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Issues about the Dynamic Behaviour of Rigid Free-Standing Blocks under Earthquake Ground Motion

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ABSTRACT: The rocking response of rigid free-standing blocks under the action of ground motion is quite different from the typical response associated with a structural system, either elastic or ductile. Thus, the validated theories on the seismic behaviour of general structural systems cannot be applied directly to the safety of rigid systems subjected to overturning. In this paper, aimed at overcoming these difficulties, possible simplifications of the equation governing rocking behaviour are firstly summarised. However, taking the self-balanced effect of the earthquake as a reference point, elementary schemes characterised by zero final velocity and displacement, are introduced to represent the seismic input as a combination of them. The dynamic response of a rigid free-standing block is then analysed, focusing attention on the possible risks of divergence and on the maximum angular displacement involved. Appropriate diagrams show the influence of such parameters as the size and slenderness of the block, the coefficient of restitution and the features of the acceleration pulses representing earthquakes.

1 INTRODUCTION

Historically, the rocking response of rigid blocks subjected to earthquake ground motion has been a field of interest to researchers for over a century. Nevertheless, it was Housner (1963) who first gave the problem a modern treatment. He showed that despite the apparent simplicity of a single rocking block dynamics, a non-trivial behaviour was present and a number of unexpected results emerged. Basically, the stability of a block subjected to a particular ground motion does not necessarily increase monotonically with the increasing size or decreasing slenderness ratio. Nor does overturning of a block by a ground motion of particular intensity imply that the block will necessarily overturn under the action of more intense ground motion.

Thus, in order to simplify the analysis Housner (1963) described the base acceleration as a rectangular or a half-sine pulse and expressions were derived for the minimum acceleration required to overturn the block, as function of the duration of the pulse. Experimental and numerical analysis were later developed by Yim et al. (1980), showing that, in contrast with the response to a single pulse, the response to more irregular but simplified accelerograms is very sensitive to the geometrical parameters of the block, as well as to the details of ground motions and to the coefficient of restitution. Therefore, they used a probabilistic approach to identify certain statistically recurrent properties of the response. Aslam et al. (1980) also analysed, both numerically and experimentally, dynamic behaviour under harmonic excitations and simulated accelerograms, confirming the difficulty in providing prediction criteria for the response.

The works in this field also highlight the importance of the coefficient of restitution by means of the measurement of the energy loss due to impact, first theoretically provided by Housner (1963). Housner's coefficient, depending on the slenderness ratio of the block, was then adopted by Giannini (1984), who also introduced the dependence on the application point of the resulting impulsive forces due to impact. Comparisons between numerical analysis based on the

Giannini's formulation and experimental tests have been carried out, among other authors, by Liberatore et al. (2001a, b). Prieto et al. (2004) introduced the damping effects non longer by means of a coefficient of restitution but understood as the presence of impulsive forces (Dirac-delta ones). And Sinopoli (1989) showed the importance of the possibility of sliding, together with rocking, during the impact, depending on the slenderness ratio of the block.

However, where the study of the block is referred to the masonry block, further difficulties due to uncertainties about the structural behaviour of masonry should be taken into account. This implies high levels of uncertainty for the seismic analysis of masonry structures, for which the single block is the basic reference. As a consequence, it is evident that the current seismic rules on masonry structures are lacking in scientific robustness and, therefore, many indications can be accepted only as prudential criteria, based on experience and on rough evaluations.

For these reasons, it is necessary to tackle the problem again, devoting greater attention to the parameters that influence the block response. Particularly, it is necessary to tackle the difficulties in defining a reliable response spectrum for the rigid block such as that for elastic systems. These difficulties are due to: 1) the lack of explicit damping factor, at least for rigid blocks, with the consequence that the response is always affected by the natural motion; 2) the lack of the natural frequency, with the consequence that an harmonic excitation cannot have the same important role as for elastic systems; 3) the load-displacement characteristic that is completely different from the more common structural systems. Therefore, in order to overcome these difficulties, this paper focuses on the earthquake ground motion which can be represented by a combination of elementary schemes, and some general observations on the structural response are presented in terms of stability of the block.

