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Disgregative Phenomenon of Antique Mortars

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ABSTRACT: The problem of the conservation of the mortars, particularly of those present in the antique buildings positioned in the historical centres of big tourist and cultural attraction, is strongly connected to the daily overlapping, to the intrinsic problems due to the degradation, of phenomenon's of durable and uniform vibrations, like, ad example, those connected to the urban traffic, to the use of perforating and demolishing machines, to the underground mining and finally to the quantities of energy that are given out from subsoil by means of the so called micro seisms that accompany the bradyseism.

1 DISGREGATIVE PHENOMENON OF ANTIQUE MORTARS

1.1 Mogelling

The work takes his steps from the following conservative requirement: to define a mechanic micro model for making a dynamic analysis congruent with the representation mortar as granulated highly cohesive agglomerate.

The above said problem appears very topical, with reference to the old buildings of historical centres and particularly to the monumental ones, especially where it is possible that dangerous situations can overlap to previous deficiency due to neglect and dereliction if not to impressive episodes of damage or disruption.

The present work refers to an elementary physic model, whose mechanic behaviour can be related with that of highly cohesive granulated agglomerate, and so with that of a mortar which, in such a way, can be assimilated to it.

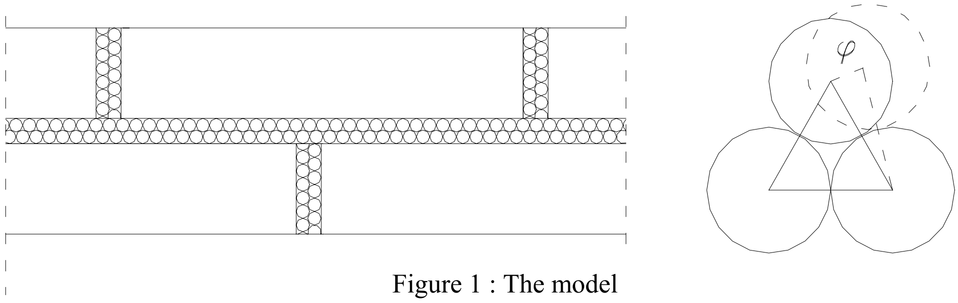

Figure 1 : The model

Assuming that the material is composed by element of not too variable dimension, in such a way as to be able to consider them like an aggregate of spheres with the same radius r and the same specific weight γ, it was analysed the elementary mechanic system composed of three smooth and infinitely rigid spheres disposed in the position of minimal density i. e. the centres of the spheres are placed at the vertexes of an equilateral triangle (Fig. 1).

The upper sphere is subjected to a vertical load N, to a horizontal load, to the own weight, to a force exerted by an elastic tie rod which simulates, with good adherence to the reality, the cohesive bond between the three spheres, and finally to a dynamic impulse applied by means of a displacement having sinusoidal law. Referring to the above mentioned loads it have been obtained the law of the motion using as characterizing parameter of the mechanism the angle φ (Fig. 1).

1.2 Dynamic analysis

The dynamic analysis is made assuming uniform agglomerate. The rigid spheres are connected by means of elastic tie rod which simulates, with good adherence to the reality, the cohesive bond between the three spheres. The referring g model is represented in Fig. 2.

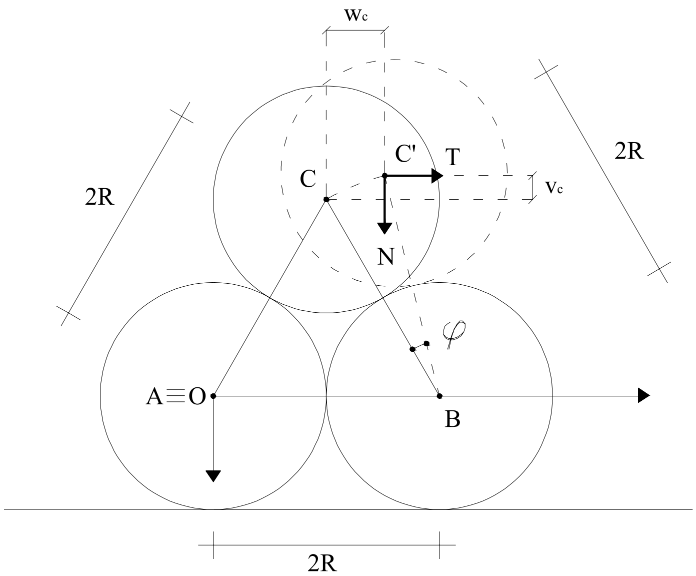

Figure 2 : The trispheric model

it is analyzed applying a seismic impulse characterized by a displacement of the upper sphere having sinusoidal law as follows:

