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In-Plane Collapse Behaviour of Masonry Walls with Frictional Resistances and Openings

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ABSTRACT: The paper discusses the limits of validity and applicability of a simplified procedure to assess the lateral capacity of masonry walls loaded in-plane. Limit state analysis is applied to masonry panels simulated as dry assemblage of regular blocks governed by static friction. The friction forces are calculated for various geometric conditions of inclination of cracks, and collapse load factors are computed by use of minimization routines. By consideration of how friction forces develop in relation to the relative shape factors of units and panels an upper and lower thresholds for the possible collapse load factor are identified and by comparison with a discrete element analysis is shown that they do bound the range of existence of the load factor. The procedure is then extended to walls with openings by considering the restraining effect of the spandrels.

1 INTRODUCTION

In the past twenty years, several approaches for modeling masonry structures have been developed to evaluate the seismic vulnerability of historic masonry buildings. Among these, many studies are based on the direct observation of recurrent damage and collapse mechanisms in seismic scenarios and are aimed at calculating the ultimate load factors by means of limit-state analysis. The mechanical model generally chosen for masonry is based on the assumption of an assemblage of dry rigid blocks with frictional behaviour. In this framework, the macro element approach, based on equilibrium equations is a simple way to define the collapse load factor (Lagomarsino 1998, D'Ayala 1999, D'Ayala and Speranza 2003, Lagomarsino and Podestà 2004). Computational methods based on the more accurate discrete element analysis developed in the last decade (Begg and Fishwick 1995, Baggio and Trovalusci 2000, Ferris and Tin-Loi 2001, Orduna and Lourenço 2003, Gilbert et al. 2003, Gilbert et al. 2006). The computational effort is generally quite considerable, especially when a non-linear analysis is developed. The goal of this paper is to demonstrate that using the macro element approach is possible to define a range of existence of the collapse load factor that bounds the solution obtained with the discrete element analysis approach, and hence that it is reasonable and safe to use the simplified method in the context of vulnerability assessment when large number of buildings need to be checked with limited resources. This comparison is herein developed for in-plane loaded masonry walls.

In general for a model considering a constitutive law of pure contact among the blocks, governed by friction, failure will occur for incipient rotation of a portion with respect to the other, for pure sliding of blocks along a crack, or for a combination of the two modes.

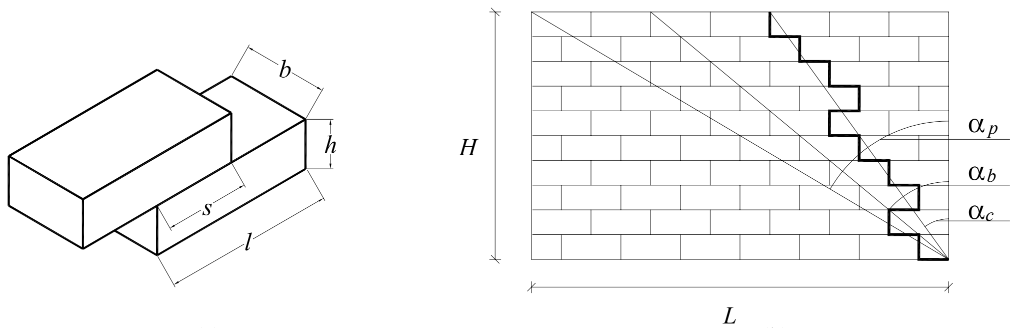
The resistant forces are gravity loads and friction forces on contact surfaces, function of the friction coefficient and the stresses normal to the contact surfaces. With the hypothesis that normal stresses are constantly distributed on contact surfaces at a limit state, it is possible to assume that the friction material is compliant (Casapulla and Jossa 2001).

The calculation of the maximum friction forces along a given crack only depends on the angle of crack α_c (inclination of the crack from the vertical) in relation to the shape ratio of the unit and of the entire wall. (Casapulla and Jossa 2001, D'Ayala and Speranza 2003).

