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# Three-Dimensional Finite Element Analysis of Taj Mahal Structure

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ABSTRACT: The paper involves 3-D finite element modeling and analysis of the famous Taj Mahal structure using standard commercial package- SESAM (Super Element Structural Analysis Module). The stresses have been found to be small compared to the compressive strength of masonry giving a factor of safety of the order of 4 to 8. The vertical stresses in the columns are primarily compressive with indications of small zones of insignificant tensile stresses on one of the faces. Further, under vertical loads, displacements of domical portion and floor system are almost uniform, while the substructure deforms in natural bowl shape.

## 1 INTRODUCTION

The Taj Mahal monument is the most important Archaeological survey of India monument in India. It is essentially a double domical structure supported on eight massive columns with arches spanning between these columns. The entire structure rests on 33.5 m thick raft with about 18.3 m thickness of the raft above the ground level. The raft rests on a thin sandy layer of 9.1 m thickness, followed by a 63.4 m thick clayey strata underlain by rock. The entire superstructure and the raft are built in brick masonry consisting of 3.8 cm thick fire burnt clay bricks and mortar joints of varying thickness upto as much as two inches. The geometry of the monument is very complex indeed. The exact dimensions of the monument and the sub-structure are unknown and were obtained from the data supplied by Archaeological survey of India, New Delhi. The analysis was necessitated due to the proposal of creating an artificial reservoir by constructing a weir across the river Yamuna behind the Taj structure.

The Taj Monument has been analysed by Finite Element Method (FEM) (Zienkiewicz 1971) as a three dimensional solid continuum using two types of solid elements namely 20 Noded Hexahedral elements and 15 noded Triangular prisms. The monument has been analysed for vertical loads only. The analysis for wind loads has not been considered necessary, as the monument is extremely stable because of its gigantic dimensions and its location in a zone of moderate wind velocities. The bore-hole observations have indicated a water table level which submerges all the sandy and clayey soil strata beneath the monument. The elastic constants of the masonry used in the construction were obtained using chunks of masonry taken from the ruins of contemporary structures across the river Yamuna and those of the soil, using the undisturbed soil samples extracted during the borings.

## 2 SUPER ELEMENT STRUCTURAL ANALYSIS MODULE (SESAM)

SESAM is a large, general purpose Computer Software based on the finite element analysis. It contains super-element capabilities for the modeling of the structure and uses substructure technique for the analysis of large and complex structures. It has a very powerful interactive preprocessor for modeling and an equally powerful post processor for the interpretation of the output. It performs static, dynamic, linear and non-linear stress analysis of any type of structure. A unique multi-level super-element technique is available for stress analysis. The SESTRA (Super-element Structural Analysis) program module, which performs the stress analysis, is the heart of SESAM. SESTRA is based on the displacement formulation of FEM with elements available for trusses, beams, membranes, shells, axi-symmetric solids, 3 Dimensional solids and transition elements between shells and solids. Direct analysis or the multilevel super-element (i.e. substructure) technique may be applied in static analysis. The code implementation is performed by utilizing Cholesky's method for the solution of linear equations, combined with the sub-matrix approach for data storage and matrix operations.

## 3 DETAILS OF MODELING

A three dimensional finite element analysis of the complete structure including the foundation and the participating soil mass was undertaken for the 'exact' stress analysis. The element type chosen for the structural idealization permits simulation of curved edges/boundaries as well.

### *3.1 Geometric data*

The geometric details of the Taj structure are available in many texts (Begley et al.1989, Nath 1972). In addition, the Archaeo1ogical Survey of India (ASI), New Delhi has also prepared the drawings of the monument on the basis of actual site measurements and have been adopted in this study.

Taj Mahal is built on a square solid platform measuring 98.2 m x 98.2 m in plan and 33.5 m thick, out of which, a thickness of 15.2 m is buried in the ground. This platform on a 9.1 m thick sand layer which is followed by 63.6 m thick clay layer underneath. Natural rock is located at a depth of 88.0 m below the ground level. The main musoleum, octagonal in plan, is symmetrical about two axes and rests centrally on the platform. The roof of the main circular hall is a double domical structure which rests on eight inner columns of irregular shape. The thickness of the outer dome varies from 3.7 m at the springing level to 3.4 m at the crown with a maximum thickness of 4.3 m in between. The inside diameter of the outer dome is 19.5 m. The inner dome is a1most spherical with an average thickness of 1.8 m and the base diameter of 16.8 m.

