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Damage Model with Crack Localization – Application to Historical Buildings

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ABSTRACT: A model for the analysis of large structures, able to reproduce the mechanical degradation of the material and to predict the potential collapse under certain load conditions, is presented. The model is based on the so-called *smeared-crack scalar damage* model, modified in such a way that it can reproduce localized individual (discrete) cracks. This is achieved by means of a local *crack tracking* algorithm. The model has been used to analyze the response of the structure of Mallorca Cathedral under gravity and seismic forces. Compared with the traditional smeared cracking approach, the tracking method has shown a better capacity to predict realistic collapsing mechanisms; the resulting damage in the ultimate condition appears localized in individual cracks, which behave similarly to real plastic hinges; the computed ultimate loads become less sensitive to the variation of the tensile strength and other material parameter.

1 INTRODUCTION

Models describing damage as a smeared variable distributed over the volume of the structure are normally used for the analysis of reinforced concrete structures and they provide acceptable results. However, the analysis of unreinforced structures, and particularly, the study of unreinforced masonry ones requires more sophisticated approaches able to describe the localized damage which can be normally observed in these structures both in service and at the ultimate condition.

In the so-called *smeared-crack scalar damage model* (Cervera, 2003), the damage in a point is defined by means of a scalar value (called damage variable) that affects the elastic constitutive matrix and represents the level of degradation of the material, ranging from intact material (elastic) to completely damaged material with no stiffness.

The first results obtained by the application of this model to the finite element analysis of masonry buildings showed that damage is simulated in an unrealistic way involving significant volumes spreading over large regions of the structure. As a result of this, a modification was implemented to describe localized damage in the form of discrete cracks. This is achieved by means of a *crack tracking* algorithm. This algorithm is able to detect the point of the boundary of the structure where a crack is originated and then it lets the crack develop as a function of the direction of the main tensile stress. The algorithm marks a track of finite elements pertaining to the crack path which can experience potential damage.

As part of a larger project involving different approaches, the structure of Mallorca Cathedral has been studied using both the smeared-crack scalar damage model and the crack tracking algorithms, for both gravitational and seismic loads. Also, some studies have been carried out using both 2D and 3D structural models. The comparison of the results obtained from the different approaches considered illustrates the capacity of the crack tracing approach to provide a more realistic simulation of the strength response of masonry historical buildings.

2 TENSION COMPRESION DAMAGE MODEL

The model is based on the effective stress concept ($\overline{\boldsymbol{\sigma}}$), defined as the stress associated to elastic strains $\boldsymbol{\varepsilon}$; or $\overline{\boldsymbol{\sigma}} = \mathbf{D} : \boldsymbol{\varepsilon}$, where $\mathbf{D}$ is the elastic constitutive tensor. The tensor $\overline{\boldsymbol{\sigma}}$ is divided into a positive or tensile part $\overline{\boldsymbol{\sigma}}^+$ and a negative or compressive part $\overline{\boldsymbol{\sigma}}^-$, in the following way:

$$\overline{\boldsymbol{\sigma}}^+ = \sum_{i=1}^{3} \langle \overline{\sigma}_i \rangle \, \mathbf{p}_i \otimes \mathbf{p}_i \qquad \overline{\boldsymbol{\sigma}}^- = \overline{\boldsymbol{\sigma}} - \overline{\boldsymbol{\sigma}}^+ \tag{1}$$

where $\overline{\sigma}_i$ is the i-th principal stress of $\overline{\boldsymbol{\sigma}}$; $\mathbf{p}_i$ represents the unit vector associated with the principal direction i and $\langle \bullet \rangle$ are the Macaulay brackets.

Two internal variables of damage, each one associated with a sign of the stresses are also defined: d^+ for tensile damage and d^- for compressive damage. Under these considerations, the constitutive equation is written as:

$$\boldsymbol{\sigma} = \left(1 - d^+\right) \overline{\boldsymbol{\sigma}}^+ + \left(1 - d^-\right) \overline{\boldsymbol{\sigma}}^- \tag{2}$$

These variables state the level of damage reached at each integration point, in such a way that $d^{\pm} = 0$ means that the material is intact and $d^{\pm} = 1$ indicates the total material failure. The use of a different variable for each sign of the stresses implies that, for example, a material previously damaged to traction would recover its original behaviour if it is put under compression, and vice versa.

