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The Rankine-Type Criterion Aimed at describing Masonry Orthotropy

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ABSTRACT: An orthotropic material model for the analysis of masonry problems in the plane stress state is presented. In order to model orthotropic masonry behaviour, a maximum principal stress failure criterion of Rankine is adopted by incorporating the second order strength tensor. Within the framework of the finite element method for the elastoplasticity theory of small strains for softening/hardening materials, two yield surfaces resulting from the orthotropic principal stress criterion are described and implemented at the integration point level into the commercial nonlinear finite element program by means of user-defined subroutines. The basic behaviour of the model is demonstrated in numerical examples.

1 INTRODUCTION

The unreinforced masonry may be assumed to be a homogeneous but obviously anisotropic material, which exhibits distinct directional properties due to the influence of the mortar joints acting as planes of weakness. The highly anisotropic behaviour of masonry panels may be also obtained if fibre reinforced polymers are used for strengthening deteriorated walls. A numerical model must reproduce an orthotropic material with different tensile and compressive strengths along the material axes as well as different inelastic behaviour for each material axis. A reduced number of orthotropic material models specific for masonry have been proposed. A very few of them have been successfully implemented, see e.g. Lourenco (1996) where the plane stress continuum model formulated has been successfully implemented into a proprietary finite element code. Following the verifiable implementation, the model proposed in Sections 2 and 4 of this paper is also based on two yield surfaces, comprising of an anisotropic Rankine yield criterion for both tension and compression regimes. For the compression regime, the anisotropic version of the maximum principal compressive stress criterion is proposed which may be regarded as a version of the Rankine criterion. Thus, both of failure surfaces may be named Rankine-type failure criterion, since they are derived from the original Rankine yield criterion.

Additionally, apart of a generalization of the principal stress criterion, a simple formulation is presented in Section 3. Here, the simplified form of the Rankine criterion is combined with the criterion resulting from the shear failure mechanism.

An example of the graphical representation of the criteria for the plane stress in comparison with relevant experimental results from a literature is provided, illustrating the orientation-dependent nature of strength characteristics.

The paper is concluded by presenting some preliminary test results of a finite element analysis verifying own implementation. Here, the model for compression regime is a novel development.

2 ORTHOTROPIC RANKINE-TYPE FAILURE CRITERION

The fracture of isotropic brittle materials under tensile and small compressive stresses is best described by the maximum tensile stress criterion of Rankine. This criterion assumes that failure occurs when the maximum principal stress at a point inside the material reaches a value equal the tensile strength f_t of the material as found in a simple tension test. Consider a physical plane specified by a unit normal vector **n** and denote the stress state by $\boldsymbol{\sigma}$. The traction vector is $\mathbf{t}=\boldsymbol{\sigma}\mathbf{n}$ $(t_k= \sigma_{kl}n_l)$ and the normal stress vector is

$$\mathbf{t}^n = (\mathbf{n} \bullet \boldsymbol{\sigma}\,\mathbf{n})\mathbf{n} \qquad \vee \qquad t_i^n = (n_k \sigma_{kl} n_l) n_i \tag{1}$$

where the dot between two vectors signifies their scalar product. The magnitude of the normal stress component, i.e. the projection of the vector (1) on the direction **n** is dependent on the orientation of the physical plane. At the same time, the tensile strength of the isotropic material is assumed orientation-independent. The generalization of the Rankine failure criterion is possible to be done by assuming that the tensile strength also depends on the orientation of the physical plane, i.e. $f_t=f_t(\mathbf{n})$. The condition at the tension failure may be then described by employing the maximization procedure

$$\underbrace{\max}_{n}\left[\mathbf{t}^n \bullet \mathbf{n} - f_t(\mathbf{n})\right] = 0 \quad \text{subjected to constraint } \mathbf{n}\bullet\mathbf{n}=1. \tag{2}$$

