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Analysis of Vaulted Masonry Structures Subjected to Horizontal Ground Motion

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ABSTRACT: Three methods for analyzing vaulted unreinforced masonry structures subjected to horizontal ground motion are presented and compared: real time thrust-line analysis, rigid body dynamics (RBD), and discrete element analysis. These methods are used to analyze three structures of increasing complexity: the single rocking block, the masonry arch, and the masonry arch on buttresses. Comparison of the results verifies the accuracy of the methods as well as the advantages of each method.

1 INTRODUCTION

The dynamic response of unreinforced masonry structures to earthquake excitation is of critical importance when assessing the structural safety of many historic constructions. Several analysis techniques have been used to evaluate structural safety, with the ultimate goal of predicting the magnitude of force (or earthquake excitation) which would cause structural failure. However, these techniques vary tremendously in the assumptions which they are built upon, leading to a wide range of analysis complexities, computational time and effort, and therefore economic investment. Additionally, the accuracy of these methods for predicting collapse is still in question. With this in mind, the structural engineer charged with the task of evaluating structural safety is faced with the dilemma of determining which analysis technique is optimal for a given application. One must ask, “Is it feasible to spend a large amount of time and money on every masonry structure that stands vulnerable to collapse in an earthquake? On the other hand, will any cost be spared for the preservation of the world’s most acclaimed monuments?” Although several methods of analysis have been applied to determine the safety of unreinforced masonry structures subjected to earthquake loading, the goal of this research is to focus on a few analysis tools and to conduct a first order evaluation of their capabilities and validity.

However, before introducing the analysis methods used in this research, it is first necessary to take a step back and review the methods which have been traditionally used to analyze unreinforced masonry structures under static loading. Typically, three simplifying assumptions regarding the behavior of unreinforced masonry have been applied to make analysis more feasible. These assumptions were originally introduced by Couplet, but are stated clearly in the more recent work of Heyman (1995) as follows:

- masonry has no tensile strength;
- stresses are so low that masonry has effectively an unlimited compressive strength; and
- sliding failure does not occur.

These basic assumptions provide the criteria used for a first order analysis of masonry structures within the framework of limit analysis. Certainly each of these assumptions must be critically addressed for any given construction, especially under dynamic loading, but they are the foundation upon which this research is built.

The following methods of analysis will be applied to evaluate the safety of unreinforced masonry structures in seismic areas: (1) thrust line analysis using graphic statics, (2) rigid body dynamics, and (3) discrete element analysis. These methods will be introduced independently and then applied to different problems for evaluation and comparison.

1.1 Thrust line analysis using graphic statics

Although the primary focus of this research is the dynamic analysis of unreinforced masonry, it is valuable to begin with an 'equivalent static analysis', meaning the analysis of structures subjected to constant horizontal acceleration (first order earthquake simulation) and constant vertical acceleration (gravity). Equivalent static analysis is essentially a stability analysis which is solely based on geometry and is independent of scale.

Under the assumptions of limit analysis, individual blocks are not free to slide or crush, but are only free to separate, or hinge. Hinges form when the 'thrust line' can no longer be contained within the masonry and reaches the surface of the masonry. At this point, the masonry can no longer support the forces, and the structure is no longer in equilibrium without hinging. However, when hinging occurs, the collapse of the structure is not necessarily eminent. Take for example, the specific case of an arch on spreading supports studied by Ochsendorf (2006). Three hinges must form immediately upon support displacements, but the thrust line remains within the masonry and the structure therefore remains stable until four hinges are formed at the point of collapse.

Although the static analysis of arched structures has been studied for several years, Block (2005) recently developed a useful tool (http://web.mit.edu/masonry/interactiveThrust/) which uses graphic statics to achieve a rapid first order assessment of the stability of various masonry structures. The real time graphic statics framework allows the effect of geometrical changes such as arch thickness, buttress width, etc., to be readily evaluated. For the current paper, this tool was extended to simulate structures on tilting surfaces, an effective way to conduct an 'equivalent static analysis' (http://web.mit.edu/masonry).

1.2 Rigid body dynamics

While applying a constant horizontal acceleration to a structure is a valuable first step in simulating response to an earthquake, the resulting allowable horizontal ground accelerations are clearly conservative. If instead, the duration of the horizontal ground acceleration is reduced, then higher accelerations can be withstood. As a result, analytical rigid body dynamics solutions have been used to characterize this dynamic behavior of rigid body problems.