Next, the equivalent effective stress norm is defined. This is a scalar positive value that is used to compare different stress states in three dimensions, and it is useful to unify load, unload and reload concepts. The equivalent norms for tensile effective stress (τ^+) and for compressive effective stress (τ^-) have the form:

$$\tau^{\pm} = \left(\overline{\boldsymbol{\sigma}}^{\pm} : \mathbf{C}^{\pm} : \overline{\boldsymbol{\sigma}}^{\pm} \right)^{1/2}$$

[U3]

(3)

where the two non-dimensional fourth order metric tensors $\mathbf{C}^+$ and $\mathbf{C}^-$ are identical and equal to the inverse of tensor $\left(\mathbf{D}/E\right)$, being E the Young's modulus of the material..

Starting from the previous definitions, two different criteria of damage can be introduced, namely, g^+ and g^-, defined as

$$g^{\pm}\left(\tau^{\pm}, r^{\pm}\right) = \tau^{\pm} - r^{\pm} \le 0$$

(4)

where r^+ and r^- are internal variables that control the size of the damage surface in the stresses space in every time step. Their initial values are $r_0^{\pm} = f_e^{\pm}$, where f_e^+ and f_e^- are the strength of material—the stresses at which the material fails and damage appears. In this sense, the explicit definition of these internal variables has the form:

$$r^{\pm} = \max\left[r_0^{\pm}, \max\left(\tau^{\pm}\right) \right] \tag{5}$$

The criteria stated above imply that damage evolution occurs when condition (4) is not satisfied. In this case, $r^{\pm}$ is updated using (5), until damage criterion is satisfied again.

Finally, the damage variables d^+ and d^- are defined explicitly as a function of their respective internal variables $r^{\pm}$. They are monotonic increasing functions of the form $0 \le d^{\pm}\left(r^{\pm}\right) \le 1$. The post-peak behaviour is defined by means of the tensile and compressive fracture energy of the material $G_f^{\pm}$. This parameter is normalized respect to the characteristic length of the finite elements, to ensure objectivity respect to the mesh size.

3 TRACKING ALGORITHM

The damage model formulated above is not able to simulate individual discrete cracks under tensile stresses. The model is based on continuous displacements and strain fields, which can not describe the discontinuity inherent to the crack. For the present application, this limitation is overcome by forcing the crack to develop along a single row of finite elements by means of a tracking method. The fracture energy normalization respect to the characteristic length ensures that dissipation will be element-size independent.

The proposed method is applied at every time step during the analysis, just before the stress evaluation. The method works with a flag system, where finite elements are labelled to delimit the zones where cracks will appear or develop. The criteria used to define these zones depend on the magnitude and direction of the principal stresses at each element. The algorithm has been implemented for 2D problems using three-noded triangular elements.

The procedure is divided into two steps. First, new cracks are detected by checking the stress values at every finite element located on the boundary of the structure. Those elements where the tensile strength of the material is reached are labelled as a crack root. When several neighbour elements reach the tensile strength at the same time step, the *initial radius* criterion is applied. This radius (defined by the user) is the minimum distance between two crack root elements, and is used to guarantee the creation of separated discrete cracks.

At the beginning of the second step, existing cracks from previous time steps are checked to determine the elements located at the crack tips. Then, the following procedure is applied on each crack tip element and each new crack root element:

1. Determine the *exit point* coordinates. A vector is drawn from the *entrance point* coordinates (defined below), using the direction perpendicular to the principal tensile direction of the element. The exit point is defined as the intersection of that vector with the corresponding face of the element (see Fig. 1.a).