If the variation of the strength parameter $f_t(\mathbf{n})$ is known, the solution to the problem (2) provides the orientation of the plane and allows obtaining the failure condition caused by the tension. For the orthotropic materials with three mutually orthogonal privileged directions that correspond to the principal X, Y and Z axes of material orthotropy and when referred to an orthonormal $\{x_i\}$ frame coaxial with the principal axes, the simplest form of the strength distribution function $f_t(\mathbf{n})$ can be written following Geniev and Malyszko (2002) as

$$f_t(\mathbf{n}) = f_{tX}\, n_X^2 + f_{tY}\, n_Y^2 + f_{tZ}\, n_Z^2 \tag{3}$$

where the direction $\mathbf{n}=(n_X , n_Y , n_Z)$.

The parameters f_{tX} , f_{tY} and f_{tZ} denote the tensile strengths that may be determined from three simple tension tests along the direction of the material axes. The solution to the problem (2) with the variation of the strength parameters in the form (3) gives the following failure criterion

$$\begin{aligned}&(f_{tX} - \sigma_x) - (f_{tY} - \sigma_y)(f_{tZ} - \sigma_z) - (f_{tX} - \sigma_x)\tau_{yz}^2 - \\ &(f_{tY} - \sigma_y)\tau_{xz}^2 - (f_{tZ} - \sigma_z)\tau_{xy}^2 - 2\tau_{xy}\tau_{xz}\tau_{yz} = 0\end{aligned} \tag{4}$$

referred to the material axes of the orthotropic medium.

This criterion can be also obtained by assuming that the strength condition depends on the structural second-order symmetric tensor **S**, represented in an arbitrary right-handed Cartesian coordinate system $\{x_i\}$ by the components S_{ij}

$$S_{ij} = f_{tX}\, m_{ij}^{(1)} + f_{tY}\, m_{ij}^{(2)} + f_{tZ}\, m_{ij}^{(3)} \tag{5}$$

where the components $m_{ij}^{(k)} = e_i^{(k)} e_j^{(k)}$ of the respective structure-orientation tensors are dependent on three unit vectors $\mathbf{e}^{(k)}$, $k=1, 2, 3$ specifying the privileged directions of the material.

Let $\mathbf{S}^n$ denote the vector associated with the direction **n** by the tensor **S**. In the $\{x_i\}$ frame coaxial with the principal axes of material orthotropy the projection of the vector $\mathbf{S}^n$ on the direction **n** is given by the distribution (3). Instead of the maximization procedure (2), one may determine a critical principal direction $\overline{\mathbf{n}}$ of the tensor $\mathbf{N} = \boldsymbol{\sigma} - \mathbf{S}$ for which the principal value is equal to zero. Principal values and principal directions of the tensor **N** are determined by the equations

$$(N_{ij} - N\delta_{ij})\,\overline{n}_i = 0 \tag{6}$$

where N is the magnitude of the principal values. In order to have a nontrivial solution, the determinant of coefficients must vanish. Expanding the determinant leads to the characteristic equation. Taking $N=0$ in the characteristic equation leads to the requirement that determines the failure condition

$$\det[\mathbf{N}] = \det[\sigma_{ij} - S_{ij}] = 0 \tag{7}$$

Consider now the uniaxial tension test to the value $\sigma_1 = f_{tn}$ along the direction $\mathbf{n}=(n_X , n_Y , n_Z)$, referred to the principal axes of material orthotropy. In the $\{x_i\}$ frame coaxial with the axes of orthotropy, the expression of the failure condition (7) for this test takes the form

$$\det\left(f_{tn}\begin{bmatrix} n_X^2 & n_X n_Y & n_X n_Z \\ n_X n_Y & n_Y^2 & n_Y n_Z \\ n_X n_Z & n_Y n_Z & n_Z^2 \end{bmatrix} - \begin{bmatrix} f_{tX} & 0 & 0 \\ 0 & f_{tY} & 0 \\ 0 & 0 & f_{tZ} \end{bmatrix} \right) = 0 \tag{8}$$