The elevation of the monument is shown in Fig. 1. The clear storey height of the first and the second storey are 8.5 m and 9.0 m respectively. The average thickness of the floors including the vaults on passages is 1.5 m. The entire structure is supported on 24 huge columns of irregular shape.

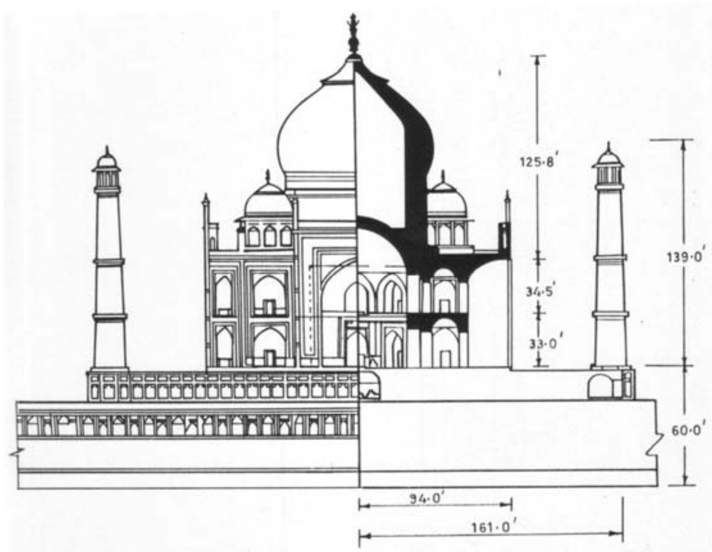


Figure 1 : Part elevation and part sectional elevation of the Taj Mahal (1 ft = 0.3048 m)

### *3.2 Structural idealization*

To enable the geometric modelling of Taj structure using SESAM, entire structure has been divided into five super elements at the first level. These are: dome, floor system and the three super elements in the foundation namely- floor foundation, foundation core and the foundation extension, as described later. The minarets which are relatively much lighter in weight were excluded in idealization. Taking full advantage of the symmetry of the structure and the loads, it has been decided to analyse only a quarter of the structure with appropriate boundary conditions along the planes of symmetry. Further, in view of the built-in facility to generate repetitive geometry using appropriate copy command, only one 15° sector of the dome, $1/8^{th}$ of the floor system and one quarter of the foundation super elements have been modeled in the first instance. The 'PREFEM' module of the software has been used for this purpose.

After the geometry was generated, the super-elements were discretized using 20/15 noded iso-parametric three dimensional elements. The material properties, load case definitions and the boundary conditions were also defined at the first level. The nodes, where one super-element joins the other super-element, were identified and declared as the super nodes. Only such declared super nodes were available for the manipulation at higher levels. The 'PREFRAME' module was used to prescribe the boundary conditions on the first level super-elements. Table 1 gives the discretization details of the five super elements followed by brief description and salient features.

Table 1 : Discretization details for five first level super elements

| Super-element | No. of elements | No. of nodes | No. of super nodes | Load case | Band width optimization using BPOPT | |
|---|---|---|---|---|---|---|
| | | | | | Before | After |
| Dome | 24 | 196 | 58 | 1 | 154 | 54 |
| Floor System | 58 | 488 | 117 | 1 | 444 | 188 |
| Foundation Floor | 126 | 769 | 261 | 1 | 411 | 176 |
| Foundation Core | 27 | 197 | 62 | 1 | 123 | 101 |
| Foundation Extension | 87 | 618 | 150 | 1 | 403 | 97 |

a. Dome:
Using the step by step procedure of defining the points, lines, surfaces and then the bodies, a 15° sector of the dome has been modeled. By copying this sector with appropriate transformations, a 45° sector ($1/8^{th}$ of the complete dome) was generated and discretized using eighteen, 20 noded and six, 15 noded three dimensional elements. The Isometric views of the idealized dome are shown in Fig. 2.
b. Floor System:
The first floor slab has been modeled first and the same has been copied at the second floor level by translating it vertically upwards by 9.0 m. Three columns connect the second floor and the first floor slabs and these columns protrude below from the bottom of the first floor slab. One eighth sector of the complete floor system was modeled.
c. Foundation:
Three super elements have been used to model the quarter portion of the total foundation including the soil medium up to the significant width and depth. The first super element of the foundation defines the portion vertically below the floors right up to the top of the rock
d. Assembly:
The assembly of different super elements was performed in PRESEL module. To model one quarter portion of the complete structure, first the dome super-element and the floor super-element, each one eighth of the complete component, were assembled. The front and the isometric views of the super structure assembly are shown in Fig. 3. The three foundation super-elements were then combined to give one quarter of the total foundation. Separate assemblies of

the super structure i.e. the dome, floor and the foundation were then combined to give the complete one quarter structure lying in the first quadrant as shown in Fig. 4.