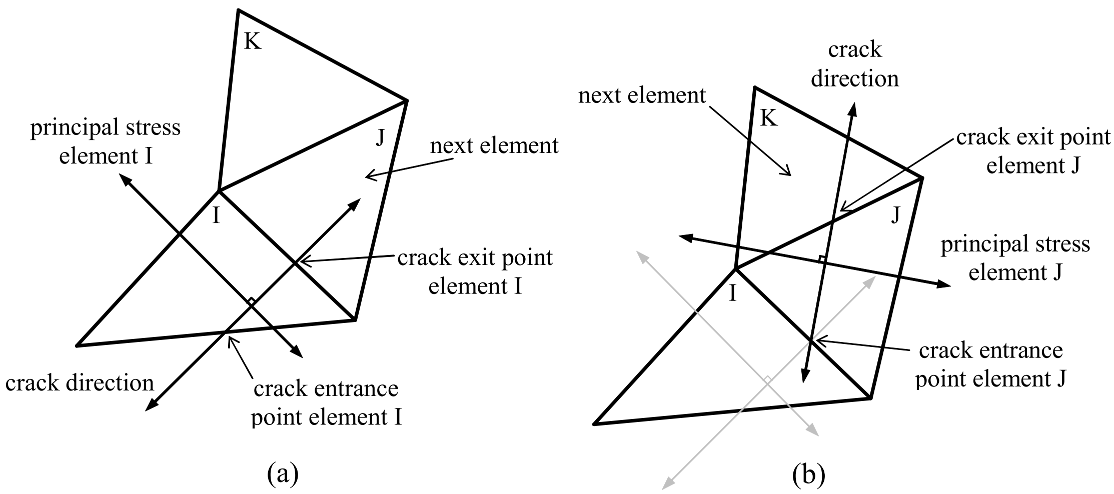


Figure 1 : (a) Exit point of element I (b) New potential element (J) and its entrance point

2. Determine the next (potential) element on the current crack. It would be the neighbouring element whose face in common with the current element corresponds with the face where its exit point is located. The element is marked as a potential element belonging to this crack.

3. Determine the *entrance point* coordinates. The entrance point of the new element on the crack is located at the same coordinates of the exit point of the previous element (see Fig. 1.b).

4. Repeat from step 1, taking the new potential element as the crack tip element.

For each crack, the previous procedure is repeated until one of the following criteria is satisfied:

Stress threshold criterion. Element tracking and labelling is stopped when the principal tensile stress is lower than a threshold defined by the user. Experience has demonstrated that 75% of tensile strength usually works well.

Crack meeting criterion. The procedure stops when a previously damaged element, or an element marked as a potentially cracking one, is found along the current crack. This means that two cracks have met, and from then on they will be considered as a single one.

Boundary criterion. When the exit point of an element is on the boundary of the structure, the cracking process is considered finished.

Once any of the previous criteria is reached, the current crack is considered totally developed and the next one is studied by restarting the cycle. Finally, after applying this procedure to all the cracks, each element will have one of the three following labels:

- Intact element, not able to damage (out of potential crack track; it will keep elastic behaviour during the current time step)
- Intact element, able to damage (in a potential crack track; it will initiate inelastic behaviour if the material strength is reached)
- Damaged element (belonging to a crack consolidated in previous time steps; it will develop inelastic behaviour during the rest of the calculations)

The analysis procedure recognizes these labels and activates the corresponding constitutive response in each element for the current time step. Also, once the stresses have been updated (and therefore the damaged indexes are known), the elements with potential cracking that really suffer damage are relabelled as included in a consolidated crack for the rest of calculations. Finally, elements potentially cracking that do not suffer damage are restored to their original status, unlabelled.

4 NUMERICAL ANALYSIS OF MALLORCA CATHEDRAL

The Cathedral of Palma de Mallorca, built during the 13th to the 15th centuries, is a three-nave building composed of a central nave spanning 19.9 m, 43.9 m high to the vaults keystone, and two collateral nave spanning 8.72 m each and 29.4 m high. The central nave is sustained on octagonal piers with a circumscribed diameter of 1.6 or 1.7 m and a height of 22.7 m to the springing of the vaults. The central vaults transfer the lateral thrust towards the buttresses by means of a double battery of flying arches (see Fig. 2). The main distresses observed on the building are large lateral deformation in the piers and vertical cracks at their base. More information on the building and previous analytical or numerical analyses can be found in Rubió (1912), Mark (1982) and Salas (2002).