Expanding Eq. (8) leads to the following expression of the directional tensile strength

$$\frac{1}{f_{tn}} = \frac{n_X^2}{f_{tX}} + \frac{n_Y^2}{f_{tY}} + \frac{n_Z^2}{f_{tZ}} \tag{9}$$

In the $\{x_i\}$ frame coaxial with the axes of the principal stresses, the expression of the failure condition (7) can be obtained if in the expression (5) the respective structure-orientation tensors $m_{ij}^{(k)}$ are known. In the case when one of the axes of the $\{x_i\}$ frame, say x_2 , i.e. σ_2, is coaxial with one of the principal axes of the material orthotropy, e.g. with the second Y-axis, the vectors $\boldsymbol{e}^{(k)}$ take the forms $\boldsymbol{e}^{(1)}=(\cos\alpha,\ 0,\ -\sin\alpha)$, $\boldsymbol{e}^{(2)}=(0,\ 1,0)$ and the failure condition (7) can be then expanded to the following form

$$\left(\frac{\cos^2\alpha}{f_{tX}} + \frac{\sin^2\alpha}{f_{tZ}}\right)\sigma_1 - \frac{\sigma_1\sigma_3}{f_{tX} f_{tZ}} + \left(\frac{\cos^2\alpha}{f_{tZ}} + \frac{\sin^2\alpha}{f_{tX}}\right)\sigma_3 = 1 \tag{10}$$

where the angle α measures the rotation between the first axis x_1, i.e. the axis of the first principal stress σ_1 and the first material X-axis

In the case of the plane stress state in the plane XZ, where X and Z are the principal axes of material orthotropy, the failure criterion can be written as

$$(f_{tX} - \sigma_x)(f_{tZ} - \sigma_z) - \tau_{xz}^2 = 0 \tag{11}$$

When the failure condition (7) is satisfied, the normal direction $\overline{\mathbf{n}} = (\overline{n}_i)$ of the physical critical plane in which the failure caused by tension takes place, can be directly determined from Eq. (6). In the plane stress state we have $\overline{\mathbf{n}} = (\overline{n}_X, 0, \overline{n}_Z)$ with the constraint $\overline{n}_i\overline{n}_i = 1$ and

$$\overline{n}_X^2 = \frac{f_{tZ} - \sigma_Z}{(f_{tX} - \sigma_X) + (f_{tZ} - \sigma_Z)} \quad \text{or} \quad \tan 2\overline{\varphi} = \frac{(\sigma_1 - \sigma_3)\sin 2\alpha}{(\sigma_1 - \sigma_3)\cos 2\alpha - (f_{tX} - f_{tZ})} \tag{12}$$

where $\overline{\varphi}$ denotes the angle between the normal to the tension failure plane and the first material X-axis.

If the tensile strengths f_{tX} , f_{tY} and f_{tZ} are replaced by the compressive strengths with positive values, i.e. by the strengths $-f_{cX}$, $-f_{cY}$ and $-f_{cZ}$, the failure condition caused by the compression can be formulated in the same manner. For instance, instead of Eqs. (11 and 10) we have

$$(f_{cX} + \sigma_x)(f_{cZ} + \sigma_z) - \tau_{xz}^2 = 0 \tag{13}$$

and

$$\left(\frac{\cos^2\alpha}{f_{cX}}+\frac{\sin^2\alpha}{f_{cZ}}\right)\sigma_1+\frac{\sigma_1\sigma_3}{f_{cX}f_{cZ}}+\left(\frac{\cos^2\alpha}{f_{cZ}}+\frac{\sin^2\alpha}{f_{cX}}\right)\sigma_3=-1 \tag{14}$$

As one can see from Eqs. (10-14), a representation of an orthotropic failure surface in term of principal stresses only is not possible. For plane stress situation, a graphical representation can be obtained either in terms of the full stress vector (σ_X ,σ_Y and τ_{XY}), referred to the material axes, or in terms of principal stresses and the angle α. It is obvious that it is not possible to formulate the failure criterion of orthotropic material in terms of the principal stresses only, since they are isotropic functions of the stress state. In Fig. 1, a comparison of the proposed criteria with experimental data of masonry specimens subjected to biaxial tests are presented. The material parameters are taken from Page (1981, 1983). The comparison shows quite good agreement in the tensile regime, less good agreement in compressive regime and a discrepancy in the shear regime. Some improvements are needed which may be done either by an introduction of a correction factor in the Rankine criterion or by addition third criterion specified the for shear failure. The former method is used in Section 4. The latter one is discussed bellow within simplified forms of failure criterion.