2 THE EQUATION OF MOTION FOR A RIGID FREE-STANDING BLOCK

The equation of rocking motion for a rigid free-standing block before the first impact, with positive signs of forces and angles as in Fig. 1a, is obtained from D'Alembert's Principle:

$$Mg\,R\cos(\alpha+\theta)+I_0\ddot{\alpha}=M\ddot{y}R\sin(\alpha+\theta) \tag{1}$$

where M is the mass, $I_0=(4/3)MR^2$ the corresponding moment of inertia (with respect to O), g is the acceleration of gravity and $\ddot{\alpha}$ is the relative angular acceleration of the block.

The coefficient of friction is assumed to be sufficiently large as to prevent sliding between the block and the supporting base. If sliding motion is also negligible during impact, Eq. (1) is generally representative of the motion between two impacts. Actually, the friction coefficient does influence the size of the coefficient of restitution, as properly examined later.

We now present some simplifications where Eq. (1) assumes small values of α.

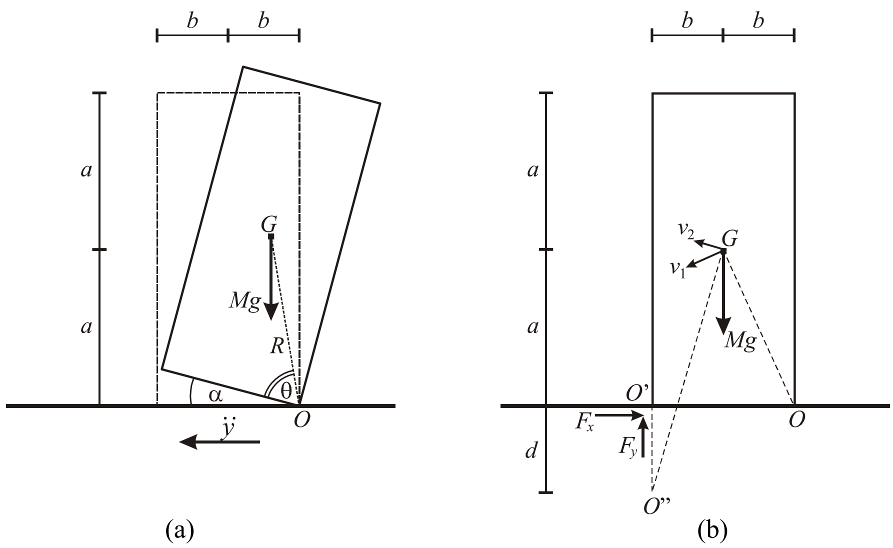


Figure 1 : (a) Rocking rigid block system; (b) velocities of the mass centre during impact.

2.1 First approximation of the equation of motion

For slender blocks, i.e. $\lambda = a/b \geq 4$, the sine and cosine functions in Eq. (1) can be replaced by their mean values in the interval $0 - \alpha_c$, with $\alpha_c = \arctan(b/a)$ the critical angle of overturning, i.e.:

$$\sin(\alpha + \theta) = \frac{1 + \sin\theta}{2}; \qquad \cos(\alpha + \theta) = \cos\theta(1 - \alpha/\alpha_c) \tag{2}$$

Eq. (1) now becomes:

$$-Mgb\frac{\alpha}{\alpha_c} + I_0\ddot{\alpha} = M\ddot{y}R\frac{1 + \sin\theta}{2} - Mgb \tag{3}$$

which can be written in the form:

$$I_0\ddot{\alpha} - P\alpha = +Q - S \tag{4}$$

where:

$$Mgb\frac{1}{\alpha_c} = P; \qquad M\ddot{y}R\frac{1 + \sin\theta}{2} = Q; \qquad Mgb = S \tag{5}$$

Eq. (4) is a standard differential equation and has the solution:

$$\alpha = Ae^{\sqrt{\frac{P}{I_0}}t} + Be^{-\sqrt{\frac{P}{I_0}}t} + \frac{Q - S}{P} \tag{6}$$

2.2 Second approximation of the equation of motion

For very small rotations of the block, which we are most interested in, a closer approximation of Eq. (1) is:

$$Mgb + I_0\ddot{\alpha} = Ma\,\ddot{y} \tag{7}$$

which represents a uniform accelerated motion between two impacts.

Eq. (7) will be used later in this paper for the dynamic analysis of the block.