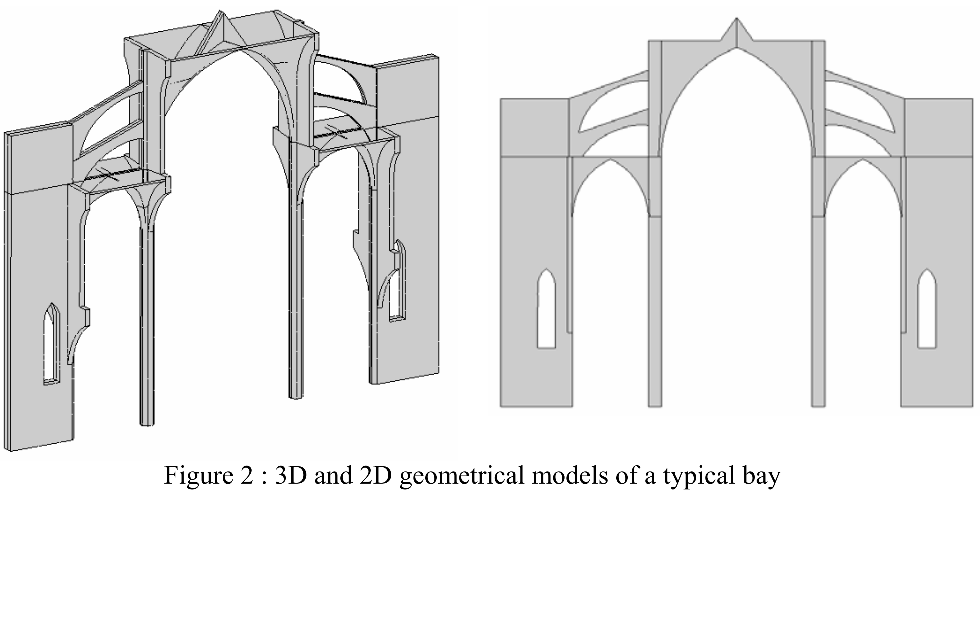


Figure 2 : 3D and 2D geometrical models of a typical bay

4.1 Numerical model

The analyses are carried out on the typical bay of the building using both 2D (plane stress) and 3D models (Fig. 2). As stated before, the localized damage model can only be applied on the 2D model, its extension to 3D problems being considered as a future development. The 3D finite element mesh is composed of 49979 four-node tetrahedral elements and 14689 nodes. The 2D finite element mesh is composed of 16430 three-node triangular elements and 8824 nodes.

Two main materials are defined. The first type of material is considered to model the soft limestone masonry found in most of the structure: buttresses, vaults and walls. Flying arches and piers are defined with a second type of material representing the harder limestone masonry found in piers, arches and flying arches. The material properties are estimated based on experiments carried out on core specimens taken from the building. The parameters used are shown in Table 1. In all cases a large value of the material fracture energy is considered to simulate the formation of "plastic" hinges with considerable ductility. Previous parametric studies have proved that this value has influence on the collapse load factor, but its study escapes of the reach of the present paper.

Table 1 : Material parameters

	Structural elements	Young's modulus MPa	Comp. strength MPa	Tens. strength MPa
Material 1	buttresses, vaults, walls	2000	2	0.1
Material 2	flying arches, arches, piers	8000	8	0.4

4.2 Analysis of the structure subjected to gravity loads

Analyses under gravitational loads are carried out using both the smeared crack and the localized damage models. The former is performed on the two and three-dimensional models; the results obtained can be used to compare the accuracy of the 2D plane stress approximation. It is important to highlight that the results from the 2D analysis were obtained after an intricate calibration process of the thickness of the different structural Theelements. The localized damage analysis latter is performed only on the 2D model. Three different initial radii are defined: 1 m, 2 m and 3 m. In all analyses, gravity is increased until a total collapse of the structure is reached.

The damage distribution and the collapse mechanisms obtained for every case studied are showed in Figs. 3-4. All cases show similar failure modes. Failure is controlled by damage on the buttress, at the bottom left side of the window. Also, a dislocation occurs in the main vault, and two hinges are clearly formed on each flying arch.

It is observed that the 2D approximation is acceptable due to its similarities with the original 3D model.