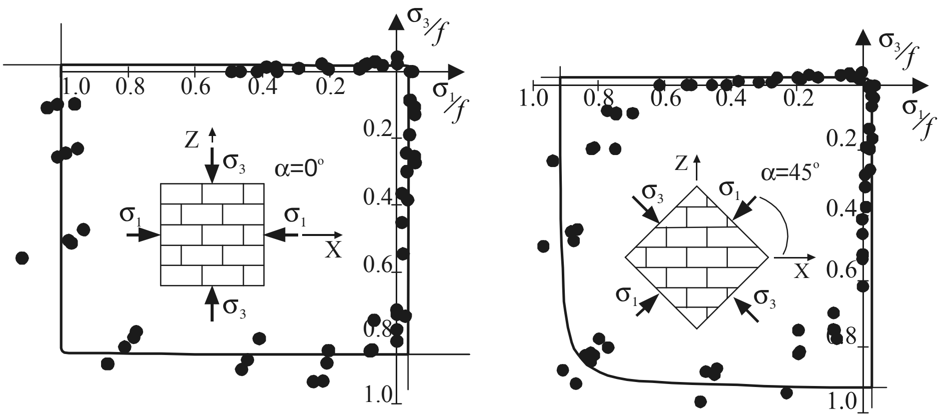

Figure 1 : Contours of generalized failure criterion of Rankine for strength parameters:
f_{tX}=0.3, f_{tZ}=0.15, f_{cX}=10.0, f_{cZ}=8.5 [MPa]

3 SIMPLIFIED FORM OF ORTHOTROPIC FAILURE CRITERION

The directional tensile and compressive strengths may be used to formulate the simplified principal stress failure criteria, see e.g. Malyszko (2002). Following the Eq. (9), the tensile strength f_{ti} along the direction of the principal stress axis σ_i=f_{ti} that is not coincided with the material axes, can be written as

$$\frac{1}{f_{ti}}=\frac{n_{Xi}^2}{f_{tX}}+\frac{n_{Yi}^2}{f_{tY}}+\frac{n_{Zi}^2}{f_{tZ}} \tag{15}$$

where the direction cosines between the principal stress axis (σ_i) and the material axis are denoted as n_{Xi} , n_{Yi} and n_{Zi}. Using Eq. (15), the failure criterion due the tension may be specified by

$$f_{ti}-\sigma_i=0 \quad \text{for} \quad i=1,2,3 \tag{16}$$

Also, the failure condition caused by the compression can be formulated in the same manner as

$$f_{ci}+\sigma_i=0 \quad \text{for} \quad i=1,2,3 \tag{17}$$

where the compressive strength f_{ci} along the direction of the principal axis $\sigma_i = f_{ci}$ that is not coincided with the material axes is written as

$$\frac{1}{f_{ci}} = \frac{n_{Xi}^2}{f_{cX}} + \frac{n_{Yi}^2}{f_{cY}} + \frac{n_{Zi}^2}{f_{cZ}} \tag{18}$$