In the following first the friction force F for various conditions of relative values of unit's and wall's shape ratios are derived. Additional loads due to horizontal structures and live loads are also considered. Two classes of collapse load factors are derived using standard minimization methods. Sets of curves of the collapse load factor are compared with solutions obtained using a discrete element analysis. The procedure is extended to walls with openings.

2 FRICTION FORCES ALONG A CRACK

The unit's shape factor is defined as the angle $\alpha_b = \arctan(s/h)$ where s is the length of superposition between two units of length l belonging to two adjacent courses, h is the height of the brick and s/h is the shape ratio of the unit, having assumed $s = l/2$ (Fig. 1a). It is also assumed that all units have same dimensions and are laid with the same staggering. The following considerations are limited to angles of crack $\alpha_c \le \alpha_b$, as mechanisms with a greater angle of cracks are unlikely to occur under the current assumptions.



(a) (b)

Figure 1: (a) Block dimensions; (b) inclinations for the unit and wall shape ratios and for the variable angle of crack.

Therefore, having defined $\alpha_p = \arctan(L/H)$ the shape factor of the wall of width L and height H (Fig. 1b), two cases can be identified as follows:

1) $\alpha_b \le \alpha_p$

In this case, the total friction force is independent of the angle of crack, and will be constant and equal to the product of the weight of trapezoid OABC in Fig. 2 times the friction coefficient:

$$F_1 = \left[pH \tan\alpha_b + \frac{(H \tan\alpha_b + s)H}{2}\gamma \right] bf \tag{1}$$

In Eq. (1), besides the geometric parameters already defined and shown in Figs. 1 and 2, p is a uniformly distributed load for unit of surface, γ is the specific weight of the material and f is the friction coefficient.

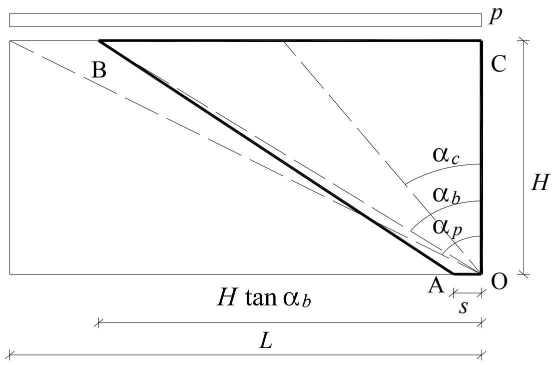


Figure 2 : Masonry wall with $\alpha_b \le \alpha_p$. The solid line shows the equivalent area of wall considered for the calculation of the friction force.

2) $\alpha_b \ge \alpha_p$. When this is the case then two different instances need to be considered:

2a) $\alpha_c \le \alpha_p$ (Fig. 3a)

In this case the portion of wall to be considered is identified by the two areas OABCD and O'A'CD' in Fig. 3a, where lines AB and A'C follow the inclination of the unit's shape ratio. The force is obtained as:

$$F_2 = \left[pL + \left(2H - \frac{L}{\tan\alpha_b} \right) \frac{L-s}{2} \gamma + sH\gamma + p(H\tan\alpha_b - L) + (H\tan\alpha_b - L + s)\left(H - \frac{L}{\tan\alpha_b} \right) \frac{\gamma}{2} \right] bf = \\ = \left[pH\tan\alpha_b + \frac{(H\tan\alpha_b + s)H}{2}\gamma \right] bf = F_1 \tag{2}$$

It should be noted that as $\alpha_c \le \alpha_p$ the number of surfaces crossed by the crack line does not change with α_c and it is always equal to the total number of courses in the panel considered, hence the total friction force is equal to case 1.