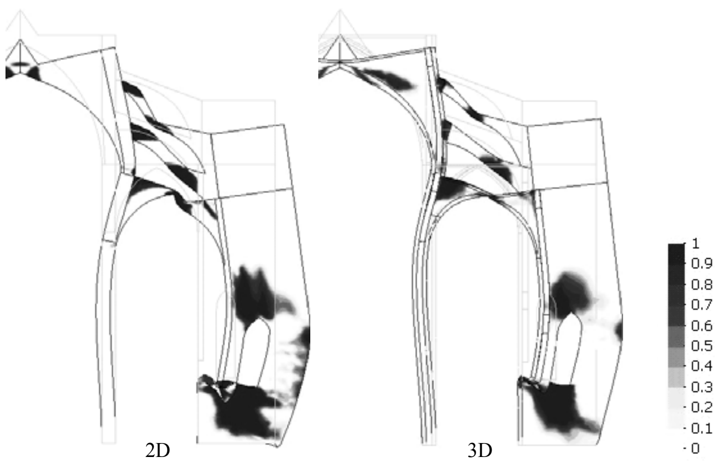
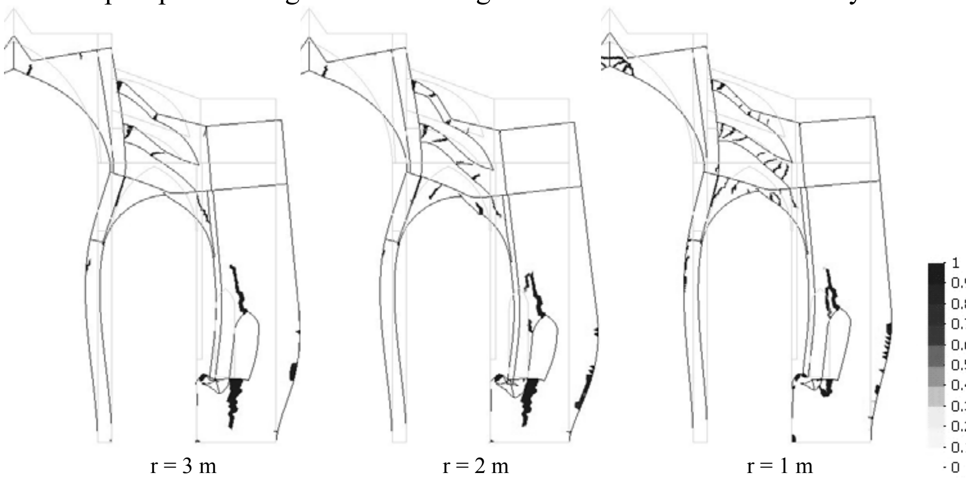


Figure 3 : Collapse mechanism and distribution of the tensile damage index under gravity loads with the smeared crack model

With regard to the results of the localized damage model (Fig. 4), it can be observed that varying the initial radius does neither influence the collapse mechanism, nor the failure load. However, it is interesting to highlight that the generation of localized cracks, acting as plastic hinges, represensrepresents more realistically the behaviour of the structure in the ultimate condition.

Fig. 5 shows the comparison of the curves relating the load factor with the horizontal displacement on top of piers throughout the loading obtained for the different analyses carried out.



r = 3 m r = 2 m r = 1 m

Figure 4 : Collapse mechanism and distribution of the tensile damage index under gravity loads with the localized damage model and different initial radius (r) values

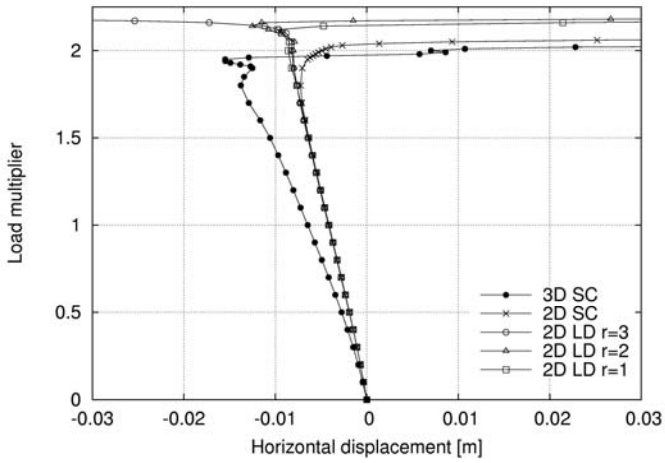


Figure 5 : Collapse load multiplier under gravity loads (SC: smeared crack model; LD: localized damage model; r: initial radius)

Fig. 5 shows the comparison of the curves relating the load factor with the horizontal displacement on top of piers throughout the loading obtained for the different analyses carried out.