The simplified form of the Rankine criterion, Eqs. (16, 17), is next combined with the criterion resulting from the shear failure mechanism. In the case of the shear failure in the XZ-plane, the critical plane direction ought to lie between the direction of the plane with the minimal shear strength (C_Z) and the direction of the principal shear stress $\tau_{13} = (\sigma_1 - \sigma_3)/2$. Following the expression (3), the function of the shear strength distribution $C(\mathbf{n}) = C_{13}(\alpha)$ may be written as

$$C_{13}(\alpha) = 0{,}5\left[(C_X + C_Z) - (C_X - C_Z)\left|\sin 2\alpha\right|\right] \tag{19}$$

where α denote an angle between the normal to the critical plane and the principal stress direction. The criterion resulting from the shear failure mechanism can be written as

$$\tau_{13} = C_{13}(\alpha) - \mu\frac{\sigma_1 + \sigma_3}{2} \quad \text{or} \quad \tau_{xz} = C_Z - \mu\sigma_z \tag{20}$$

where μ is the friction coefficient. For $|\alpha| = \pi/4$ both expressions in Eq. (20) mean the same restrictions. The shear criterion requires the material parameters μ, C_Z that may be obtained from triplet test. The shear strength parameters C_X, C_Z may be also obtained from the tests with the predetermined shear failure plane along the material axes in direct shear as proposed by Malyszko (2004).

In Fig. 2, a comparison of the proposed criteria with experimental data of masonry specimens subjected to biaxial tests are presented. The material parameters are: $\mu=1.0$, $C_X=1.45$ *MPa*, $C_Z=0.725$ *MPa*, $f_{tX}=0.3$ *MPa*, $f_{tZ}=0.15$ *MPa*, $f_{cX}=10.0$ *MPa* and $f_{cZ}=0.15$ *MPa*. The lines *1* parallel to the principal stresses axes are drawn according to Eqs. (16, 17) and correspond to the tension and compression failures.

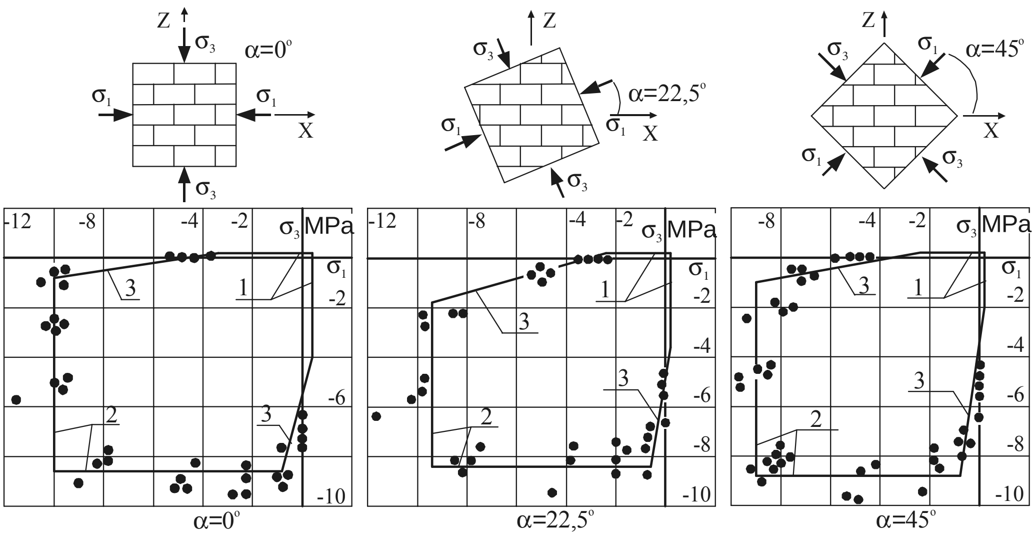

Figure 2 : Comparison of simplified criteria with experimental data from Page (1981, 1983)

Predictions according to the straight lines *3* are drawn according to the shear failure, described by Eqs. (20). Again, according to different values of α (0^0, 22.5^0 and 45^0), different graphical representations of an orthotropic failure surface are found. The comparison confirms ability of the proposed criteria to reproduce the strength of masonry. Globally, good agreement is found.