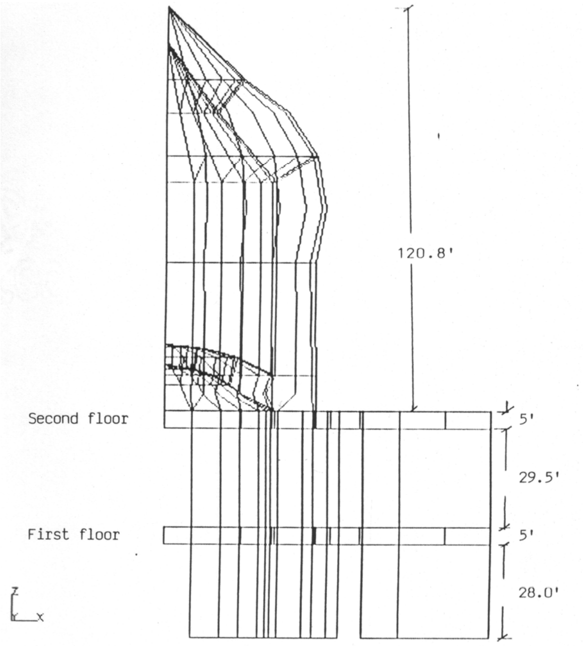


Figure 2 : Discretized view of dome super-element

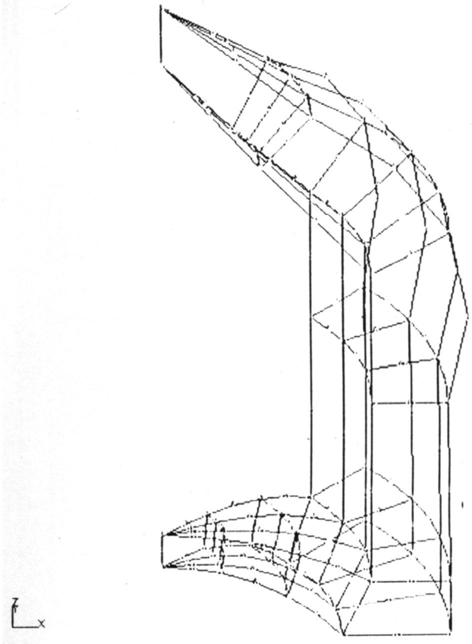

Figure 3 : Side view of super structure assembly (1 ft = 0.3048 m)

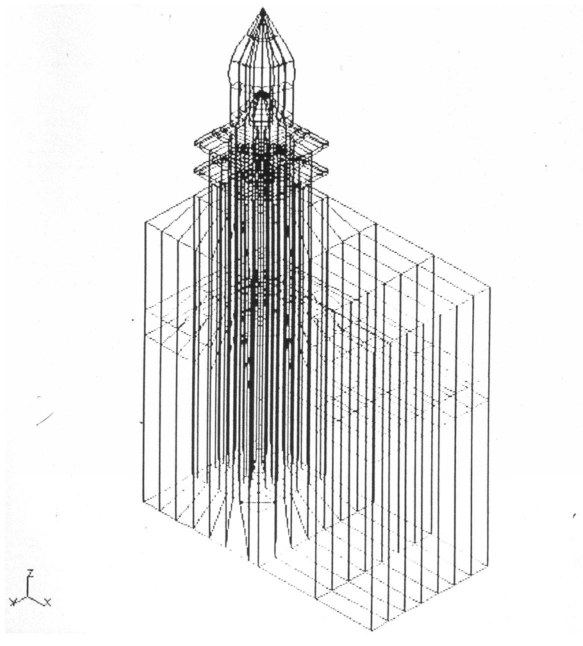

Figure 4 : View of idealized quarter structure of Taj Mahal

*3.3 Material properties*

a. Masonry:

The properties of the masonry (brick work in lime mortar) as determined from the laboratory tests conducted on specimens brought from the site and described in Vol. II (Agra Heritage Project 1991) and used in the analysis are presented below in Table 2.When listing facts use either the style tag List summary signs or the style tag List number signs.

b. Soil Layers:

The details of the geotechnical studies conducted at the site and the subsequent laboratory investigations of the two soil layers are reported in Vol. III (Agra Heritage Project 1991) of the report. The soil properties used in the analysis are presented in Table 3.