A collapse gravity multiplier load of 2.0 and 2.05 is obtained for the 2D and the 3D analyses respectively. In the localized damage analysis, the failure load factor is 2.15[U12]. Thus, the computed ultimate load is close in all cases.

4.3 Sensitivity analysis

Given the difficulty found in the experimental determination of the average tensile strength of masonry, numerical analysis normally adopt empirical values based on the experience or on very general correlations. In the present study, the tensile strength f_e^+ is taken as 5% of the compressive strength. A parametric study is carried out to study the influence of this parameter on the ultimate capacity of the structure. For that purpose, both the 2D localized damage model with r = 3 m and the 2D smeared crack model are analyzed with four different tensile strength values (f_e^+, $f_e^+/2$, $f_e^+/4$, $f_e^+/10$). Fig. 6 shows the curves relating the collapse load factor with the tensile strength value used. The smeared crack model shows a significant sensitivity to the value adopted for the tensile strength, showing a variation of the ultimate load of about 30% with respect to the value calculated for the initial value f_e^+. The variation found with the localized damage model is only 5%.

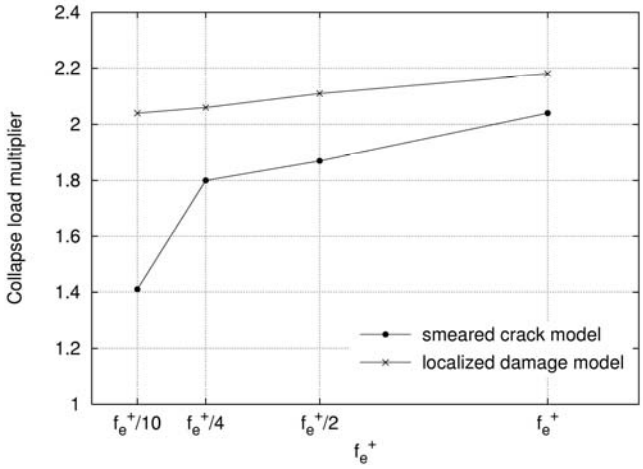


Figure 6 : Variation of the collapse gravity load multiplier in function of the tensile strength f_e^+

4.4 *Analysis of the structure subjected to seismic loads*

The seismic performance of the building is assessed by means of a push-over analysis consisting of the gradual application of a system of lateral equivalent static forces on the structure. The analysis is divided into two steps. The gravity load is applied in the first step. In the second step, the lateral seismic forces are applied and increased gradually until reaching failure.

In Fig. 7 the seismic load factor (defined as the ratio between the lateral unit force and gravity) is plotted against the horizontal displacement on the top of piers. The smeared crack model causes failure at a load factor of 0.1. This value is the same for both models (2D and 3D). The localized cracking model produces a higher failure load factor as should be expected due to the restrictions that the model imposes to the propagation of damage. In that case, the building fails at load multipliers 20% higher for r = 1 m and r = 2 m. The difference is 40% (load factor 0.14) for an initial radius of r = 3 m.

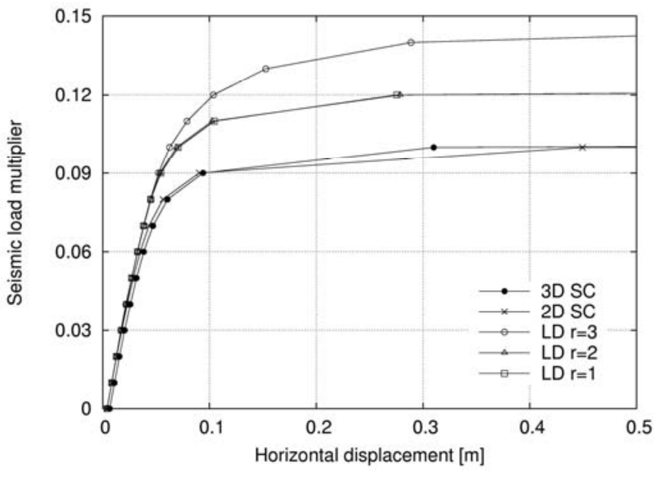


Figure 7 : Collapse load multiplier under seismic loads (SC: smeared crack model; LD: localized damage model; r: initial radius)