4 IMPLEMENTATION IN A FINITE ELEMENT CODE

Following the verifiable implementation with given source code from DIANA, two yield surfaces resulting from the orthotropic principal stress criterion (from Section 2) are implemented within the framework of the finite element method for the elastoplasticity theory of small strains for softening/hardening materials. The implementation is restricted to the integration point level and to the state of plane stress parallel to the *XZ*-plane with the assumption that the principal axes of orthotropy are coincided with the frame of reference. The constitutive relation between stresses $\boldsymbol{\sigma} = (\sigma_x, \sigma_z, \tau_{xz})$ and strains $\boldsymbol{\varepsilon} = (\varepsilon_x, \varepsilon_z, \gamma_{xz})$ are given in a material point. Within the framework of the elastoplasticity the orthotropic Rankine-type failure criteria serve as the yield surfaces and have to distinguish between domains of the elastic and plastic material response. The geometrical nonlinearity like cracking is accounted for by the introduction of loading functions f_β of the stress $\boldsymbol{\sigma}$ and the internal state parameter κ_β, where $\beta=t$ for tension and $\beta=c$ for compression regime. Within the yield surface ($f_\beta < 0$), the material behaves elastically according to the linear Hooke's law

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}^{el} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{pl}) \tag{21}$$

where in accordance with the usual approach of the flow theory of plasticity, the basic assumption of additive strain decomposition of the strain tensor $\boldsymbol{\varepsilon}$ into an elastic part $\boldsymbol{\varepsilon}^{el}$ and an irreversible plastic part $\boldsymbol{\varepsilon}^{pl}$ is made. For an orthotropic or transversely isotropic material in a so-called generalized plane stress problem in the *XZ*-plane, the elastic parameters are represented by the matrix $\boldsymbol{D}$ that contains four material constants: two Young's moduli E_x and E_z, one shear module G_{xz} and Poisson's ratio ν.

On the yield surface ($f_\beta = 0$), the material begins to yield. The inelastic strain $\boldsymbol{\varepsilon}^{pl}$ may be nonzero. The inelastic strain rate in the intersection of the different yield surfaces is obtained from a linear combination of the plastic strain rate of the tensile and compressive yield surfaces according to Koiter's generalization

$$\dot{\boldsymbol{\varepsilon}}^{pl} = \dot{\boldsymbol{\varepsilon}}_t^{pl} + \dot{\boldsymbol{\varepsilon}}_c^{pl} = \dot{\lambda}_t \frac{\partial g_t}{\partial \boldsymbol{\sigma}} + \dot{\lambda}_c \frac{\partial g_c}{\partial \boldsymbol{\sigma}} \tag{22}$$

where $\dot{\lambda}_t, \dot{\lambda}_c$ are inelastic multipliers and g_t, g_c are plastic potentials in tension and compression regimes, respectively .

The inelastic behaviour can be described by a strain softening hypothesis given by the maximum and minimum principal plastic strains in tension and compression regime, respectively. Thus, the particularly simple expression $\dot{\kappa}_\beta = \dot{\lambda}_\beta$ may be recovered for the internal state parameters κ_β.

The model has been coded in Fortran programming language and next implemented in a proprietary finite element code using the user-supplied subroutine *usrmat*. The subroutine lets the user specify a general nonlinear material behaviour by updating the state variables over the equilibrium step $n \rightarrow n+1$ in the iterative local Newton-Raphson procedure. The implicit Euler backward algorithm is used. In the presence of plastic flow, the return mapping algorithm for the corner regime reduces to the following system of five nonlinear equations containing five unknowns (the $\boldsymbol{\sigma}_{n+1}$ components, $\Delta\kappa_{t,n+1} = \Delta\lambda_{t,n+1}$ and $\Delta\kappa_{c,n+1} = \Delta\lambda_{c,n+1}$)

