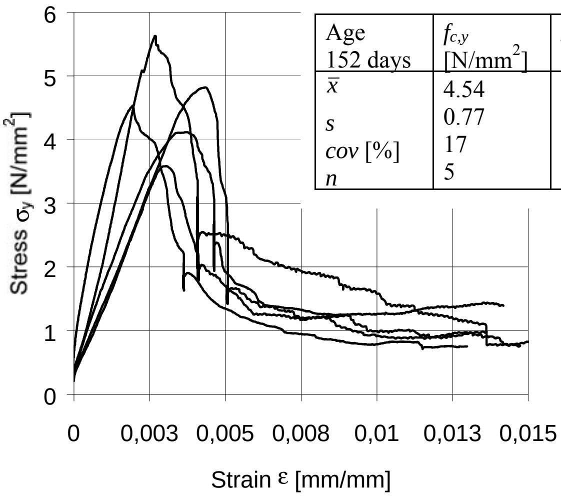


Figure 1 : Masonry pillars – stress-strain relationship and statistical summary



Age 152 days	$f_{c,y}$ [N/mm^2]	E_y [N/mm^2]	$G_{f,c}$ [Nmm/mm^2]
$\bar{x}$	4.54	1690	1.35
s	0.77	372	1.02
cov [%]	17	22	76
n	5	5	5

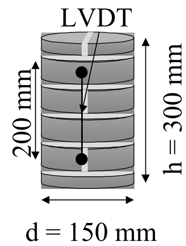
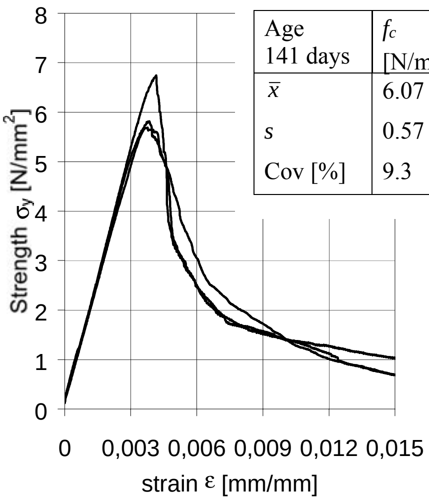


Figure 2 : Masonry cores-stress-strain relationship and statistical summary



Age 141 days	f_c [N/mm^2]	E_y [N/mm^2]	[illegible]	G_{fc} [Nmm/mm^2]
$\bar{x}$	6.07	1642	0.19	2.73
s	0.57	140	0.06	0.60
Cov [%]	9.3	8.5	31	22

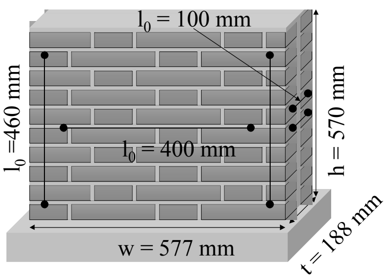


Figure 3 : Masonry wallets – stress-strain relationship and statistical summary

The third series of samples are masonry wallets. The three small masonry wallets have nominal sizes: $w \times d \times h = 600 \times 188 \times 600$ *mm*. The sample mean of the compressive strength is relatively high (*6.07 N/mm²*). The spread on the compressive strength is very low. This might indicate that taking larger samples, a more homogenized material is obtained, resulting in lower spread on the material properties. The masonry wallets behave less brittle than masonry prisms or cores, Fig. 3.

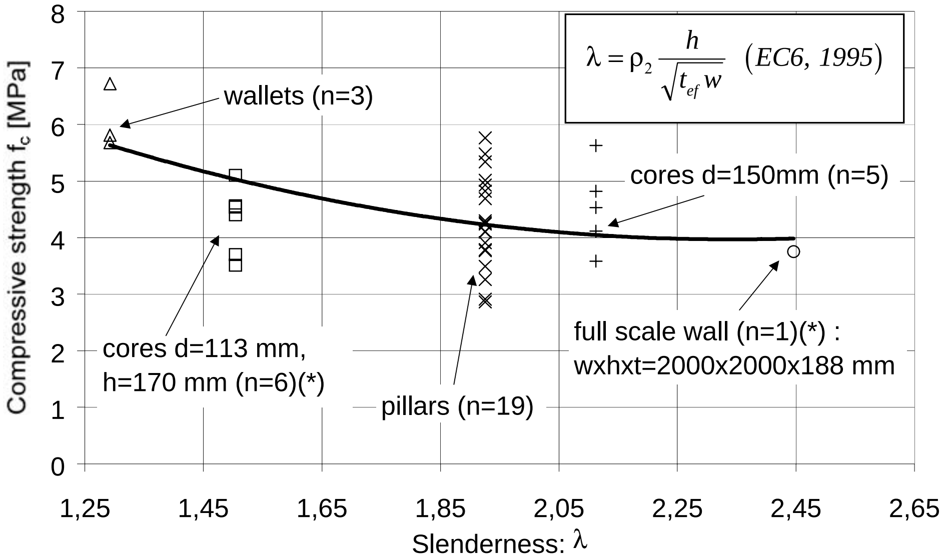


Figure 4 : Compressive stress versus slenderness

As found in previous tests (Schueremans, 2001a), the mean compressive strength seems to decrease as the "slenderness" (8) increases (EC6, 1995). Therefore, the strength values of pillars and cores are more representative for masonry full scale walls than small masonry wallets, having a limited slenderness, Fig. 4. As pillars and cores are far more easily to build and test, these are preferred. In addition, cores can be taken more easily from an existing building representing the original state of the masonry.