$$\Delta w_p = \Delta\eta \sin\omega_p t \tag{1}$$

With small displacement hypothesis it is possible to write the components of the displacement v_c and w_c of the centre C, and the lengthening

$$\Delta l = AC - AC'. \tag{2}$$

in fact:

$$\begin{aligned} v_c &= -2R\sin\left(\frac{\pi}{3}+\varphi\right)+2R\sin\frac{\pi}{3} = -2R\sin\frac{\pi}{3}\cos\varphi-\cos\frac{\pi}{3}\sin\varphi+ \\ &+2R\sin\frac{\pi}{3} = -2R\frac{\sqrt{3}}{2}\cos\varphi-\frac{1}{2}\sin\varphi+ \\ &+2R\frac{\sqrt{3}}{2} = R\left[\sqrt{3}\left(1-\cos\varphi\right)-\sin\varphi\right] \end{aligned} \tag{3}$$

$$\begin{aligned} w_c &= 2R\cos\frac{\pi}{3}-2R\cos\left(\frac{\pi}{3}+\varphi\right) = \\ &= 2\frac{R}{2}-2R\left[\cos\frac{\pi}{3}\cos\varphi-\sin\frac{\pi}{3}\sin\varphi\right] = \\ &= R-2R\left(\frac{\cos\varphi}{2}-\frac{\sqrt{3}}{2}\sin\varphi\right) = R\left(1-\cos\varphi+\sqrt{3}\sin\varphi\right) \end{aligned} \tag{4}$$

with small displacement hypothesis it is possible to write:

$$\begin{aligned} \cos\varphi &\cong 1-\frac{\varphi^2}{2} \\ \sin\varphi &\cong \varphi \end{aligned} \tag{5}$$

and from (3), (4) we obtain

$$\begin{aligned} v_c &= R\left[\sqrt{3}\left(1-\cos\varphi\right)-\sin\varphi\right] = \left[-\varphi+0.866\varphi^2\right] \\ w_c &= R\left(1-\cos\varphi+\sqrt{3}\sin\varphi\right) = \left[1.732\varphi+0.5\varphi^2\right] \end{aligned} \tag{6}$$

Furthermore, considering that the upper sphere is subjected to a seismic impulse characterized by a given displacement $\Delta w_p = \Delta\eta\sin\omega_p t$, so (6) become:

$$\begin{aligned} v_c &= R\left[\sqrt{3}\left(1-\cos\varphi\right)-\sin\varphi\right] = \left[-\varphi+\frac{\sqrt{3}}{2}\varphi^2\right] \\ w_c &= R\left(1-\cos\varphi+\sqrt{3}\sin\varphi\right) = \left[\sqrt{3}\varphi+\frac{\varphi^2}{2}\right]+\Delta\eta\sin\omega_p t \end{aligned} \tag{7}$$

ignoring the second order terms from (7) we obtain:

$$\begin{aligned} \dot{v}_c &= R\left[-\dot{\varphi}+2\frac{\sqrt{3}}{2}\varphi\dot{\varphi}\right] = R\left[-\dot{\varphi}+\sqrt{3}\varphi\dot{\varphi}\right] \cong -R\dot{\varphi} \\ \dot{w}_c &= R\left(\sqrt{3}\dot{\varphi}+\varphi\dot{\varphi}\right)+\Delta\eta\sin\omega_p t \cong \sqrt{3}R\dot{\varphi}+\Delta\eta\sin\omega_p t \end{aligned} \tag{8}$$

and the lengthening can be written as (Fig. 3)