$$\left\{\begin{array}{l}\mathbf{D}^{-1}(\boldsymbol{\sigma}_{n+1}-\boldsymbol{\sigma}^{trial})+\Delta\lambda_{t,n+1}\dfrac{\partial g_t}{\partial\boldsymbol{\sigma}}|_{n+1}+\Delta\lambda_{c,n+1}\dfrac{\partial g_c}{\partial\boldsymbol{\sigma}}|_{n+1}=\mathbf{0}\\ f_{t,n+1}=\sqrt{\frac{1}{2}\boldsymbol{\xi}_{t,n+1}^T\mathbf{P}_t\,\boldsymbol{\xi}_{t,n+1}}+\frac{1}{2}\boldsymbol{\xi}_{t,n+1}^T\boldsymbol{\pi}=0\\ f_{c,n+1}=\sqrt{\frac{1}{2}\boldsymbol{\xi}_{c,n+1}^T\mathbf{P}_c\,\boldsymbol{\xi}_{c,n+1}}-\frac{1}{2}\boldsymbol{\xi}_{c,n+1}^T\boldsymbol{\pi}=0\end{array}\right\} \tag{23}$$

where the trial stress is

$$\boldsymbol{\sigma}^{trial}=\boldsymbol{\sigma}_n+\mathbf{D}\,\Delta\boldsymbol{\varepsilon}_{n+1} \tag{24}$$

and

$$\mathbf{P}_\beta=\frac{1}{2}\begin{bmatrix}1&-1&0\\-1&1&0\\0&0&4\gamma_\beta\end{bmatrix},\quad \boldsymbol{\pi}=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad \boldsymbol{\xi}_\beta=\boldsymbol{\sigma}-\boldsymbol{\eta}_\beta,\quad \boldsymbol{\eta}_t=\begin{bmatrix}\bar{f}_{tX}\\\bar{f}_{tZ}\\0\end{bmatrix},\quad \boldsymbol{\eta}_c=-\begin{bmatrix}\bar{f}_{cX}\\\bar{f}_{cZ}\\0\end{bmatrix} \tag{25}$$

Note that Eqs. (11, 13) are recast in a matrix form in Eqs. ($23_{4,5}$). Also, the additional parameters γ_β are introduced in the projection matrix $\boldsymbol{P}$ in Eq. (25) in order to control the shear stress contribution to failure instead of $\gamma_\beta=1.0$ being the standard Rankine value. If the standard values are used in failure criteria, the shear-compression part can be underestimated by the model in terms of biaxial loading. However, the failure criteria criteria ($23_{4,5}$) with the standard value may be taken as the plastic potentials g. If the projection matrix $\boldsymbol{P}$ with the standard value $\gamma_\beta=1.0$ is taken in the plastic potentials g and, at the same time, the value $\gamma_\beta\neq 1$ is taken in the yield criteria ($23_{4,5}$) then the non-associated flow rule is applied, both for tension and compression regimes. The value γ_β may be determined as

$$\gamma_\beta=f_{\beta X0}f_{\beta Z0}\big/\tau_\beta^2 \tag{26}$$

where τ_β are the pure shear strengths in tension and compression, respectively. The values $f_{\beta X0}$ and $f_{\beta Z0}$ are the characteristic yield values (initial or peak) of the uniaxial strengths in the direction of the material X, Z axes that may be obtained from appropriate equivalent stress-equivalent strain softening/hardening diagrams $\bar{f}_{\beta X}(\kappa_\beta)$, $\bar{f}_{\beta Z}(\kappa_\beta)$, with different fracture energies $G_{\beta X}, G_{\beta Z}$ for each yield value, see Lourenco (1996) for details. Note that in Eqs. (26) two additional strength parameters are introduced.

For the preliminary testing with Diana a single-element test was chosen under displacement control, with dimensions 100 x 100 [mm^2] (Fig. 3). The displacements in the vertical direction are not constrained. The elastic material parameters are $E_X=7500$ *MPa*, $E_Z=4000$ *MPa*, $\nu=0{,}15$ and

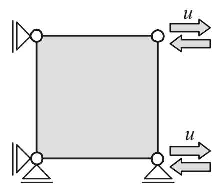

Figure 3 : Uniaxial tension and compression problem

$G_{XZ}=1400$ *MPa*. The inelastic material parameters are given in Tab. 1. The additional material parameter, the equivalent plastic strain κ_{cp} corresponding to the peak compressive strength according to softening/hardening law, is assumed to be equal to 0.002. The equivalent length, which is used to regularize the results with the respect to mesh refinement, is equal to *0.1* m.