Table 2 : Properties of the masonry.

| Property | Magnitude | Units |
|---|---|---|
| Young's modulus | 1600 | MPa |
| Poisson's ratio | 0.12 | - |
| Compressive strength | 5.69 | MPa |
| Unit weight | 20.91 | $KN/m^3$ |

Table 3 : Properties of soil layers.

| Property | Sand layer | Clay layer | Units |
|---|---|---|---|
| Young's modulus | 55.0 | 50.0 | MPa |
| Poisson's ratio | 0.25 | 0.40 | - |

It may be mentioned here that soil below the Taj structure is already in a saturated condition. No material change in mechanical properties of various soil layers would therefore occur due to further impounding of the reservoir. The self weight of soil layers and the foundation platform was neglected in the analysis as the settlement due to self weight would have already occurred much before the super structure was built due to consolidation.

## 4 RESULTS AND DISCUSSION

### *4.1 Displacements*

The plots for the pertinent displacement of interest, i.e. vertical displacement, w, in the z direction are presented in Figs.5 and 6. The original geometry of the structure (Fig. 5) is shown in the green whereas the deformed shape of the structure is shown in the red, so that the vertical distance between the two points on the original and the deformed geometry gives the vertical displacement to a scale 163: 1.

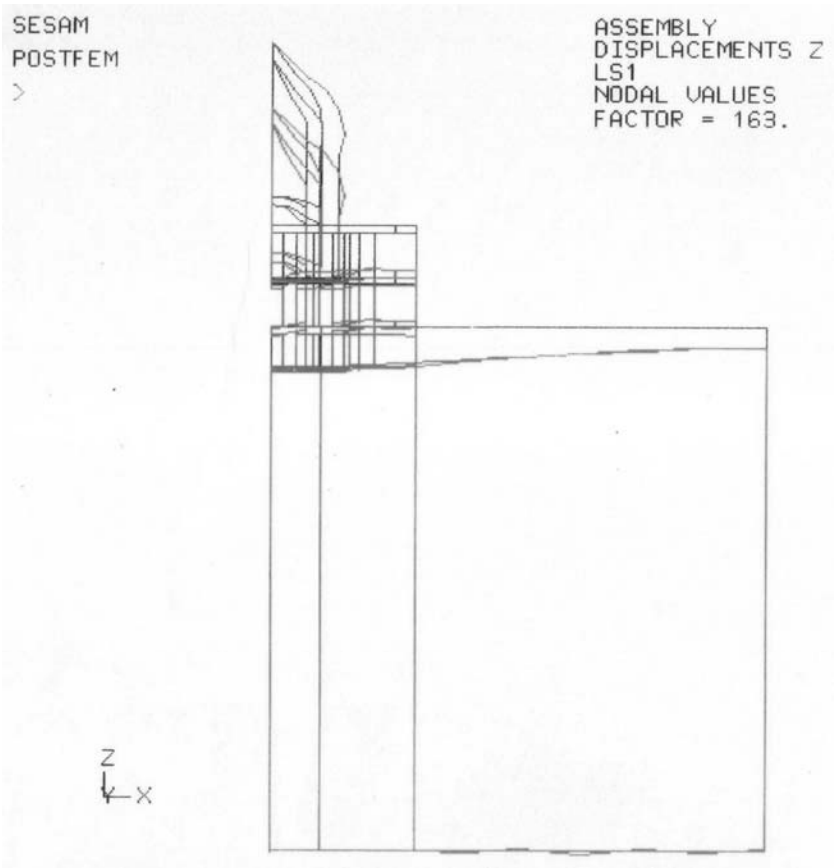

Figure 5 : Displacement, w in Z-direction for complete structure

It is seen that due to vertical loads, the displacement of the domical portion and the floor system is almost uniform at any level so that there is no distortion of shape of the super structure. The displacement in the substructure is however non-uniform, being maximum under the domical portion and reducing almost linearly towards the end of the participating soil considered in the analysis. It is seen that the displacement at the bottom of the sub structure is zero, as prescribed in the boundary condition.