The fundamental rigid body dynamics problem is that of the single rocking block under horizontal base motion, which was first investigated by Housner (1963). Housner uses the same assumption as Heyman, but must also assume a restitution coefficient to account for the dynamic behavior which causes impact during rocking. Spanos (1984) expanded on the research of Housner, and plots the stability of the rocking block subjected to harmonic accelerations of varying frequency and amplitude. The nature of the solution would logically be the same for multiple block 'structures' but the analytical solution becomes difficult to obtain. Additionally, several other researchers have noted the shortcomings of the limit analysis assumptions, and have found analytical solutions which allow for the possibility of more complex behavior such as sliding and bouncing (Augusti and Sinopoli 1992, Shenton and Jones 1991).

Rigid body dynamics was also used by Clemente (1998) and Oppenheim (1992) to study the semi-circular arch. Both of these studies effectively extend the idea of the single rocking block to a more realistic structure which still only has one degree of freedom. Their analysis also does not include the possibility of sliding or more complicated arch hinging mechanisms.

1.3 Discrete Element Analysis

Although the analytical solutions to the rigid body dynamics problems provide insight into the nature of the dynamics of masonry structures, their complexity demonstrates the need for computational tools which can correctly address the problem of rigid block dynamics. Initially, finite element programs were the computational tools of choice for most engineers, but they were optimal for problems of elasticity not stability. Advances in non-linear finite element modeling have made them more appropriate for masonry structures, but the more recent application of discrete element modeling inherently captures the discontinuous nature of masonry and allows for fully dynamic analysis with large displacements.

Discrete element programs are particularly suitable for masonry structures because they allow the definition of individual blocks within the structure. Constitutive properties of blocks and contacts must be defined and input. These programs typically solve the equations of motion of each individual block in the system using a time stepping scheme. Contact forces on each block are assumed to be proportional to the inter-penetration between blocks, which is determined using the input contact stiffness. The out of balance force after each time step is applied to the equations of motion in the next time step, making the magnitude of the time step critical for accurate modeling. Bićanić et al. (2003) applies finite element and discrete element methods to model an actual masonry arch bridge which was subjected to vertical loading until failure. Although this is a problem involving static loading, the authors conclude that discontinuous modeling frameworks provide a viable alternative for evaluating the structural integrity of masonry, and for predicting the ultimate collapse load and failure mode. In this research, the Universal Discrete Element Code (UDEC) was used and evaluated in comparison with the previously defined analysis methods (Cundall 1980).

2 THE ROCKING BLOCK PROBLEM

The first problem which will be addressed using the analysis methods already introduced is the single rocking block on a rigid half-space. As stated earlier, this problem has been extensively investigated by several researchers, but it serves as a good starting place for a comparison of the analysis methods in question.

2.1 Thrust-line Analysis

Although a thrust line analysis using a real-time graphic statics tool could be performed to achieve the constant horizontal acceleration which would cause the collapse of a single rigid block, the solution is trivial:

$$\ddot{x}_g = \lambda * g \quad where \quad \lambda = B/H \tag{1}$$

where B and H are defined in Fig. 1, and g is the acceleration of gravity. Any dynamic horizontal ground motion which does not exceed this minimum level of acceleration will cause no rocking response of the rigid block.

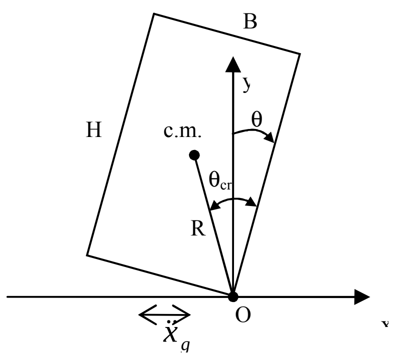


Figure 1 : Definition of rocking block problem on a rigid half-space.

