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Effect of Dome Formation in Force Flow

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ABSTRACT: In historical masonry structures, domical forms spanning a space can be a rotational dome or cloister vault dome obtained by the intersection of vaults. These forms exhibit different characteristics in distributing the imposed loads to their supports. It is important to understand force flow and to resolve stress variations for the assessment of the historical masonry dome behaviour.

In this study, the structural behaviour of different dome formations, spherical and cloister vault dome with 4, 6, 8, 16, 32, 64 and 128 slices of the same height and diameter are compared parametrically by means of the structural analysis programs. Maximum values of support reactions, nodal displacements and stresses in these masonry domical forms have been investigated under the same dead load and boundary conditions.

1 DESCRIPTION OF GEOMETRY AND ESSENTIALS OF ANALYSIS

Before any intervention to a historic structure it is important to have information about the present level of structural safety of the building. In defining the eventual state and estimating level of safety, numerical analysis is employed. But historical buildings are usually very complex structures and analytical models appropriate to the type of the structure and the type of the masonry as a composite are not easily implemented. So, good documentation and clear identification of the geometry is necessary. However these are very time consuming. Usually where the funds for the restoration are obtained before such documentation, structural analysis have to be accomplished in a rush that engineer has to idealize the geometry. This study is carried out to see how far dome geometry could be idealized.

In this study structural behaviour of several cloister vaulted dome forms under their own weight was parametrically investigated in terms of support reactions, displacements and maximum stress variations.

Cloister vaults formed by the intersection of 2, 3, 4, 8, 16, 32 and 64 barrel vaults have circular profile with 8 m radius. So the dome base is variable from square to circle in plan. The masonry material was assumed to have Young modulus E of 2 GPa and Poisson ratio ν equal to 0.2. The models were subjected to the vertical loads equal to total 10 kN/m^2 deriving from their own weight with 0.3 m thick masonry dome and the roof loads.

Static analysis of domical forms was carried out on a fairly fine 3D model based on linear thin shell theory using the SAP 2000. The 3D model (Fig. 1) was discretized with 4-node 6144 shell elements through automatic mesh generation and supported by 128 nodes. Two types of boundary conditions in the forms were considered: simply supported (pinned) and fixed supported.

2 COMPARISON OF DIFFERENT TYPES OF DOME GEOMETRIES

2.1 The support reactions and displacements in domical forms

In this parametric study, the first part of the investigation is to see the maximum support reactions in the domical forms. The boundary conditions of the forms were chosen as pinned and fixed that the real support conditions are between them. The support reactions in the edges of domical forms on the different plans are displayed in Fig. 2. The maximum values of vertical and horizontal support reactions appear near the edges. While the difference between near-edge and far-edge supports is great in 4 and 8 sliced domical forms, the difference considerably decreases for 16 and more sliced forms. For vertical support reactions the most critical domical form is the one having hexagonal base with 6 slices. For horizontal support reactions the square based form with 4 slices has the highest ones. While greater vertical support reactions occur at simply supported case, fixed supported case has greater horizontal support reactions.

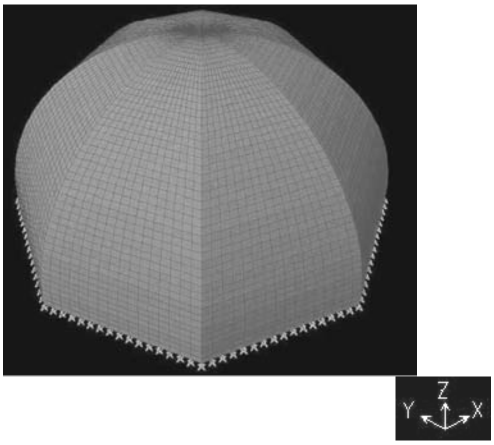


Figure 1 : Cloister vault dome with 8 slices and global axes.