2.3 Third approximation of the equation of motion

Another approximation of the equation of motion is based on the hypothesis of a continuous rounding of the two base edges due to expulsions of material during the dynamic rocking motion. This case, already presented by others (i.e. Giannini 1984), involves the possibility of defining a natural frequency for the block, provided that the instantaneous centre of rotation is inside the base. The analysis can be developed in analogy with the classical single degree of freedom elastic model.

3 IMPACT AND COEFFICIENT OF RESTITUTION

During impact the block may rotate and slide. As shown in Fig. 1b, the instantaneous centre of rotation O'' lies on the local vertical through the point O', in accordance with the assumptions that the rigid block and rigid support do not compenetrate each other and the vertex O' does not rise. Thus, with symbols indicated in Fig. 1b, the conservation of momentum and angular momentum about the point O'' of the block before and after impact can be written as follows:

$$\begin{aligned} &M(\dot{x}_{2G} - \dot{x}_{1G}) = M((a + d)\dot{\theta}_2 - a\dot{\theta}_1) = F_x\Delta t = -\beta F_y\Delta t \\ &M(\dot{y}_{2G} - \dot{y}_{1G}) = M(b\dot{\theta}_2 + b\dot{\theta}_1) = F_y\Delta t \\ &M(\dot{x}_{1G}a + \dot{y}_{1G}b) + I_G\dot{\theta}_1 = M(\dot{x}_{2G}a + \dot{y}_{2G}b) + I_G\dot{\theta}_2 \end{aligned} \tag{8}$$

where $\dot{x}_{iG}$ and $\dot{y}_{iG}$ are the components of the velocities v_i of the mass centre before ($i=1$) and after ($i=2$) impact and β is the ratio F_x/F_y. From Eqs. (8) we derive:

$$d = \frac{-\left(\dot{\theta}_2 + \dot{\theta}_1\right)b\beta + a\dot{\theta}_1}{\dot{\theta}_2} - a, \qquad \dot{\theta}_2 = \frac{M\left(a^2 - b^2\right) + I_G}{M\left[(a+d)a + b^2\right] + I_G} = \frac{\lambda^2 + 4 + 3\lambda\beta}{\lambda^2 + 4 - 3\lambda\beta}\dot{\theta}_1 \qquad (9)$$

where $d \geq 0$ for geometric compatibility. This means that $\dot{\theta}_2 / \dot{\theta}_1$ is always less than 1 and decreases when d increases. Also, an upper bound of β can be found from Eqs. (9).

The parameter $C = \dot{\theta}_2 / \dot{\theta}_1$, well known as coefficient of restitution, represents the ratio between the angular velocity just after and before impact, when the block and the support are assumed rigid. Actually, at least for masonry structures with mortar, the real coefficient of restitution is much smaller because of some velocities transmitted to the ground and local plastic deformations of the block. It also decreases when the size of the block increases.

4 PULSE SEQUENCES REPRESENTING EARTHQUAKE GROUND MOTION

It is well known that if damping is not considered in the analysis, the spectral response of elastic systems is very sensitive to little changes of the natural period of the systems. Also, earthquakes with the same peak ground acceleration provide spectra very different from each other.

Thus, when studying the seismic rocking motion of a rigid free-standing block, characterised by no explicit damping and no unique period of free vibration, the prediction of its response becomes more difficult because it strongly depends on the intensity of the ground acceleration changing at each cycle of rocking. Moreover, the rigid block has a load-displacement characteristic that is completely different from the more common structural system where seismic response is based on the concepts of flexibility and ductility.

Therefore, one of the possibilities of reaching any reliable result is to choose as a reference point a particular aspect common to all earthquake ground motions: the self-balanced effect of the earthquake resulting from the null integral of the displacements, velocities and accelerations of the ground over the time of its duration. Thus we may deduce that the general earthquake ground motion can be simplified as the combination of elementary schemes characterised by zero final velocity and displacement.

The assembly unit shown in Fig. 2a, consisting of three acceleration pulses, is the simplest scheme obeying to this property of ground motion.

Let us now start by studying this single unit. Two limiting conditions are clear straightaway:

1) if $T \to 0$ the unit tends to a zero pulse and, consequently, its effect on the block is zero.

2) if T is very large, during the time between two pulses the motion of the block is of natural type, with several impacts damping the effects of the pulses.

An intermediate more realistic case is characterised by T equal to the duration of the first half-cycle of the block, i.e. from $t = 0$ to the time of the first impact.