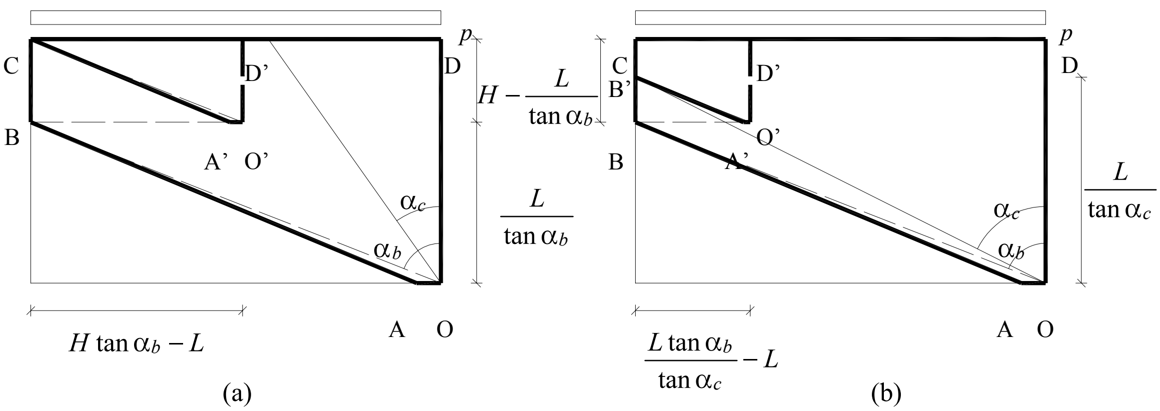


Figure 3 : Masonry wall with $\alpha_b \ge \alpha_p$ and (a) $\alpha_c \le \alpha_p$ and (b) $\alpha_c \ge \alpha_p$ Layout of the portions of wall considered for the calculation of the friction forces.

2b) $\alpha_c \ge \alpha_p$ (Fig. 3b)

In this case the surface areas of wall to be considered are OABCD e O'A'B'CD' and the resultant friction force is:

$$F_3 = \left[\tan\alpha_b (p + H\gamma) - \frac{\gamma}{2}\left(\frac{L\tan\alpha_b}{\tan\alpha_c} - s \right) \right] \frac{L}{\tan\alpha_c} bf \tag{3}$$

As it can be seen F_3 is dependent on α_c.

In summary two equations are needed to define the friction force in relation to the inclination of the crack and the geometry of the problem:

$$\begin{aligned} &\alpha_c \le \min(\alpha_b, \alpha_p) && F = F_1 = F_2 \\ &\alpha_p \le \alpha_c \le \alpha_b && F = F_3 \end{aligned} \tag{4}$$

As the value of the friction force on a given surface varies linearly with the height of wall above it the overall resultant of the friction force for a given crack line will be applied at 1/3 of the height of the cracked part from the base of the panel (see later Figs. 4a, b).

3 COLLAPSE LOAD FACTORS FOR OVERTURNING MECHANISMS

The assumption of a model of masonry with dry blocks would imply that the identification of the collapse mechanism relies on the analysis of all possible relative movements between any two blocks, so as to identify the position of the cracks associated with the minimum collapse load factor. Various formulations of such an approach can be found in literature, and one procedure, developed by the first author with others (Gilbert et al. 2006) will be used here to evaluate some of the results of the procedure presented in this paper. The aim of the work presented here is to develop a relatively simple approach which can allow for fast parametric analysis of many different configurations and eventually the definition of fragility curves for given masonry types and walls opening layout.. The assumption is that the wall reaches collapse due to the formation of a single crack which separates it into two macro blocks and the mechanism of rotation of one block with respect to the other is considered for the standard application of virtual work to the incipient collapse configuration. The virtual work equation can then be solved for the load factor λ and differentiated with respect of the angle of crack in order to find the value of the latter which yields the minimum value of λ.

The present approach is based on the fact that, whatever the type of relative mechanisms between each couple of blocks on opposite face of the critical crack, the set of values formed by the collapse load factors of the real mechanisms in such walls is bound from above by the assumption of full development of the friction force on every contact surface (calculated as F_1 or F_3 dependent on the geometric parameters), and from below by the total absence of friction.