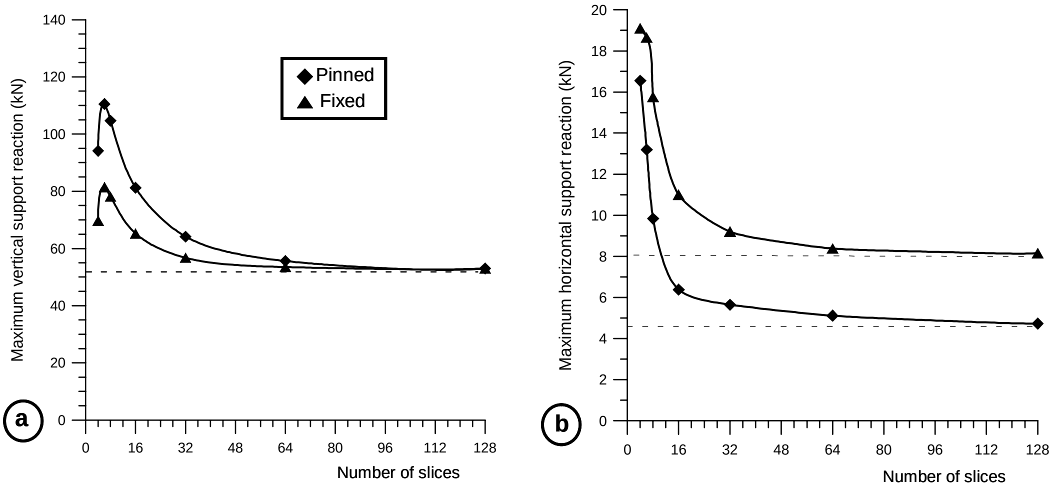


Figure 2 : Variations of (a) maximum vertical reactions (b) maximum lateral horizontal reactions according to number of slices and boundary conditions.

Second parameter investigated is the maximum vertical and lateral displacements of domical forms under vertical loads of 10 kN/m^2. These parameters are displayed in Figures 3, 4 and in Tables 1, 2 according to the boundary conditions. Maximum vertical displacement can be assumed constant 3 mm in pinned supported forms, as for fixed supported forms the maximum displacement slightly increases with increase of the slice number and constant following the 16 sliced form. The location of maximum vertical displacement is at a $0.35l$-$0.45l$ ($\alpha \cong 65^{\circ}$) horizontal distance from centre for quadrilateral and hexagonal forms, in the centre for the other forms.

In pinned and fixed supported quadrilateral forms under vertical loads, the lateral displacement along the radial line reaches to the maximum value of 4.2 mm (α=23^{o}) and 2.6 mm (α=28^{o}), respectively. Following octagonal form the angle can be assumed as α=13^{o} for pinned supported domes and α=17^{o} for fixed ones.

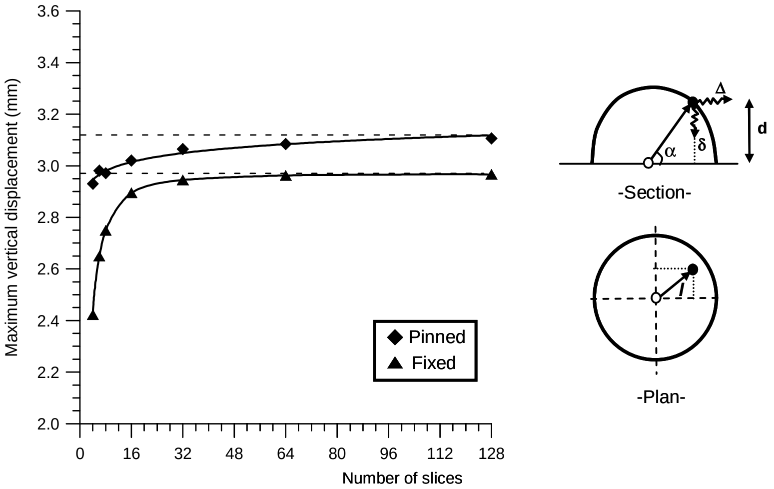


Figure 3 : Values and locations of maximum vertical displacements according to number of slices.