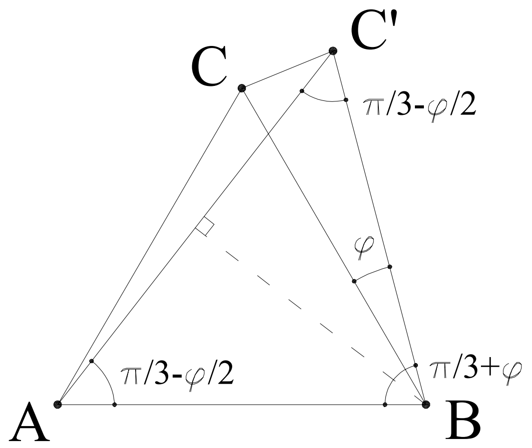

Figure 3 : △l calculated like AC'-AC

$$\begin{aligned}\Delta l &= 2\cdot 2R\cos\left(\frac{\pi}{3}-\frac{\varphi}{2}\right)-2R=\\ &=2R\left(2\cos\frac{\pi}{3}\cos\frac{\varphi}{2}+2\sin\frac{\pi}{3}\sin\frac{\varphi}{2}-1\right)=\\ &=2R\left(2\cdot\frac{1}{2}\cos\frac{\varphi}{2}+2\cdot\frac{\sqrt{3}}{2}\sin\frac{\varphi}{2}-1\right)=\\ &=2R\left(\cos\frac{\varphi}{2}+\sqrt{3}\sin\frac{\varphi}{2}-1\right)=\\ &=2R\left(1-\frac{1}{2}\left(\frac{\varphi}{2}\right)^2+\sqrt{3}\frac{\varphi}{2}-1\right)=2R\left(\frac{\sqrt{3}}{2}\varphi-\frac{1}{8}\varphi^2\right)=\\ &=R\left(\sqrt{3}\varphi-\frac{1}{4}\varphi^2\right)\cong\sqrt{3}R\varphi\end{aligned} \quad . \tag{9}$$

The lagrangian function is

$$\mathscr{L}=T-E_t \tag{10}$$

where E_t is the potential total energy and T is the Kinetic energy being:

$$\begin{aligned}T&=\frac{1}{2}mv^2=\frac{1}{2}m\dot{v}_c+\frac{1}{2}m\dot{w}_c=\\ &=\frac{m}{2}\left(4R^2\dot{\varphi}^2+\Delta\eta^2\omega_p{}^2\cos^2\omega_p t+2\sqrt{3}R\Delta\eta\omega_p\dot{\varphi}\cos\omega_p t\right)\end{aligned} \tag{11}$$

and

$$E_t = L - P \quad . \tag{12}$$

Potential energy can be written as:

$$\mathrm{P} = -\mathrm{N}\cdot \mathrm{v}_\mathrm{c} - \mathrm{T.w}_\mathrm{c} \tag{13}$$

while deformation energy is

$$\mathrm{L} = \frac{1}{2}\mathrm{k}\Delta \mathrm{l}^2 + \mathrm{C}_0\Delta \mathrm{l} \tag{14}$$

where k is the elasticity of the tie rod that represent the cohesion. From (12), (13), (14), considering the (7), (9) we have:

$$\begin{aligned} E_t &= -Nv_c - Tw_c + \frac{1}{2}k\Delta l^2 + C_o\Delta l = \\ &= -NR\left(-\varphi + \frac{\sqrt{3}}{2}\varphi^2\right) - TR\left(\sqrt{3}\varphi + \frac{\varphi^2}{2}\right) + \\ &+ T\Delta\eta \sin\omega_p t + \frac{3}{2}kR^2\varphi^2 + \sqrt{3}C_0 R\varphi \end{aligned} \tag{15}$$

Known E_t and T it is possible to write Hamilton equation:

$$\begin{aligned} &4R^2 m\ddot{\varphi} + \left(3kR^2 + \sqrt{3}C_0 R - \sqrt{3}NR - TR\right)\varphi = \\ &= \sqrt{3}mR\Delta\eta\omega_p^2 \sin\omega_p t + T\Delta\eta\omega_p \cos\omega_p t - NR + \sqrt{3}TR \end{aligned} \quad . \tag{18}$$

Imposing

$$\begin{aligned} m &= \frac{\gamma}{g}\frac{4}{3}\pi R^3 \\ N &= 4R^2\sigma \\ T &= 4R^2\tau \end{aligned} \tag{19}$$

from the (15) it is possible to obtains

$$\ddot{\varphi}^2 + \omega^2\varphi = a\sin\omega_p t + b\cos\omega_p t + d \tag{20}$$

being

$$\omega^2 = \frac{g}{\gamma R^2}\left(\frac{9}{16}\frac{k}{\pi R} + \frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2} - \frac{3\sqrt{3}}{4\pi}\sigma - \frac{3}{4\pi}\tau\right) \tag{21}$$

if $\frac{9}{16}\frac{k}{\pi R} + \frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2} - \frac{3\sqrt{3}}{4\pi}\sigma - \frac{3}{4\pi}\tau > 0$, or