Table 1 : Inelastic material parameters

Tension regime					Compression regime				
f_{tX0} MPa	f_{tZ0} MPa	γ_t	G_{tX} kNm/m^2	G_{tZ} kNm/m^2	f_{cX0} MPa	f_{cZ0} MPa	γ_c	G_{cX} kNm/m^2	G_{cZ} kNm/m^2
0.35	0.25	1.0	0.05	0.015	10.0	8.8	1.0	20.0	15.0

The stress-strain response for the tensile loading along the *X*-axis is shown in Fig. 4. When the tensile strength equal to *0.35 MPa* is reached, the material strength degrades according to the exponential tensile softening of the theoretical model. The anticipated constitutive behaviour is exactly reproduced. The stress-strain response for the compressive loading along the *X*-axis is shown in Fig. 4. Again, when the compressive strength equal to *10 MPa* is reached, the material strength degrades according to the hardening/softening law with a residual plateau from the theoretical model.

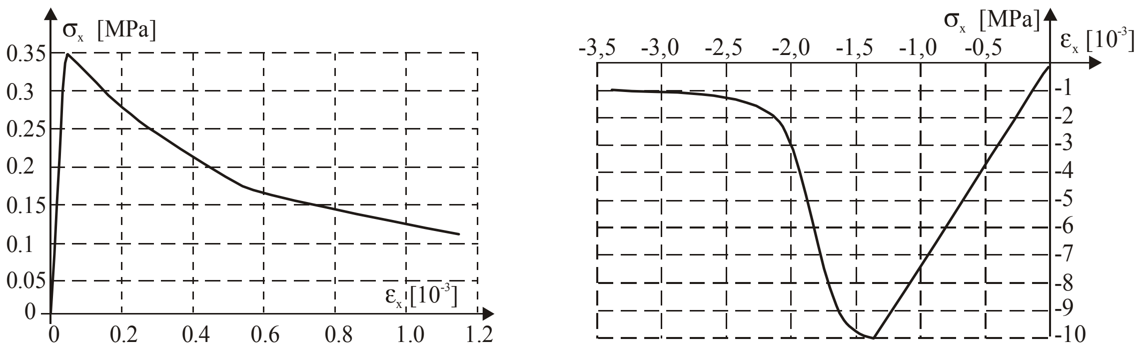

Figure 4 : Uniaxial stress-strain response in tension and compression

5 FINAL REMARKS

In the framework of the multisurface rate-independent plasticity, a plane-stress orthotropic continuum model is presented. A general formulation of the Rankine failure criteria, which serve as the yield criteria in the model, is provided in Section 2. The conditions at failure are described by invoking both a critical plane approach and the existence of the second order strength tensor. In addition, a simplified form of Rankine failure criteria is formulated in Section 3. This form may be used to calculate masonry ultimate strength in a simple way. In Section 4, as illustrative examples, the simple numerical tests have been done in order to show the basic behaviour of the model after the own numerical implementation in proprietary finite element code. Following the verifiable implementation, the tension softening and compression hardening/softening that may capture the total degradation process are taken into account with different values of the fracture energy along each material axis. The main strength parameters are the tensile (f_{tX0} , f_{tZ0}) and compressive (f_{cX0} , f_{cZ0}) strengths in both directions, while the softening parameters are tension (G_{tX}, G_{tZ}) and the compression (G_{cX}, G_{cZ}) fracture energies and subsequent softening in compression (κ_{cp}). Two additional strength parameters (γ_β) which determine the shape of the yield functions and control the shear stress contribution to the failure are used in order to proper estimate shear-compression and shear-tension regimes of the model.

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