2.2 Rigid Body Dynamics

The RBD solution of the rigid block subjected to harmonic horizontal ground motion is not quite as straightforward. Spanos and Koh (1984) present the full derivation and come to the following linearized equations of motion:

$$I_o\ddot{\theta} + MR\ddot{x}_g + MgR(\theta_{cr} - \theta) = 0, \quad for\ \theta > 0$$
$$I_o\ddot{\theta} + MR\ddot{x}_g - MgR(\theta_{cr} + \theta) = 0, \quad for\ \theta < 0 \tag{2}$$

where I_o is the moment of the inertia of the block about point O, M is the mass of the block, and where the horizontal base acceleration can be defined as:

$$\ddot{x}_g = A\theta_{cr} g \cos(\omega t) \tag{3}$$

where A is an acceleration amplitude coefficient, and ω is the frequency of the harmonic acceleration.

Solving these piecewise linearized equations of motion, the rotation angle as a function of time can be obtained. The resulting solution for the specific case of a block with H=4 m, B=1 m, a density of 2000 kg/m^3, a restitution coefficient of 0.925, an excitation frequency of 10.88 rad/s, and an acceleration amplitude of 0.5*g is plotted in Fig. 2. A block aspect ratio of 4:1 was selected so that neglecting both sliding and bouncing behaviour is more reasonable.

2.3 Discrete Element Analysis

The same problem can also be solved using UDEC, however, different material properties are required. For this analysis, the density of masonry was assumed to be 2000 kg/m^3, and the joint normal and shear stiffness' were assumed to be 200 GPa/m. Rayleigh damping was used with a minimum damping ratio (ξ_{min}) of 0.01% occurring at a frequency of 0.1 Hz. This limited the damping ratio to less than 0.5% in the frequency range from 0.001 to 10 Hz. A large coefficient of friction was used to prevent any sliding.

Figure 2 displays that the results of analysis using UDEC and RBD compare quite well. This is not a surprise, but it is a critical first step in determining the validity of UDEC as an analysis method. These results are supported by the results of Papantonopoulos (1997), who demonstrated good correlation between the solutions yielded by UDEC and RBD for a rocking block subjected to simulated earthquake ground motion. The results shown in his paper were not known to the authors of this paper at the time of investigation, and therefore show an independent verification for a similar problem.

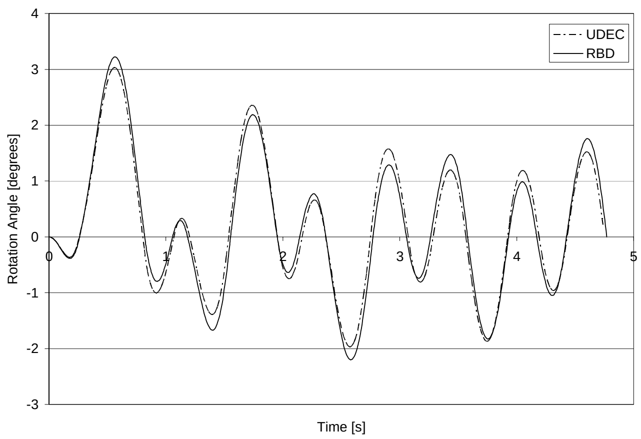


Figure 2 : Comparison of the predicted rotation angle (θ, see Fig. 1) of a rocking block on a rigid half-space using both UDEC and rigid body dynamics.

3 THE SEMI-CIRCULAR MASONRY ARCH

While the analysis of the rocking block serves as a good verification problem with an achievable analytical solution, the analysis of the masonry arch provides results that are more applicable to real structures. For this case study, the masonry arch studied by Oppenheim (1992) will be investigated, which has the following characteristics: seven voussoirs, inclusion angle (β) of

157.5 degrees, center radius of 10 m, and a thickness/radius ratio of 0.15. Using this geometry, the methods of analysis will be applied and compared.

3.1 Thrust-line Analysis

Thrust-line analysis is used to rapidly determine the constant horizontal acceleration which would cause collapse, and to determine the hinge locations at the point of collapse. Figure 4a shows the thrust-line (dashed) of the arch at the point just before collapse, when tilted to simulate a constant horizontal acceleration. If a larger constant horizontal acceleration is applied, hinges will form at the four locations where the thrust line touches the extrados or the intrados of the arch, and the arch will collapse. The maximum allowable constant acceleration was found to be 0.37*g from thrust-line analysis.