The contours of the vertical displacement w in the z direction for the complete structure are shown in Fig. 6. It is seen that equal value displacement occurs at greater depths in the foundation below the monument. Thus, all the slope contours slope upwards from the centre line of the foundation to the outer edges. The above displacements have been calculated for the dead load of the monument and the live load on the floors and domical surfaces. These are the instantaneous values if the monument were to be suddenly placed on the existing foundation. Obviously, these displacements have already occurred and are of no significance as regards the future deformations in the monument.

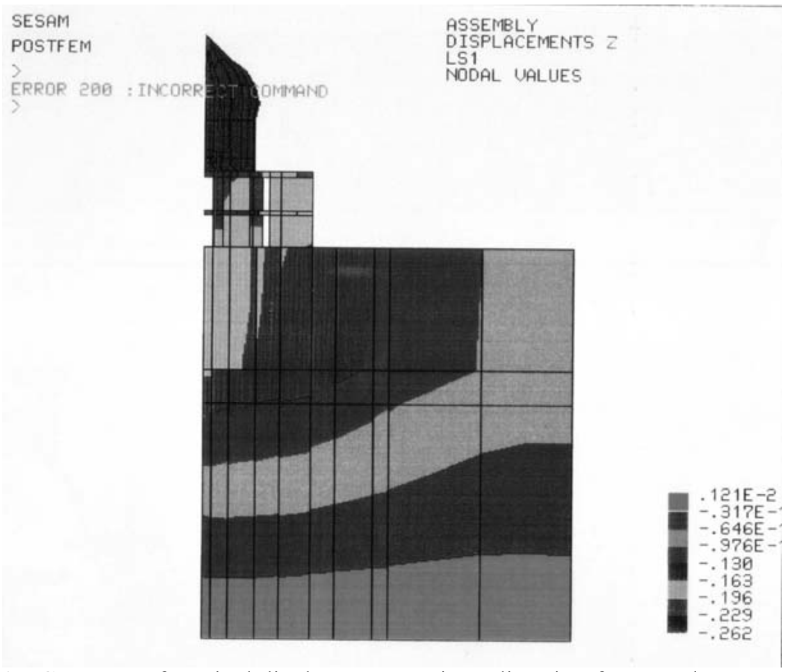


Figure 6 : Contours of vertical displacement, w in Z-direction for complete structure

## 4.2 *Stresses*

Figs. 7 and 8 display the stress contours for stresses $\sigma_x$ and $\sigma_y$ in the domical portion. It is seen from Fig. 7 that the maximum compressive stress of 0.43 MPa develops in the outer edge of the lower dome which is in contact with the supporting column.

It is seen from Fig. 8 that the maximum compressive stress in the domical portion of 0.66 MPa occurs in the same zone as the maximum stress, $\sigma_x$. Since the domical portion is symmetrical, $\sigma_x$ and $\sigma_y$ stresses represent also the hoop stresses which are critical for the safety consideration of the masonry dome. The maximum compressive and the tensile strength of masonry as determined from the sample of masonry collected from the contemporary monuments have been found to be 5.69 MPa and 0.57 MPa respectively. It is seen that both compressive and the tensile hoop stresses induced in the dome are of magnitude as to yield a factor of safety of 8.6 and 4.75 in compression and tension respectively.

Fig. 9 shows the stress contours for the vertical stress, $\sigma_z$ for the complete structure analysed. It is seen that $\sigma_z$ is compressive in the entire structure except for a tiny area at the junction of the lower dome, the first floor and the columns, as discussed above. The maximum compressive stress in the foundation is of the order of 0.33 MPa, which is very small. It may also be seen that there is a tendency for small tensile stresses to develop in the portion where columns rest on the ground or in the portion farther away from the monument area.

