Limit Analysis of Multiple Span Masonry Portal Frames

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ABSTRACT: In this paper, the behaviour of multi-bay masonry portals under horizontal loads is studied. The structural scheme derives from the natural extension of the single portal frame, already analyzed by the authors in previous studies, and this longitudinal development represents a structural configuration commonly found in monumental masonry buildings. Under the classical hypothesis of limit analysis as applied to masonry structures, possible collapse mechanisms are detected and relevant load multipliers derived through equilibrium conditions written using the principle of virtual works for holonomic systems. Simple expressions for the collapse load are then proposed and discussed. The adopted approach is also compared with some non linear FEM analyses on study cases drawn from real structures.

1 INTRODUCTION

In masonry buildings the use of repetitive elements, characterized by the recurring geometrical proportions, is fairly common. In particular, multiple span portals can be found both in ordinary buildings, whose façades are often regardable as an assembly of such elements, and monumental buildings like churches, where internal longitudinal macroelements show similar partitions. Thus, in a “macro-elemental” approach, now widely used in research works (Abbruzzese et al. 1990, Ciampoli et al. 2001) it is of interest to evaluate the horizontal capacity of such structural schemes. In this paper the collapse load multiplier is derived using the well known limit analysis approach, somehow extending the results already obtained by the authors regarding the single span portal.

2 THE SINGLE PORTAL FRAME

Previous studies carried out by the authors on the single portal frame (De Luca et al., 2005) have led to the selection of four collapse mechanisms (Fig. 1) on account of mechanical and engineering considerations.

Frame mechanism  Mixed mechanism I  Mixed mechanism II  Storey Mechanism

Figure 1: Masonry portal – collapse mechanisms.
Exact expressions of the multiplier for each of such mechanisms, with the ratios B/D, H/D and t/H (Fig. 2) regarded as governing parameters, were derived. Subsequently, a simplified expression with a clear physical meaning was proposed (1). It splits up the multiplier in three factors of immediate mechanical significance: the first term represents the pier overturning load; the second one the stabilizing effect of the beam weight and the third one the opening percentage effect.

\[
\frac{F}{W_{\text{tot}}} = \frac{B}{2 \cdot h} \left( 1 + \frac{W_{\text{beam}}}{W_{\text{tot}}} \right) \left( 0.50 + \frac{B}{D} \right)
\]

where: \( F/W_{\text{tot}} = \) collapse multiplier; \( B = \) pier width; \( h = \) pier height; \( W_{\text{beam}} = \) weight of the beam; \( W_{\text{tot}} = \) total weight; \( D = \) Length of two piers with the central opening.

The formula, tested over more than 80 cases, provided results with generally less than 6% scatters.

3 THE MULTIPLE SPAN PORTAL FRAME

The extension of the single bay frame to the multi-bay frame geometry is shown in Fig. 2. The main fundamental ratios – B/D, H/D and t/H – used for the simple portal will be still regarded as parameters, while the number of bays \( n \) is introduced.

Figure 2 : Extention of the single to the multiple portal frame.

Under the same hypotheses about the hinge formation, i.e. hinges due to cracking only at intersections pier-spandrel, it is easy to demonstrate, and for brevity reasons we omit it, that the number of hinges to be formed can be expressed as the summation of the parts forming the mechanism and the number of bays. The chosen mechanism shapes for the single portal can be extended to the multiple span portal, at the cost of some necessary variations that are shown in the following.

3.1 Expressions for kinematic multipliers

The frame mechanism can be naturally extended to a multi-bay portal. Geometry and load condition of a two-bays portal is shown in Fig. 3a.

Horizontal forces can be applied as a resultant to one side of the structure or distributed over the piers. While this consideration does not make any considerable difference for single portal, the derived expressions provide different collapse values due to non equal displacements of the load application points in the kinematic chains (Fig. 3b).

In order to derive the expression of the collapse multiplier, from the kinematic chain simple geometrical considerations provide:

\[
\varphi_1 \cdot d_1 = \varphi_3 \cdot d_3; \quad \varphi_2 \cdot B = \varphi_4 \cdot L; \quad \varphi_3 \cdot d_3^* = \varphi_5 \cdot d_5; \quad \varphi_5 \cdot B = \varphi_4 \cdot L; \tag{2}
\]

\[
\frac{H}{\varphi} = \frac{HL}{t} = d_1 = d_3^*; \quad \Rightarrow \quad d_3 = \frac{HL}{t} - B - L = \frac{HL - Bt - Lt}{t} = d_5 \tag{3}
\]

substituting the expressions of \( d_1 \) and \( d_3 \) in (2) and fixing the quantity \( \psi \).
\( \varphi_t \frac{HL}{L} = \varphi_t \frac{HL - Bt - Lt}{L} ; \psi = \frac{HL}{HL - Bt - Lt} \)  \hspace{1cm} (4)