$$\omega^2 = -\frac{g}{\gamma R^2}\left(\frac{9}{16}\frac{k}{\pi R} + \frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2} - \frac{3\sqrt{3}}{4\pi}\sigma - \frac{3}{4\pi}\tau\right) \tag{22}$$

if $\frac{9}{16}\frac{k}{\pi R} + \frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2} - \frac{3\sqrt{3}}{4\pi}\sigma - \frac{3}{4\pi}\tau < 0$

and still

$$a=\frac{\sqrt{3}}{4}\frac{\Delta\eta\omega_p^2}{R}$$
$$b=\frac{3}{4\pi}\frac{g}{\gamma R^3}\Delta\eta\omega_p\tau \quad . \tag{23}$$
$$d=-\frac{g}{\gamma R^2}\left(\frac{3}{4\pi}\sigma-\frac{3\sqrt{3}}{4\pi}\tau\right)$$

The general integral of the (17), assuming the initial conditions

$$\varphi_0=\varphi(t=0)=0$$
$$\dot{\varphi}_0=\dot{\varphi}(t=0)=0 \tag{24}$$

is

$$\varphi=-\frac{1}{\omega^2-\omega_p^2}\left(b\left(\cos\omega t-\cos\omega_p t\right)+a\frac{\omega_p}{\omega}\left(\sin\omega t-\frac{\omega}{\omega_p}\sin\omega_p t\right)\right) \tag{25}$$
$$+\frac{d}{\omega^2}\left(\cos\omega t-1\right)$$

if $\frac{9}{16}\frac{k}{\pi R}+\frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2}-\frac{3\sqrt{3}}{4\pi}\sigma-\frac{3}{4\pi}\tau>0$ or

$$\varphi=\frac{1}{\omega_p^2-\omega^2}\left[\left(d+b\right)\cosh\omega t+a\left(\frac{\omega_p}{\omega}\sinh\omega t-\sin\omega_p t\right)\right]+$$
$$+\frac{1}{\omega_p^2-\omega^2}\left[d\left(\frac{\omega_p^2}{\omega^2}\left(\cosh\omega t-1\right)-1\right)-b\cos\omega_p t\right] \tag{25'}$$

if $\frac{9}{16}\frac{k}{\pi R}+\frac{3\sqrt{3}}{16\pi}\frac{C_0}{R^2}-\frac{3\sqrt{3}}{4\pi}\sigma-\frac{3}{4\pi}\tau<0$

1.3 Applications

Known the function φ graph it is possible to obtain its maximum value $\varphi_{\max}$. The limit value of φ, named $\varphi_{\lim}$, i. e. the value of φ that imply the breaking of the agglomerate, is the minimum value between $\frac{\pi}{6}$ (overriding of the upper sphere with respect to the inferior fixed spheres) and the value of φ that imply the breaking of the tie rod. Known $\varphi_{\max}$ and $\varphi_{\lim}$ it is easy to make a mechanic analysis of the agglomerate under seismic load, verifying if φ exceeds its limit value $\varphi_{\lim}$, using an appropriate security factor.

Like example we have considered the case of a muddy agglomerate with elements having diameter of one millimetre under one's own weight and under seismic load. Imposing c = 10 kg/cmq , $\Delta\eta=0{,}5mm$ the function φ depend only on the time and on the depth z. So, fixed z = 1, we have graphed the functions φ and $\dot{\varphi}$ (Figs. 4, 5).

We have furthermore calculated the maximum value of φ, named $\varphi_{\max}$, with respect to the depth z