Two different possibilities are herein examined. The first pulse of the unit is applied to the starting point of the motion with initial conditions of angular displacement and velocity $\alpha_0 = 0$ and $\dot{\alpha}_0 = \dot{\alpha}_0$, respectively. The difference between the two schemes in Figs. 2a,b is only the instant of application of the second pulse (point 1), which is just before (scheme A) and just after (scheme B) impact.

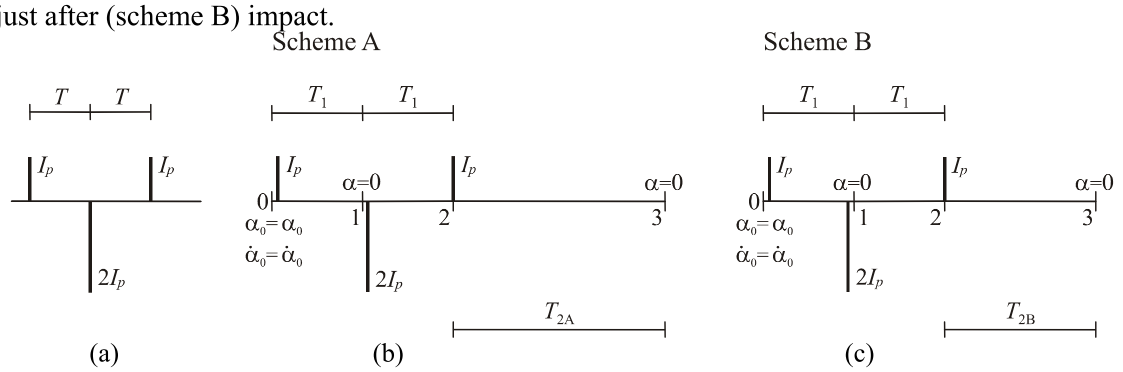


Figure 2 : (a) Assembly unit chosen to represent earthquake ground motion, with the second pulse acting: (b) just after impact (scheme A) and (c) just before impact (scheme B).

There are two parameters useful to compare the two schemes, depending on the interest either on the risks of divergence (progressive increase of angular displacement) or on the maximum angular displacement produced by the schemes. These parameters are:

1) angular velocity at point 3 ($\dot{\alpha}_3$) with respect to initial velocity $\dot{\alpha}_0$;
2) maximum resulting angular displacement.

5 STABILITY CONDITIONS OF THE BLOCK SUBJECT TO AN ASSEMBLY UNIT OF EARTHQUAKE GROUND MOTION

This section is aimed at analysing the effects of the two presented assembly units of ground acceleration on the rocking motion of a rigid free-standing block, with a general coefficient of restitution *C*. The natural motion of the block is easily defined. Eq. (7), subject to initial conditions $\alpha_0 = \alpha_0$, $\dot{\alpha}_0 = \dot{\alpha}_0$ at $t = 0$, has the solution:

$$\alpha = \alpha_0 + \dot{\alpha}_0 t - \frac{1}{2}\frac{Mbg}{I_0}t^2 = \alpha_0 + \dot{\alpha}_0 t - 0{,}5 a_g t^2 \tag{10}$$

where:

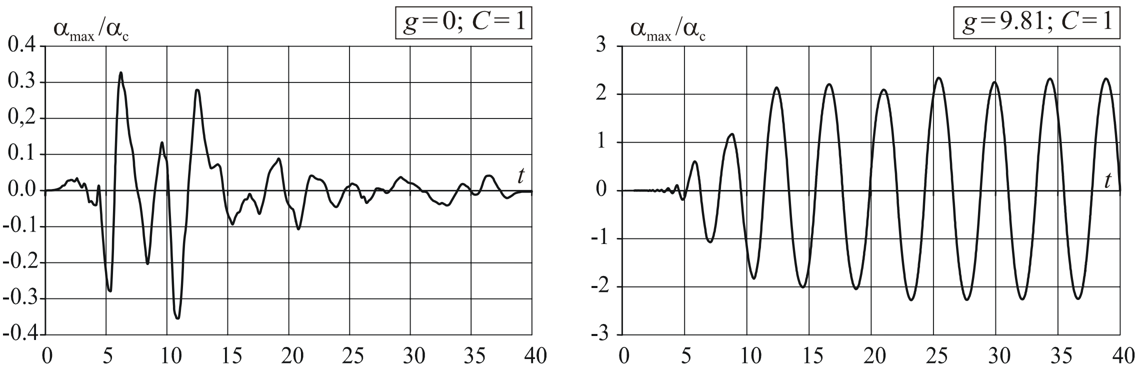
$$a_g = \frac{3}{4}\frac{b}{a^2 + b^2}g = \frac{3}{4b}\frac{1}{\lambda^2 + 1}g \tag{11}$$