3.2 Rigid Body Dynamics

Oppenheim (1992) used RBD and also found the constant horizontal acceleration necessary to cause collapse to be 0.37*g, as well as the same hinge locations shown in Fig. 4a. However, the amount of effort required to obtain the solution is significantly larger. Additionally, using thrust-line analysis, the geometry of the arch can be updated in real time to rapidly determine the safety of various arch geometries. The RBD solution verifies the thrust-line analysis solution, but also highlights the usefulness of thrust-line analysis for a rapid first order assessment of the stability of an arched structure.

The advantage of using RBD is that the dynamic behaviour of the arch can be studied. Oppenheim (1992) studied the dynamic behaviour of the arch during the first half cycle of oscillation under an idealized horizontal ground impulse. However, in order to write the single degree of freedom equations of motion, the assumption was made that, under impulse loading, the hinge locations will form and remain at the same locations as found during constant horizontal acceleration loading. This assumption, while necessary, unrealistically limits the motion of the arch but provides a way of obtaining a closed form solution.

3.3 Discrete Element Analysis

The impulse ground motion implemented by Oppenheim (1992) was also used for UDEC analysis in order to compare results (Fig. 3a). The same material properties stated in section 2.3 were used for analysis, and the hinge locations were limited to those found by Oppenheim (1992). The resulting collapse mechanism can be seen in Fig. 4b. Using both methods, the arch could withstand a 1.0*g acceleration impulse for 0.42 seconds, but longer impulse durations were found to cause collapse (Fig. 3b). The results compare remarkably well with those obtained by Oppenheim (1992), and again verify the accuracy of UDEC in predicting analytical rigid body dynamic behaviour.

Although we have some confidence in the solutions, the applicability of these solutions is still in question. What would happen if hinges were allowed to form anywhere? What would happen if sliding were permitted? What would happen *after* the first half cycle of oscillation? This is where the framework of discrete elements extends our ability to solve these problems.

Using UDEC, it was determined that the actual critical collapse would occur during the second half-cycle of oscillation resulting in the ultimate collapse mechanism shown in Fig. 4c, but the hinge locations would change continuously during motion prior to collapse. Assuming no sliding, the arch under investigation would actually collapse under an idealized impulse of magnitude 1.0*g for only 0.23 seconds, instead of 0.42 seconds. Additionally, if sliding were allowed, then sliding failure would occur before failure due to instability for friction coefficients (μ) less than 0.53.

More generally, the stability of the structure can now be determined for a variety of combinations of acceleration magnitudes and durations. The resulting regions for which the arch does not hinge at all, recovers, and collapses are shown in Fig. 5. The results of Oppenheim clearly overestimate the actual capacity of the arch to withstand the given impulse, due to the assumptions which were made.

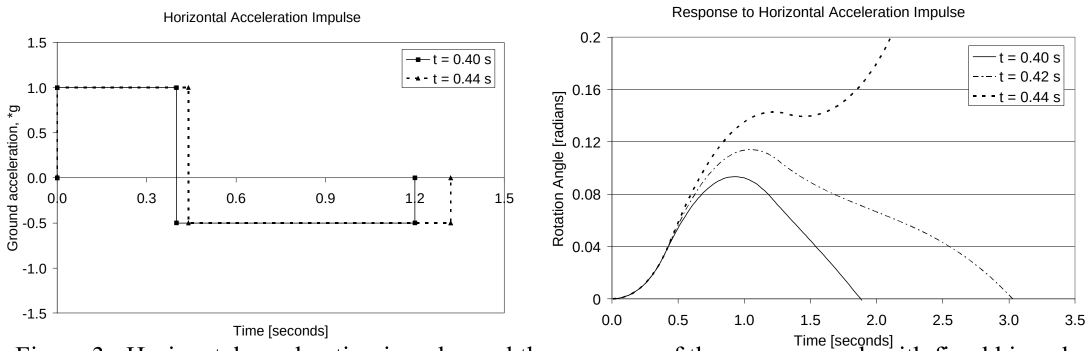


Figure 3 : Horizontal acceleration impulse and the response of the masonry arch with fixed hinge locations using UDEC (rotation angle indicates rotation of the right arch segment about point A in Fig. 4b).