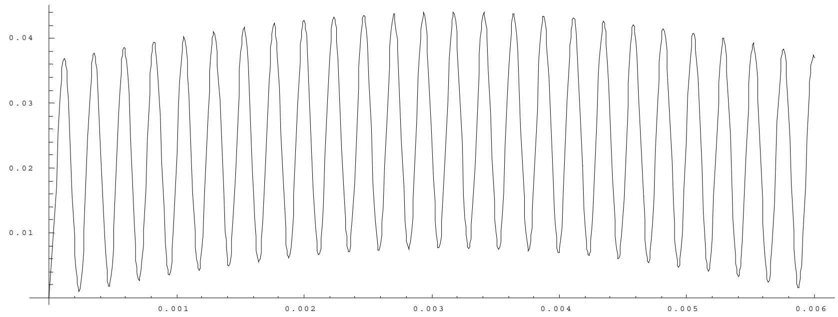

Figure 4 : function φ with respect to the time, fixed the depth z

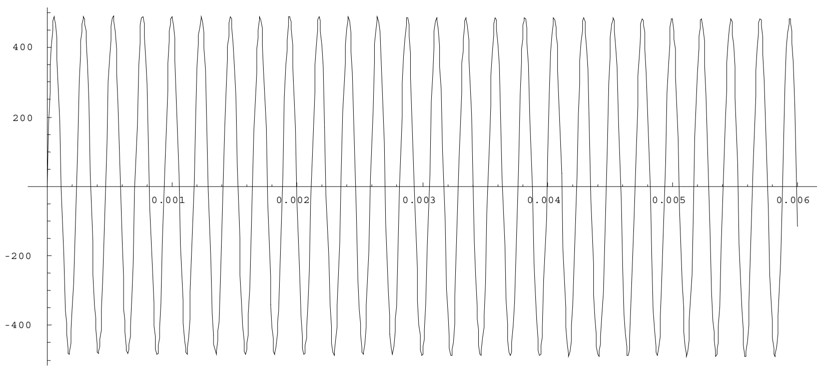

Figure 5 : Function $\dot{\varphi}$ with respect to the time fixed the depth z

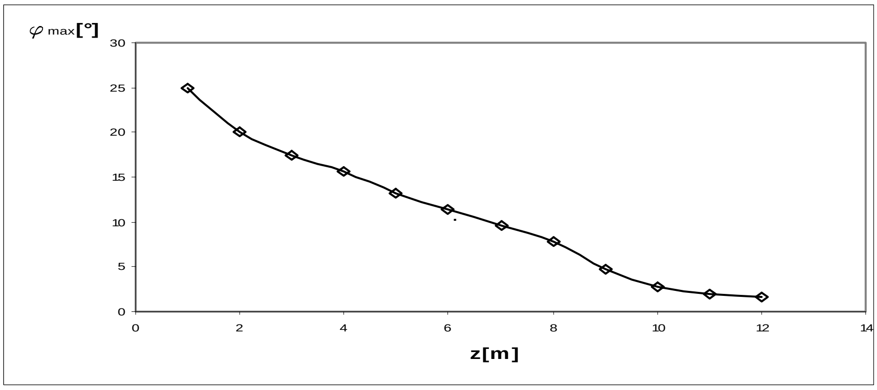

Figura 6: Function $\varphi_{\max}$ with respect to the depth z

We have, furthermore, calculated the maximum value of φ, named $\varphi_{\max}$, with respect to the depth z (Fig. 6). Of course $\varphi_{\max}$ decreases with respect to the depth in fact agglomerate, increasing depth, stiffen by means of one's own weight.

We have also graphed the impressed displacement $\Delta\eta$ with respect to the desgregation frequency, see Fig. 7.

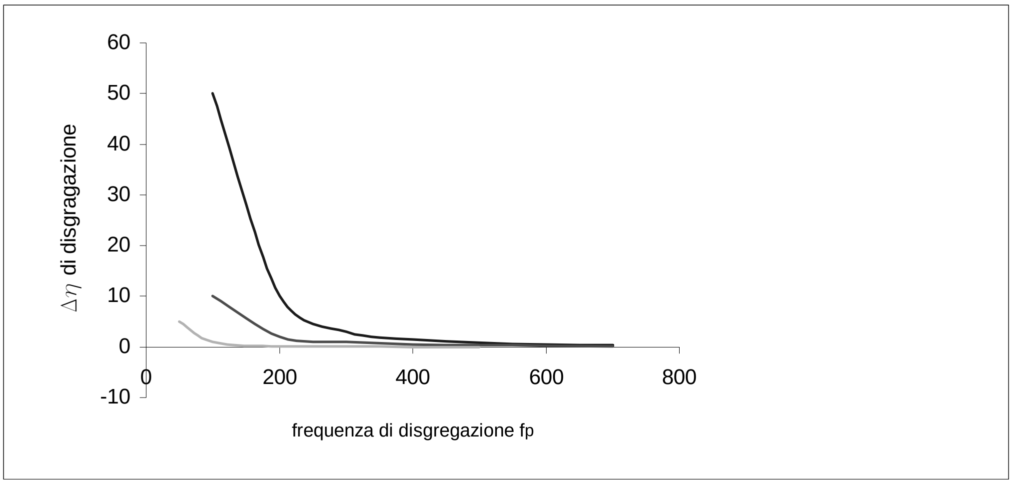

Figure 7 : Impressed displacement vs. desgregation frequency

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