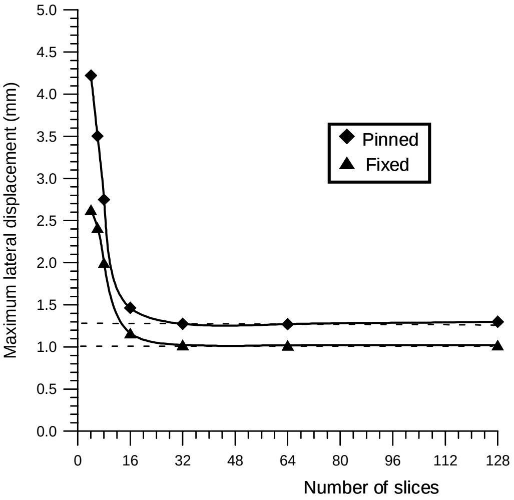


Figure 4 : Values and locations of maximum lateral displacements according to number of slices.

Table 1 : Characteristics of maximum vertical displacements in domical forms.

Forms (no of slices)	Maximum vertical displacement		Vertical distance from centre		Horizontal distance from centre		Angle	
	δ (mm)		d (m)		l (m)		α (^{o})	
	Pinned	Fixed	Pinned	Fixed	Pinned	Fixed	Pinned	Fixed
4	2.93	2.42	6.8	6.9	3.0	2.8	66.3	67.8
6	2.98	2.65	6.9	7.1	3.5	3.3	63.4	65.2
8	2.97	2.75	8.0	8.0	0.0	0.0	90	90
16	3.02	2.89	8.0	8.0	0.0	0.0	90	90
32	3.06	2.94	8.0	8.0	0.0	0.0	90	90
64	3.08	2.96	8.0	8.0	0.0	0.0	90	90
128	3.11	2.97	8.0	8.0	0.0	0.0	90	90

Table 2 : Characteristics of maximum lateral displacements in domical forms.

Forms (no of slices)	Maximum lateral displacement		Vertical distance from centre		Horizontal distance from centre		Angle	
	δ (mm)		d (m)		l (m)		α (°)	
	Pinned	Fixed	Pinned	Fixed	Pinned	Fixed	Pinned	Fixed
4	4.22	2.63	2.32	2.82	5.41	5.29	23.2	28.0
6	3.50	2.42	2.32	2.57	6.63	6.56	19.3	21.4
8	2.75	2.00	2.07	2.57	7.13	6.99	16.2	20.2
16	1.46	1.16	1.81	2.32	7.63	7.51	13.4	17.2
32	1.27	1.02	1.81	2.32	7.75	7.62	13.2	17.0
64	1.27	1.02	1.81	2.32	7.77	7.65	13.1	16.9
128	1.30	1.02	1.82	2.32	7.79	7.66	13.1	16.9

2.2 *Maximum normal stresses in domical forms*

Normal stresses due to bending σ_{11} (S_{11}) varying linearly in the thickness direction were investigated in the top and bottom surfaces. The maximum values of compressive and tensile stresses in both surfaces of shell elements were displayed in Table 3 and Fig. 5. At the top of the surface while the compressive stresses are dominant, in the bottom surface great tensile stresses become dominant. According to derived results maximum normal stresses occurs in the pinned support conditions of the domical forms that 2, 3 and 4 vaults intersect. For this boundary condition normal stresses rapidly diminish with increasing vault intersections and converge to a nearly constant value. As for the pinned boundary conditions this reduction occurs more gradually in comparison to the fixed condition.

Table 3 : Maximum S_{11} stresses in top and bottom surfaces.

Form (no of slices)	Top surface		Bottom surface	
	Compressive stresses (MPa)	Compressive stresses (MPa)	Tensile stresses (MPa)	Tensile stresses (MPa)
	Pinned	Fixed	Pinned	Fixed
4	-0.596	-0.385	0.838	0.499
6	-0.745	-0.512	1.012	0.692
8	-0.586	-0.440	0.966	0.700
16	-0.249	-0.249	0.598	0.460
32	-0.254	-0.254	0.335	0.259
64	-0.256	-0.256	0.251	0.195
128	-0.255	-0.255	0.230	0.174