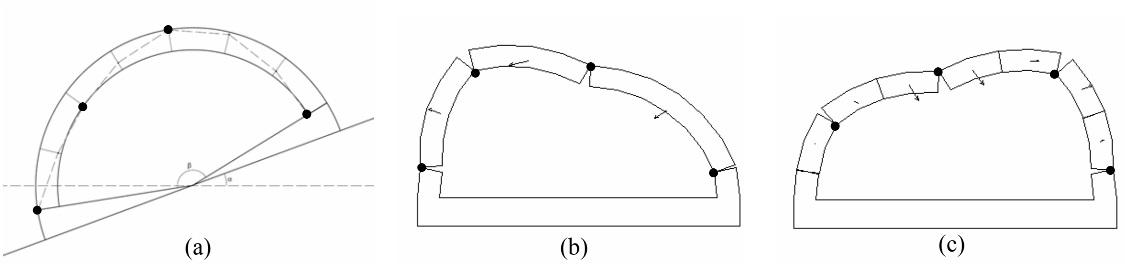


Figure 4 : Collapse mechanisms (points indicate hinge locations) which occur during: (a) tilted thrust-line analysis (thrust-line is dashed), (b) the first half-cycle of oscillation if the hinge locations are fixed (UDEC), and (c) the second half-cycle of oscillation if the hinge locations are not fixed (UDEC).

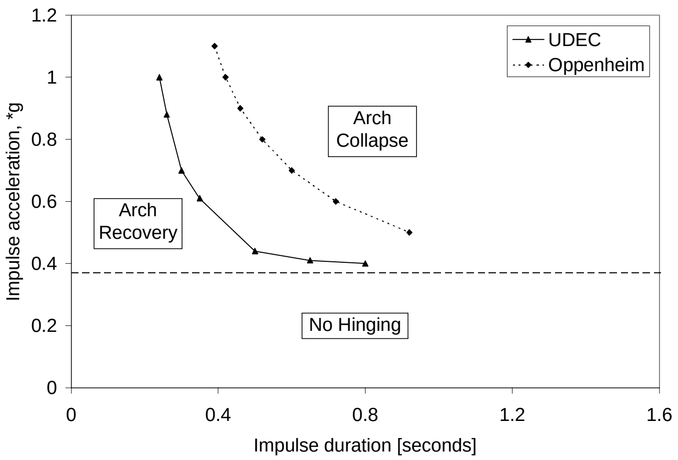


Figure 5 : Arch response as a function of the impulse duration and acceleration magnitude.

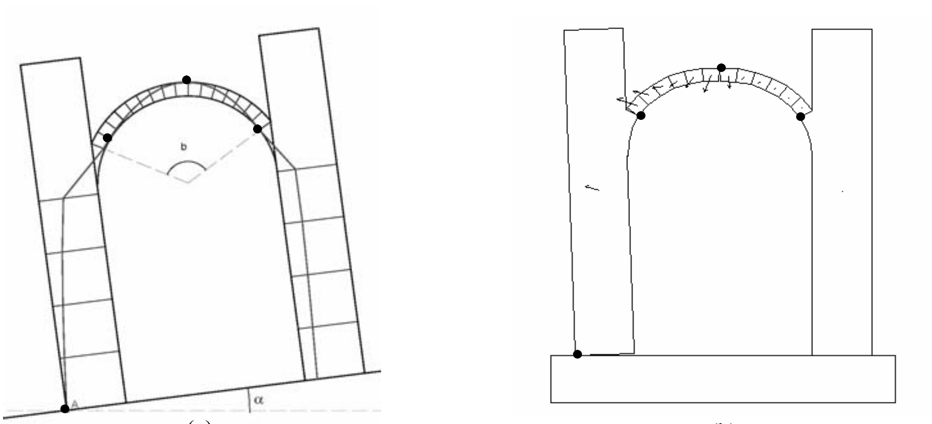
4 ARCH ON BUTTRESSES

The final problem which will be addressed is the arch on buttresses. In order to allow comparison with the previous results, an arch with the same radius (10 m), and t/R ratio (0.15) will be investigated. The buttress height and width were assumed to be 1.5 and 0.5 times the vault span, respectively. This problem was investigated using only thrust-line and discrete element analysis because single degree of freedom RBD analysis would limit the motion of the arch on buttresses unrealistically.

For equivalent static analysis, the UDEC material properties stated in section 2.3 were again used. Both thrust-line and UDEC analysis resulted in a constant horizontal acceleration necessary to cause collapse of 0.13*g. The hinge locations at collapse compared well (see Fig. 6).