For force applied at left end:

\[ \varphi_1 \text{ lagrangian parameter}; \varphi_2 = \frac{B}{L} \psi \varphi_1 ; \varphi_3 = \psi \varphi_1 ; \varphi_4 = \frac{B}{L} \psi^2 \varphi_1 ; \varphi_5 = \psi^2 \varphi_1 \]  \hspace{1cm} (5)

The equilibrium condition through the principle of virtual works gives:

\[ F \varphi_1 H = W_1 \varphi_1 \frac{B}{2} + W_2 \varphi_2 \frac{L}{2} + W_3 \varphi_3 \frac{B}{2} + W_4 \varphi_4 \frac{L}{2} + W_5 \varphi_5 \frac{B}{2} \]  \hspace{1cm} (6)

And then, with some elementary algebra:

\[ \frac{F}{W_{tot}} = \frac{B}{2H} \left( \frac{W_1 + W_5 \psi + W_5 \psi^2}{W_{tot}} + \frac{W_2 \psi + W_4 \psi^2}{W_{tot}} \right) \]  \hspace{1cm} (7)

Generalizing for \( n \) bays:

\[ \frac{F}{W_{tot}} = \frac{B}{2H} \left( \frac{\sum_{i=0}^{n} W_i \psi^i}{W_{tot}} + \frac{\sum_{j=1}^{n} W_j \psi^j}{W_{tot}} \right) \]  \hspace{1cm} (8)

where \( W_p = \text{weight of the single pier}, \ W_b = \text{weight of the single beam} \) and \( n = \text{number of bays} \).

In case of force applied at the right end, identical procedure as above gives for \( n \) bays:

\[ \frac{F}{W_{tot}} = \frac{B}{2H} \left( \frac{1}{W_{tot}} \sum_{i=0}^{n} \frac{1}{\psi^i} + \frac{1}{W_{tot}} \sum_{j=1}^{n} \frac{1}{\psi^j} \right) \]  \hspace{1cm} (9)

Figure 3: Frame mechanism – (a) geometry and loads; (b) kinematic chain.
When the force is distributed along the piers, the expression of the collapse multiplier for a two bays portal is:

$$\frac{F}{W_{tot}} = \frac{B}{2H} \frac{3}{1 + \psi + \psi^2} \left( \frac{W_p + W_b \psi + W_b \psi^2}{W_{tot}} + \frac{W_p \psi + W_b \psi^2}{W_{tot}} \right)$$  \hspace{1cm} (10)

and its generalization for n bays is:

$$\frac{F}{W_{tot}} = \frac{B}{2H} \frac{n+1}{\sum_{r=0}^{n} \psi^r} \left( \frac{W_p \sum_{i=0}^{n} \psi^i}{W_{tot}} + \frac{W_b \sum_{i=1}^{n} \psi^i}{W_{tot}} \right)$$  \hspace{1cm} (11)

with the already stated meanings of $W_p$, $W_b$ and $n$.

### 3.2 Expressions for kinematic multipliers: mixed mechanisms

The mixed mechanisms chosen for the single portal feature the formation of hinges at different height of the piers (Fig. 1 – mixed mechanisms I and II). In case of multiple bays, some issues arise about the mechanism formation. As shown in Fig. 4a and 4b, if the beam is considered as a unitary element, which is also unlikely for large spans, no contact point between the beam and the last pier exist. Considering the formation of two hinges at the transversal element, different displacements at the middle pier will occur so that neither this possibility is allowed. For these reasons, given the hypotheses made, mixed mechanism I cannot be extended to multiple bays portals. On account of similar considerations, the only possible mixed mechanism is shown in Fig. 5.

![Figure 4: Mechanism II and III: impossible mechanisms.](image)

![Figure 5: Mixed mechanism – (a) geometry and loads; (b) kinematic chain.](image)

From the kinematic chain $\phi_1=\phi_2=\phi_3=\phi$ and the principle of virtual works gives:

$$F(H-t) = W_1 \frac{B}{2} + W_2 B + W_3 \frac{B}{2} + W_4 \frac{B}{2}$$  \hspace{1cm} (12)

And then, with some elementary algebra:
Generalizing for \( n \) bays, the expression for the collapse multiplier is:

\[
\frac{F}{W_{\text{tot}}} = \frac{B}{2h} \left( \frac{(n+1)W_{\rho} + 2W_{b} - W_{c}}{W_{\text{tot}}} \right)
\]  

where \( W_{c} \) = weight of the cross node.

### 3.3 Expressions for kinematic multipliers: Mechanism III (Storey mechanism)

The third mechanism ("storey type") taken into account considers all the piers rotating of the same quantity and the entire beam translating only (Fig. 6).