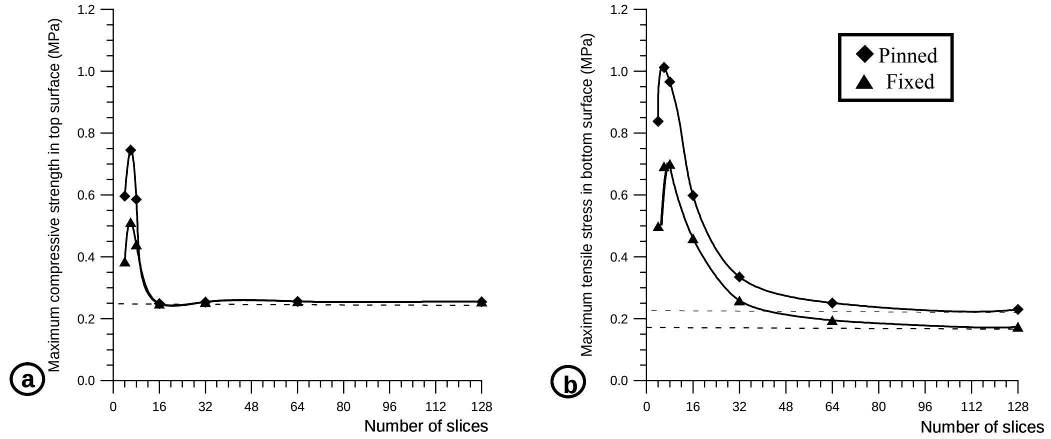
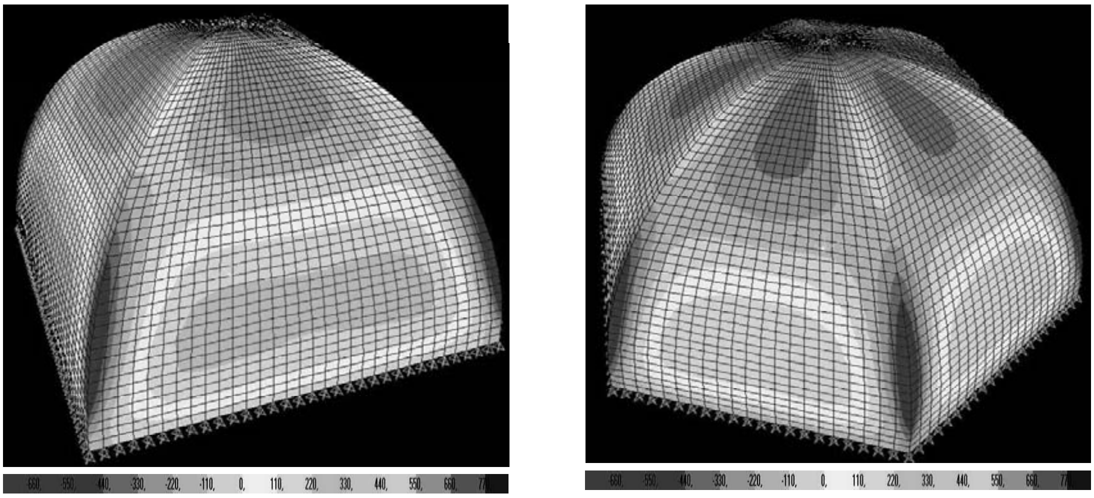
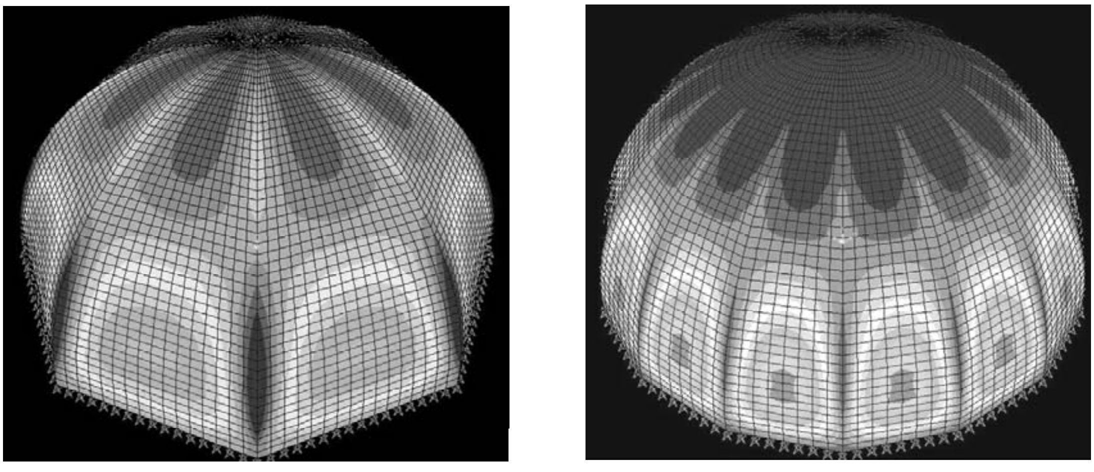