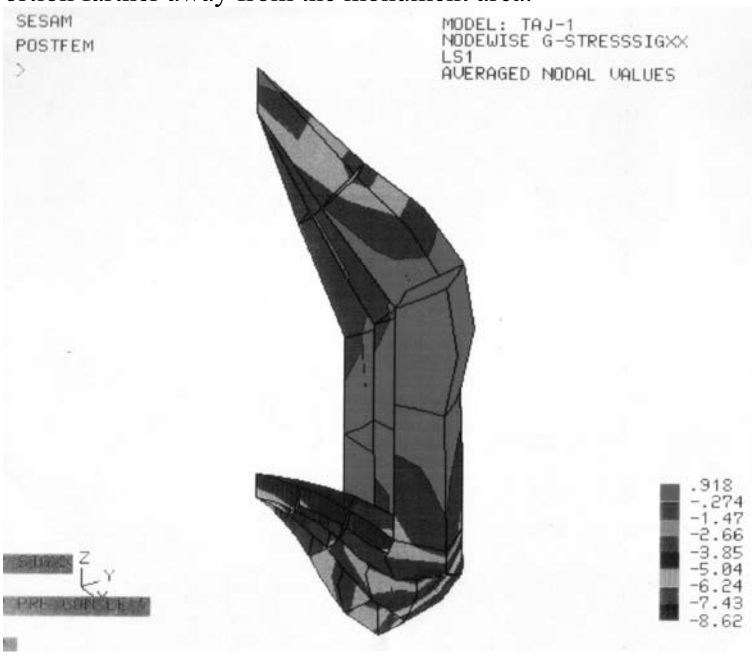


Figure 7.: Stress contours for $\sigma_x$ in domical portion

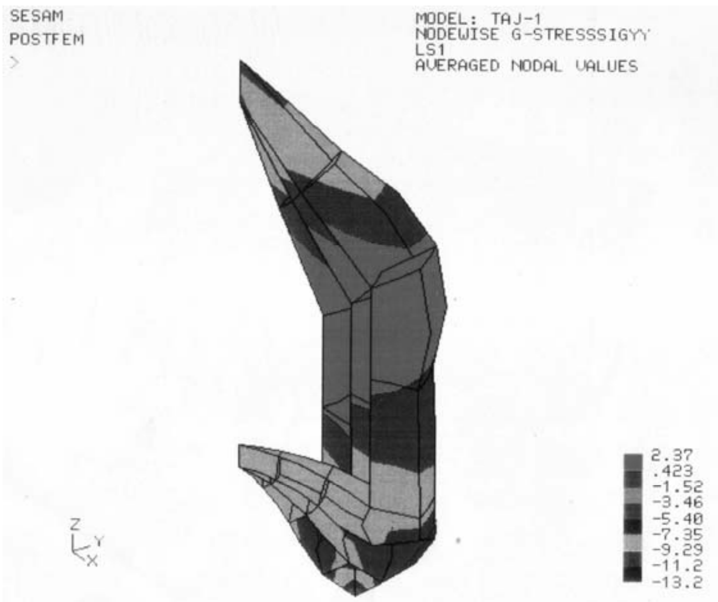


Figure 8 : Stress contours for $\sigma_y$ in domical portion

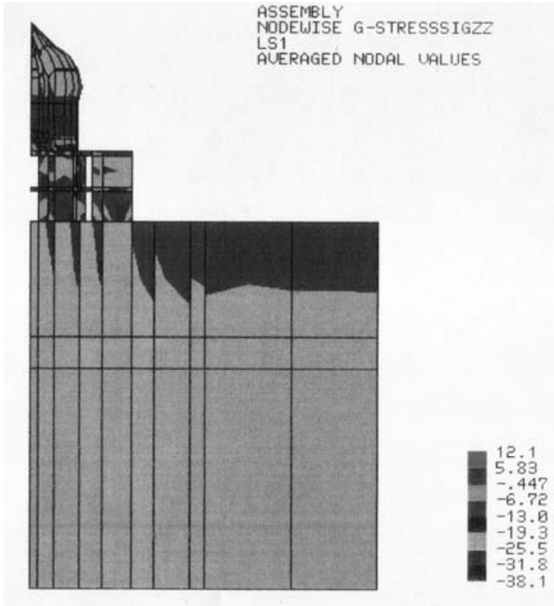


Figure 9 : Stress contours for $\sigma_z$ in domical portion

## 5 CONCLUSIONS

Detailed analysis of results from 3D FE analysis indicates that the hoop stresses in the super structure consisting of the two domes are essentially compressive in nature. The magnitude of stresses is small compared to the compressive strength of masonry giving a factor of safety of the order of 4 to 8. The vertical stresses in the columns are primarily compressive with indications of small zones of insignificant tensile stresses on one of the faces. The stress analysis also indicated that the entire foundation system is in a state of vertical compression.

Further, the displacements in domical portion and the floor system of the main structure under vertical loads are almost uniform, while the substructure deforms in natural bowl shape suggesting a well planned, constructed and healthy structure. These displacements are instantaneous and have already occurred immediately after the monument was constructed and are of no significance as regards the future deformations and the safety of the structure.

## ACKNOWLEDGEMENTS

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