It is also easy to verify that, if there is no energy loss during impact, the period of free vibration would be (Casapulla and Maione 2005):

$$T = 4\sqrt{\frac{\dot{\alpha}_0^2}{a_g^2} + \frac{2\alpha_0}{a_g}} \tag{12}$$

Before evaluating the effects of the schemes in Fig. 2, it is worth pointing out the self-balanced effects that the earthquake motion has on the block when gravity is neglected. In fact, it is easy to verify, by using Eq. (7) without gravity and with $C = 1$, that the rotations of the block due to a general earthquake are proportional to the ground displacements and therefore are zero at the end of its duration. In Fig. 3a the ratios of these rotation to the critical ones are shown with reference to the earthquake recorded in Sturno, Irpinia (Italy) 1980 (STU00, PEER strong motion database record, PGA=0.25*g*); on the contrary, Fig. 3b shows that gravity is the cause of divergence. Also, if we consider the accelerogram starting from a point later than the initial one, the self-balance property vanishes and the block response is completely different.

Now let us analyse the effects of the schemes in Fig. 2 on the block, considering the initial conditions of angular velocity only ($\alpha_0 = 0$; $\dot{\alpha}_0 = \dot{\alpha}_0$). Then, an impulsive force $I_p = M\ddot{y}\,dt$ involves the angular velocity on the block:



(a) (b)

Figure 3 : Block response to the Sturno earthquake (Irpinia-Italy, 1980, PGA=0.25*g*): (a) gravity is neglected; (b) gravity is the cause of divergence.

$$\dot{\alpha}_p = \lambda a_g \frac{I_p}{Mg} \tag{13}$$

which is added to the initial condition and gives the velocity at point 0:

$$\dot{\alpha}'_0 = \dot{\alpha}_0 + \dot{\alpha}_p \tag{14}$$

At the end of the first half-cycle (point 1 in Fig. 2b,c) we have:

$$\dot{\alpha}_1 = -\dot{\alpha}'_0, \qquad T_1 = \frac{2\dot{\alpha}'_0}{a_g} \tag{15}$$

where T_1 is the duration of the first half-cycle of the block with initial angular velocity only.

Eq. (14) being the initial velocity of the first interval 0-1, the angular velocities referred to the other intervals of the two schemes are obtained as (with $\nu = \dot{\alpha}_0 / \dot{\alpha}_p$):

$$\begin{aligned}
&\dot{\alpha}'_{1\mathrm{A}} = -\dot{\alpha}_p [C(1+\nu)+2]; \qquad \dot{\alpha}'_{1\mathrm{B}} = -\dot{\alpha}_p C(3+\nu) \\
&\dot{\alpha}'_{2\mathrm{A}} = -\dot{\alpha}_p [C(1+\nu)-2\nu-1]; \quad \dot{\alpha}'_{2\mathrm{B}} = -\dot{\alpha}_p [C(3+\nu)-2\nu-3] \\
&\dot{\alpha}'_{3\mathrm{A}} = \dot{\alpha}_p C\sqrt{C^2(1+\nu)^2 + 2C(1+\nu)+5+4\nu} \\
&\dot{\alpha}'_{3\mathrm{B}} = \dot{\alpha}_p C\sqrt{C^2(3+\nu)^2 - 2C(3+\nu)+5+4\nu}
\end{aligned} \tag{16}$$

In order to compare the two schemes in terms of velocity, let us denote ρ the ratio between the initial velocity of the third interval $\dot{\alpha}'_3$ and $\dot{\alpha}_0$. This ratio for schemes A and B respectively is expressed by the formulations:

$$\begin{aligned}
&\rho_\mathrm{A} = (C/\nu)\sqrt{C^2(1+\nu)^2 + 2C(1+\nu)+(5+4\nu)} \\
&\rho_\mathrm{B} = (C/\nu)\sqrt{C^2(3+\nu)^2 - 2C(3+\nu)+(5+4\nu)}
\end{aligned} \tag{17}$$