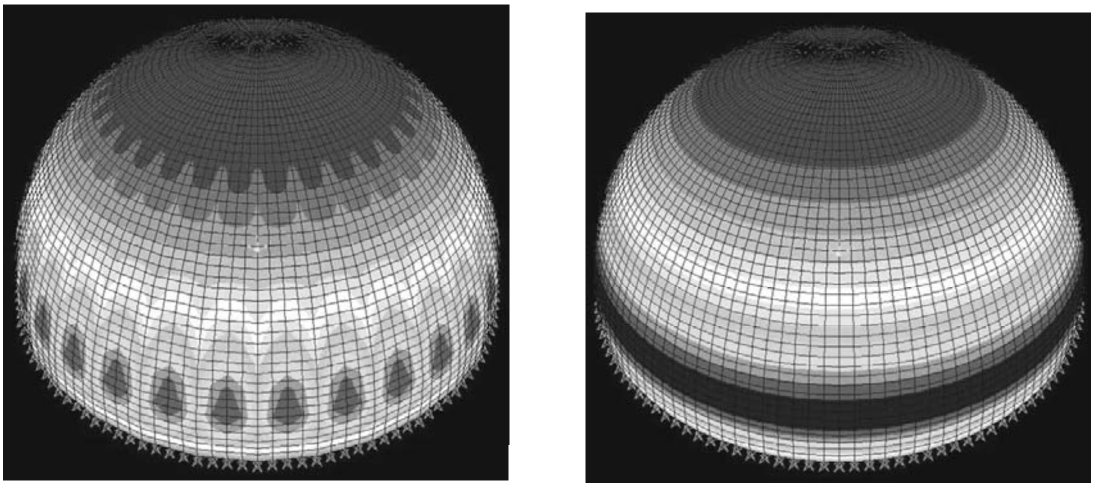
Figure 5 : Maximum compressive and tensile stresses in (a) top (b) bottom surfaces of shell elements.



4 sliced (10^{-3} MPa) 6 sliced



8 sliced (*10^{-3} MPa) 16 sliced (*10^{-3} MPa)



32 sliced (10^{-3} MPa) 128 sliced (10^{-3} MPa)

Figure 6 : S_{11} stress contours on the top surfaces of domical forms.

2.3 Normal stress contours in domical forms

For simply supported boundary conditions the normal stress (S_{11}) contours on the top surfaces of shell elements are shown in Fig. 6. The sections close to the supports along intersecting curves of the vaults are critical for tensile stresses both at the top and at the bottom. For forms having more slices than 8, the tensile stresses rapidly decrease for the forms get close to a sphere. In fixed and simply supported domical forms the angle to the vertical axis that normal stresses become zero is at 52^o.

3 CONCLUSIONS

The following conclusions can be drawn from this study:

- Under dead loads due to the own weight of masonry dome and roof loads most critical support reactions appear near the edges. Difference between near-edge and far-edge supports is great for 4 sliced and 6 sliced cloister vaults. The forms having more than 8 slices, the difference considerably decreases.
- In the forms having more than 8 slices, lateral displacements rapidly decrease and become nearly constant.
- The sections close to the supports along intersecting curves of the vaults are critical for tensile stresses both at the top and at the bottom. In the shell elements the maximum tensile stresses on the bottom surface can be absolutely greater than the maximum compressive stresses on the top surface. Maximum compressive and tensile stresses on the top and bottom surfaces appeared in 6 sliced forms. For forms having more slices than 8, the tensile stresses rapidly decrease for the forms get close to a sphere.

In the numerical analysis the surfaces having more than 16 slices can be idealized as a sphere.

REFERENCES

Wilson, E.L. and Habibullah, A. SAP 2000 Three Dimensional Static and Dynamic Finite Element Analysis and Design of Structures, Version 9, Computer and Structures, Inc., Berkeley, CA, 421 pp.