Fig. 8 depicts the deformations and damage distribution for the smeared crack 2D model and localized damage models (r = 1 m, r = 2 m and r = 3 m). As expected, the model with higher initial radius shows a lesser number of cracks. Together with the large increasseincreases produced in the load multiplier, this fact illustrates that the use of large exclusion radii is not suitable and may lead to an overestimation of the strength of the structure. Conversely, the load multiplier and the damage distribution converged for exclusion ratios between 1-2 m, showing that the result is not dependent on the value of the exclusion ration provided that this is sufficiently small.

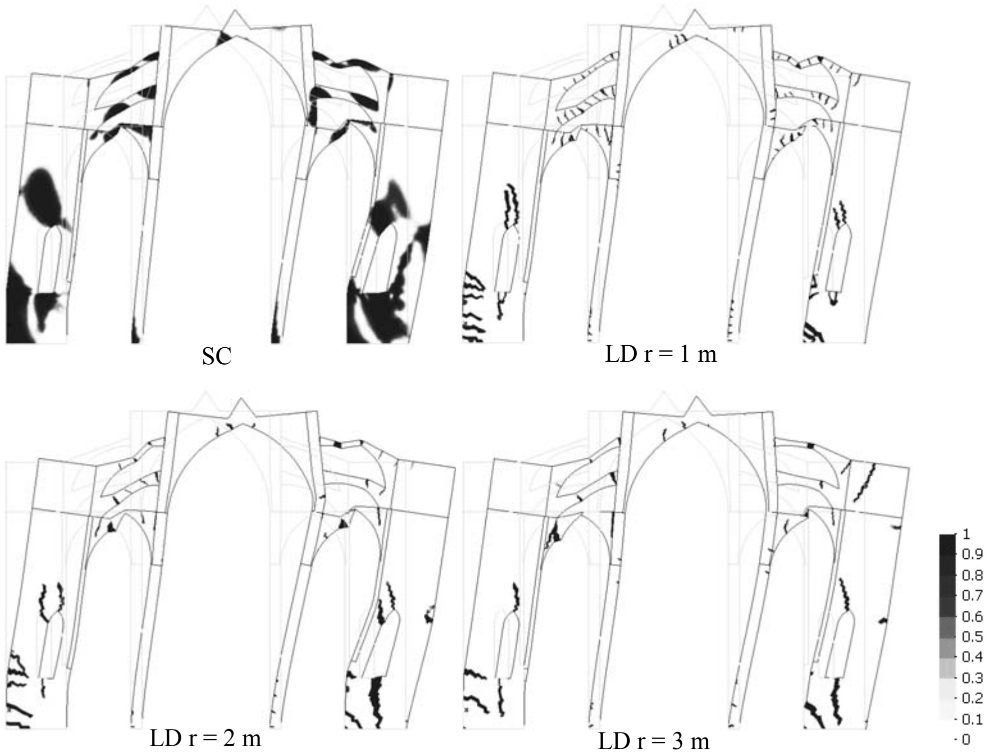


Figure 8 : Collapse mechanism and distribution of the tensile damage index under seismic loads for 2D models (SC: smeared crack model; LD: localized damage model; r: initial radius)

5 CONCLUSIONS

The crack tracking model enables the simulation of more realistic damage distributions than the original smeared-crack model. The localized cracks predicted by the crack tracking model reproduce consistently a set of expectable plastic hinges developing gradually in the structure and leading to the full collapsing mechanism. Moreover, a smaller dependency with some material parameters (especially with the tensile strength) has been found. In turn, the smeared-crack model describes damage in a widely distributed, unrealistic way. The tracking model represents a more suitable method to predict the structural behaviour of masonry buildings without requiring any significant additional computation cost.

Further developments are now being into consideration. In particular, it is intended to combine the crack tracking approach with an anisotropic constitutive equation for masonry. The resulting model would provide improved realism. In particular, it would enable the modelling of the influence of the actual orientation of the mortar bed joins on the response of the structure. Also, the extension of the tracking algorithm to three dimensions is being considerated.

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