Being $C \le 1$, it is easy to recognise from Eqs. (16) that always $\dot{\alpha}'_{1\mathrm{A}} \ge \dot{\alpha}'_{1\mathrm{B}}$, and this is because the pulse acting just after impact is not damped by the coefficient of restitution. Notwithstanding, from Eqs. (17) it results that the final effect of the two schemes in terms of velocity is almost equivalent, as also shown in Fig. 4a. Moreover, the stability of the block, guaranteed by $\rho \le 1$, decreases almost proportionately with the increasing of *C* and therefore of the slenderness implied in *C*. As an example, the results in Fig. 4a show that for $\nu = 2$ only blocks with $C < 0.5$ are stable. This observation highlights again the great importance of the coefficient of restitution in the response analysis of the block, as already discussed above.

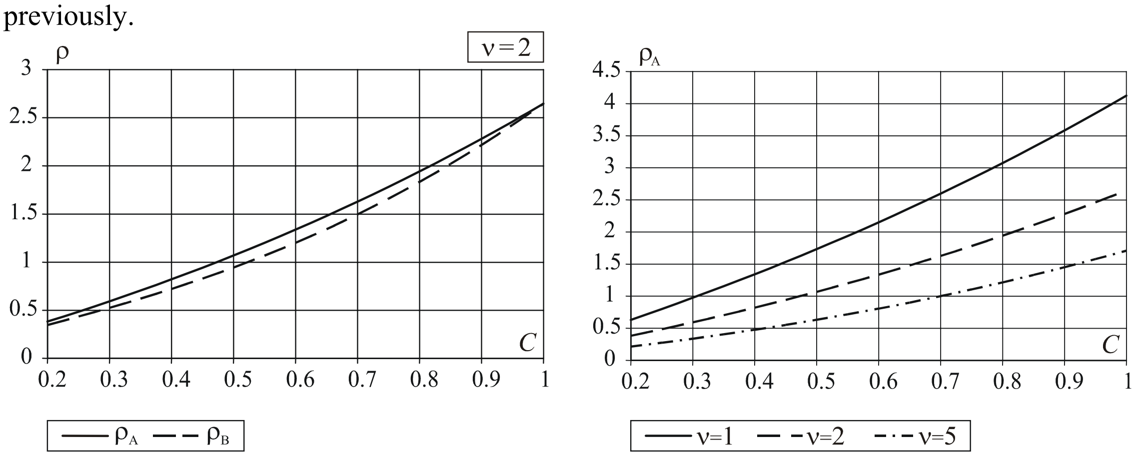
However, considering now scheme A only, the diagram in Fig. 4b shows that the divergence also increases with the increasing of the intensity of pulses and so with reduction in ν. As an example the block with $C = 0.5$ starts to diverge from $\nu = 2$, also in accordance with the diagram in Fig. 4a. However, for $\dot{\alpha}_0 = 0$ it results from Eq. (16) that is always $\dot{\alpha}'_3 > 0$ and so divergence is always present whatever the size of pulses and of *C* considered.

On the other hand, the actual difference between the two presented schemes can be better pointed out in terms of maximum angular displacement. This can occur either over intervals 0-1, 1-2 or 2-3 depending on ν and the initial velocity for each interval $\dot{\alpha}'_i$. In particular, ν influences time T_x for the maximum displacement over the second interval with respect to T_1. In fact, it is easy to verify that $T_x \le T_1$ for values of ν:

$$\nu_\mathrm{A} \ge \frac{C}{2-C}; \qquad \nu_\mathrm{B} \ge \frac{3C-2}{2-C} \tag{18}$$

for schemes A and B, respectively. For values of ν giving $T_x > T_1$, the maximum angular displacement will occur either over intervals 0-1 or 2-3. Given for example $\nu = 1$, from Eq. (18) the maximum rotation will occur over interval 1-2 whatever is *C* (as it is always $C \le 1$), provided that it does not occur over interval 0-1.