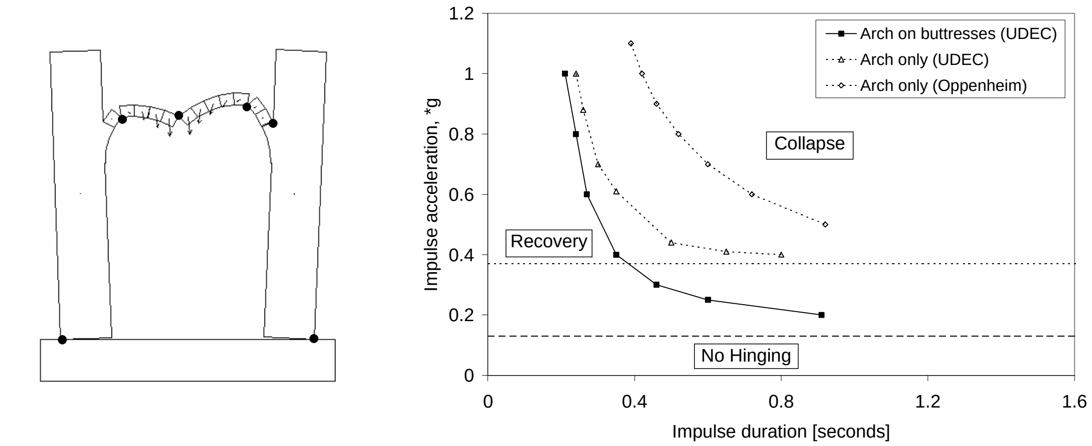
The effect of an impulse loading (see Fig. 3a) on the arch on buttresses was also investigated using UDEC. For this analysis, it was necessary to increase the joint normal and shear stiffness' to 5000 GPa/m, and the frequency of minimum damping to 1 Hz. The minimum damping ratio was effectively adjusted to less than 0.5% for the frequency range from 0.01 to 100 Hz, allowing the solution to be obtained more rapidly. Collapse occurred in the following progression: tilting of both buttresses slightly to left, tilting of both buttresses to the right, tilting of the left buttress to the left, collapse of the arch, and recovery of both buttresses. The final collapse mechanism is shown in Fig. 7a with both buttresses leaning outward while the arch collapses.

The resulting regions for which the arch on buttresses would fail are plotted in Fig. 7b, along with the results from Section 3 for comparison. For all impulse durations, the addition of the buttresses causes the structure to collapse at lower magnitudes of acceleration, effectively 'weakening' the structure. This 'weakening' effect increases with an increase in the impulse duration or a decrease in the impulse magnitude, as is expected because the buttresses (similar to the rocking block) are more susceptible to low frequency excitations (Spanos and Koh 1984), and therefore dominate the response to longer impulse durations.



(a) (b)

Figure 6: (a) Thrust-line analysis and (b) UDEC collapse mechanism of an arch on buttresses subjected to constant horizontal acceleration.



(a) (b)

Figure 7: (a) UDEC collapse mechanism under impulse loading and (b) response of arch on buttresses as a function of the impulse duration and acceleration magnitude

It also must be mentioned that the assumption of monolithic buttresses is a non-conservative one, as they have been observed to fracture under lateral loading due to the low tensile strength of masonry (Ochsendorf, 2002). Buttress cracking could be easily included by discretizing the buttresses in the UDEC model.

5 CONCLUSIONS

The primary goal of this study was to develop and evaluate analysis tools for studying masonry structures subjected to horizontal ground motions. The rocking block and masonry arch problems were used to verify the accuracy of thrust-line analysis and discrete element analysis, and display the limitations of RBD analysis. The arch on buttresses problem was used to demonstrate the capabilities of discrete element analysis for determining the collapse mechanism and collapse loadings for a more realistic masonry structure. From these example problems, the following conclusions can be drawn:

1. Thrust-line analysis is a valuable, accurate, and efficient tool for predicting the collapse mechanism and constant horizontal acceleration which will cause collapse.
2. The level of confidence in discrete element (UDEC) modeling to accurately predict the response of rigid block systems to dynamic loading has been increased.

Clearly, future work is necessary to determine model material properties which accurately simulate the behavior of actual masonry structures. This is especially necessary for the investigation of possible sliding failure. Additionally, a stability analysis such as this should always be used in combination with a strength analysis to determine if the crushing of stones is a concern.

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