4.4 Experimental verification of numerical results

For bricks, the normalized compressive strength (EC6, 1995) is used, introducing a normalizing factor ∗. The value of the coefficient *K* for group 1 masonry units equals (EC6, 1995): $K = 0.6$. The mean value and spread of the resulting lognormal distribution can be calculated on an analytical basis, Table 2.

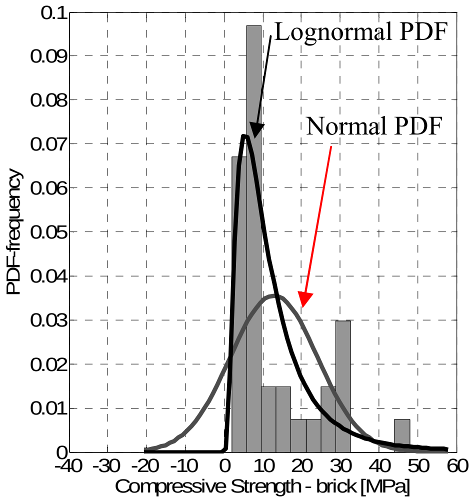
Table 2 : Probability distribution function (PDF) and parameters for masonry compressive strength

Compressive strength f_c MPa]	PDF	Average μ [MPa]	Spread σ [MPa]	Coefficient of variation Cov [%]	Test samples
Masonry	LN	4.26	0.81	19	Pillars
Experimental	/	4.53	0.77	17	Cores ϕ150
Masonry	LN	4.60	1.00	22	From brick cores ϕ50
Numerical	LN	4.55	0.84	18	From brick couplets

Comparing the numerical results with the results based on experimental research, it is clear that a good estimate of the mean value is obtained: f_c = *4.55-4.60 N/mm²* (numerical) against f_c =*4.26-4.53 N/mm²* (experimental). A very good approximation of the coefficient of variation is obtained too. In both cases, the coefficient of variation is around *20%*. The numerical model confirms the idea of obtaining a more homogeneous composite material.

CASE STUDY – DUKE'S MILL (AARSCHOT, BELGIUM)

The numerical relation outlined above and experimentally verified, is used in the analysis phase of historical construction more often (Schueremans et al., 2006a;b). It is exemplified with a case study. The construction of two separated mills at each side of the river Demer dates back to 1501-1518. They mainly served as corn and bark mills. The turbulent history of the mill called the load bearing capacity of the structural masonry to be determined (Van Balen et al., 1999). Therefore, use is made of compressive tests on bricks and mortar and the stochastic extension of the numerical relation in between the components brick and mortar and the composite masonry, eq. 3-6. The mortar strength is further validated based on a chemical analysis of the mortar, reported elsewhere (Schueremans et al., 2006b). The compressive strength of bricks and mortar samples are determined from in site specimens. Therefore, bricks have been taken from the original building at randomly chosen locations. Similarly, small mortar samples are taken from the bed joints. These are sawn into small samples with overall size $20x10x10mm^3$. Of course, the latter do not correspond with standard sizes for compressive tests of hardened mortar. The test data are summarized in Fig. 5.



Available Data :

statistics	Bricks	mortar
N	35	11
Mean [MPa]	12.90	1.64
Std [MPa]	11.23	0.89
cov [%]	87	54

Resulting parameters for PDF

Bricks: ~LogNormal PDF:
$\mu(f_b)$=9.41MPa and $f_{b,k}$= 2.58 MPa

Mortar: ~LogNormal PDF:
$\mu(f_m)$= 1.40MPa and $f_{m,k}$= 0.51MPa

Figure 5 : Data summary of tests on brick and mortar compressive strength

The histogram and fitted probability density function are shown for the brick data on the left hand side. The data statistics for bricks and mortar are summarized on the right hand side. As a result of these data, an estimate of the characteristic strength of the masonry is obtained:

$$f_k^{'} = K(\delta . f_b)^{0.65}(f_m)^{0.25} = 0.6(0.85\times 9.41)^{0.65}(1.40)^{0.25} = 2.52\,MPa \tag{8}$$

This value is used to check the remaining safety in practice and for strengthening design according to the partial safety factor format. Additionally, from this analysis, the PDF of the masonry strength can be determined according to eq. 6, with following parameters:

$$f' \approx LN\left(\mu_{f'} = 7.04MPa; \sigma_{f'}^2 = (4.03MPa)^2\right) \tag{9}$$

It is clear from Fig. 5 that using a typical normal or Gaussian distribution type for the compressive strength of the bricks this would lead to negative characteristic values, and thus hastily conclusions related to the overall compressive strength of the masonry. This is often the case for historical masonry as its average strength is usually rather limited and its spread is generally large.

5 CONCLUSIONS

In this paper, the in site strength of structural components is estimated based on non-destructive and destructive testing, material models and statistical analysis. This information is required in a structural reliability analysis to assess the remaining safety of the load-bearing system. Based on experimental results it is pointed out that a lognormal (or truncated normal) probability density function fits the masonry compressive strength well. The numerical model presented in Eurocode 6 is verified experimentally by means of compressive tests on the components brick and mortar as well as on the composite masonry. This knowledge is used in practice. The case study treated, demonstrates how the masonry compressive strength is determined based on in site material in a least-destructive manner. The latter results are used in the overall analysis of the structure and design of strengthening. From this it is clear that, in case sufficient test results are available, appropriate probability distribution functions can be assigned to the masonry compressive strength. Since this is not the only material characteristic encountered in the analysis phase, significant effort is still required for the appropriate probabilistic modelling of the increasing number of material parameters used in state of the art numerical analysis. A starting point is provided elsewhere (Schueremans, 2001a,b).

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