Thus, the diagrams in Figs. 5a,b, referred to a block with a given base width ($b = 0.2\mathrm{cm}$) and a velocity due to the pulses $\dot{\alpha}_p = 0.3$, show that the $\alpha_{\max} : \alpha_c$ ratio due to scheme A is always larger than that obtained by scheme B, whatever the values of λ and *C*. This means that pulses acting just after impact are always disadvantageous for stability in terms of displacements even if the final effect in terms of velocity is almost the same for the two schemes, as described previously.

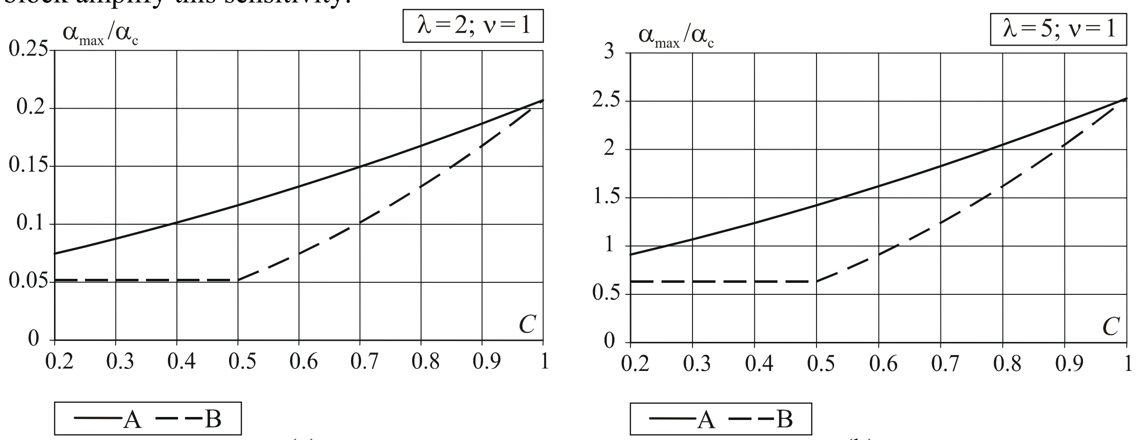


(a) (b)

Figure 4 : Stability parameter versus *C*: (a) for both schemes A and B; (b) for scheme A only.

Finally, it is worth highlighting that both schemes are very disadvantageous for slender blocks. In fact, in Fig. 5b it emerges that scheme A involves the overturning of the block with aspect ratio $\lambda = 5$ already starting from $C \cong 0.25$. Incidentally, the horizontal segments in both graphs in Fig. 5 are referred to the maximum rotation occurring over interval 0-1 for scheme B.

In conclusion, the results herein obtained do confirm the difficulties in defining reliable response spectra for the rigid block dynamic analysis: the rocking response in terms of maximum displacement is extremely sensitive to slight changes in accelerograms both in terms of intensity of single pulses and of instant of their application. The size and slenderness of the block amplify this sensitivity.



(a) (b)

Figure 5 : Maximum angular displacement involved by the two schemes for different aspect ratios.

The combination of the elementary schemes described in this paper has been assumed as sufficiently probable. Nevertheless, this does not represent the most disadvantageous one for the block. In fact, a sequence of self-equilibrated pulses can be predicted as above but which always act just before or just after impact. Therefore, simplified expressions can be derived for progressively increasing velocities and displacements. In this case the maximum angular displacement results only limited by the duration of ground motion which becomes the most meaningful parameter. This matter will be proper developed in a future work.

6 CONCLUSIONS

Many difficulties arise in defining reliable response spectra for rigid free-standing blocks subject to earthquake ground motions and these are well known in the literature.

The line suggested in this paper as an attempt in overcoming these difficulties is focused on a particular aspect that is common to all earthquake ground motions: the self-balanced effect resulting from the null integral of the displacements, velocities and accelerations of the ground over the time of its duration. Therefore, the general earthquake ground motion can be represented by a combination of elementary schemes characterised by zero final velocity and displacement.

Two assembly units, consisting of three acceleration pulses, obeying this property of ground motion are used for the dynamic analysis of a rigid free-standing block by means of a simplified equation of motion. Their effects on the rocking response are examined in terms of angular velocity and displacement. The results obtained do confirm that the stability of the block is extremely sensitive to little changes in accelerograms both in terms of the intensity of single pulses and of the instant of their application. The size and slenderness of the block and the value of the coefficient of restitution amplify this sensitivity.

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