For such mechanism, the expression of the collapse multiplier is derived with the same algebraic procedure as above, here omitted for synthesis reasons:

\[
\frac{F}{W_{\text{tot}}} = \frac{B}{2h} \left( \sum_{i=1}^{n+1} \frac{W_{\rho}}{W_{\text{tot}}} + 2 \frac{W_{b}}{W_{\text{tot}}} \right) = \frac{B}{2h} \left( 1 + \frac{W_{b}}{W_{\text{tot}}} \right)
\]  

This expression is formally the same as the one derived for the single portal frame.

### 3.4 Some conclusive remarks on collapse multiplier expressions

The expressions of the collapse multiplier for the portal frame have here been generalised to any number of bays, selecting the appropriate shapes of the failure mechanisms. The outcoming formulas appear very simple and straightforward.

It can be also noticed that, some external structural conditions may trigger the activation of a specific mechanism over the others. For example, the presence of upper masonry walls might introduce a displacement compatibility issue, so that the storey mechanisms should be more likely to take place.

As far as the load condition is regarded, quite obviously, the distributed force configuration is more representative of a seismic action, whilst the point load at one edge may represent the thrust of some other connected structures.

### 4 COMPARATIVE ANALYSES

It is immediate to recognise from the proposed expressions that the storey mechanism always provides larger values of the collapse multiplier, for the presence of the intersection element as part of the beam instead of the pier, though this difference decreases with the number of bays. Thus, in this paragraph a few comparative calculations are performed with respect to the frame and mixed mechanisms only. Such analyses have been carried varying the geometrical ratios
t/H, B/D, H/D and number of bays $n$. Fig. 7a and 7b show the collapse multipliers for portals with 1 to 10 bays, respectively with tip load and distributed forces. B/D and H/D ratios have been fixed, while ranging values from 0.1 to 0.5 of t/h have been chosen.

The expression for the mixed mechanism provides the results shown in Fig. 8, for B/D=0.1 and 0.4, with different values of t/H and fixed H/D=0.5.

In Fig. 9 the comparison of the frame and storey mechanisms is reported. Under the hypotheses made, the frame mechanism always provides smaller values of the multiplier, so that it is considered as the most likely to occur for this structural configuration, unless displacement compatibility issues arise.

5 APPLICATION AND COMPARISON WITH NON LINEAR FEA

The presence of alternating openings/walls repetitions in church buildings has suggested the possibility of applying the expressions here provided also to macroelemental structural parts extracted directly from buildings of known geometry and properties, by superimposing the multiple span portal with appropriate geometrical ratios to the actual scheme. For brevity sake, only...
two examples are here reported. Namely, a macroelement from the S. Giovanni a Mare church and one from the S. Paolo Maggiore churches have been selected. Two main issues arise about the way of proceeding. Firstly, the formation of a mechanism may be triggered by some particular boundary or loading conditions that cannot be easily accounted for in an analytical approach. For example, though the frame mechanism always provides the lowest values of the load multiplier, it may not be able to develop for the presence of platebands, reinforcing ties, or in case of very rigid upper parts, which clearly introduce a displacement compatibility constraint. In such cases the storey mechanism appears to be the most likely and the relevant values should be used. Secondly, when different dimensions of the walls width are present, an assumption must be made on the value to be used. In the following we use the maximum value along with the minimum and the medium ones to check the inherent variability.

We use also on linear FE analyses using smeared cracking approach as a term of comparison. The parameters values suggested in (Giordano et al. 2002) have been used for the analyses. In particular, as far as the uniaxial strength of the material is concerned, failure stress values of 3 MPa and 0.15 MPa have been assumed respectively for compression and tension.

The deformed shapes shown in the previous figures suggest the activation of a storey mechanism. As a matter of fact, the force-displacement curves are in good agreement with the storey mechanism collapse values, here represented by horizontal straight lines.

### 6 CONCLUSIONS

In this paper the behaviour of the multi-bay masonry portal frame is studied. Starting from a previous study on the single portal frame, the extension to repetitive elements is sought in detail. Under the hypothesis of limit analysis, equilibrium conditions of the structures are analyzed and
some mechanisms are verified. Expressions of the collapse multiplier are provided for each mechanism and the results discussed. Within the limits of the approach, the frame mechanism has turned out to be the most likely to occur for the lower values of the collapse multiplier. In real cases, though, it has to be considered that other mechanisms may be triggered by particular configurations or load conditions. In particular, the storey mechanism appears more realistic in case of rigid upper parts insisting on the multiple span portal. Applications to cases extracted from real structure are presented, and the results compared against non linear FEA, showing very close values of the obtained collapse multiplier.

In the opinion of the authors, the results here obtained can be useful from a seismic strength evaluation point of view, since they provide an estimate of the maximum horizontal load carrying capacity of very common structural elements.

ACKNOWLEDGMENTS

This research has been partially supported by RELUIS “Rete Laboratori Universitari Ingegneria Sismica” and has been developed in the context of the activities of “Linea 7”.

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