Hence the simplified approach proposed here will not define the exact collapse load factor but a range within which the load factor is contained. For this range to be relevant the assumption that the crack angle identified with this procedure should be equal or very close to the crack angle of the "real" mechanism, should also hold. This will be verified a posteriori by comparing these results with the results of the discrete analysis.

For the geometric condition $\alpha_c \le \min(\alpha_b, \alpha_p)$, as identified in Fig. 4a, the tangent of angle β is chosen as the variable parameter and relates to α_c through the following:

$$\tan\alpha_c = \frac{H\tan\beta + s}{H}. \tag{5}$$

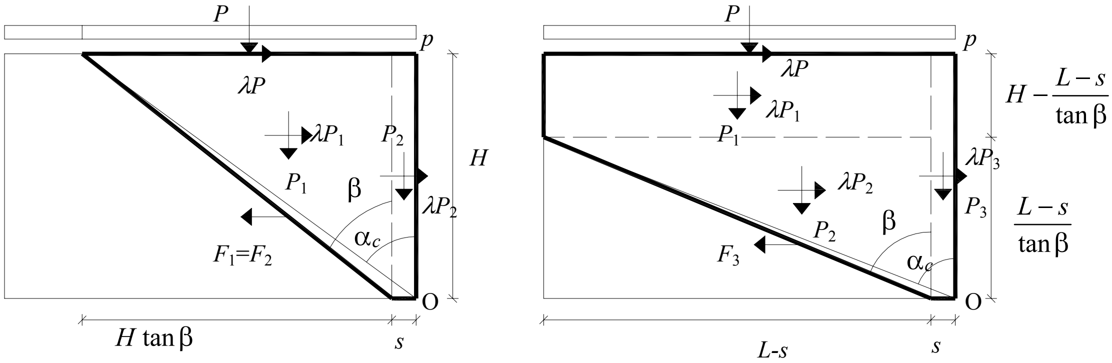
For incipient rotation around point O of the right hand part of the wall, the virtual work equation yields:

$$\lambda_1 PH + \lambda_1 P_1 \frac{2}{3}H + \lambda_1 P_2 \frac{H}{2} = P\frac{H\tan\beta + s}{2} + P_1\left(\frac{H\tan\beta}{3} + s\right) + P_2\frac{s}{2} + F_1\frac{H}{3} \tag{6}$$

Hence, introducing adimensionalised parameters and solving for λ_1 yields

$$\lambda_1 = \frac{(1+3r)\tan^2\beta + 3t(1+2r)\tan\beta + 3t^2(1+r) + 2v}{2(1+3r)\tan\beta + 3t(1+2r)} \tag{7}$$

which can be differentiated with respect to $\tan\beta$.



(a) (b)

Figure 4 : Macro block identified by the angle of crack (a) $\alpha_c \leq \alpha_p$ and (b) $\alpha_c \geq \alpha_p$.

Eq. (7) yields an upper threshold of the real solution, while the lower threshold for the same crack inclination, i.e. the same value of $\tan\beta$, is simply obtained from Eq. (7) setting the term representative of F_1 equal to zero, although this procedure not necessarily identifies the correct associated angle of crack. A similar procedure is followed to devise the collapse load factor for cases of $\alpha_p \leq \alpha_c \leq \alpha_b$ as the geometry in Fig. 4b shows.

The upper threshold of the overturning mechanism is further bound by the coefficient of friction, and a value of the upper threshold equal to the friction coefficient also indicates a failure in pure sliding. In summary, the two bounding classes for variable values of the wall shape ratio can be defined as follows:

$$\begin{aligned}\lambda_{\text{mag}} &= \min(\lambda_{1\text{mag}}, \lambda_{2\text{mag}}, f) \\ \lambda_{\min} &= \min(\lambda_{1\min}, \lambda_{2\min})\end{aligned} \tag{8}$$

The curves corresponding to Eq. (8) are shown in Fig.5.

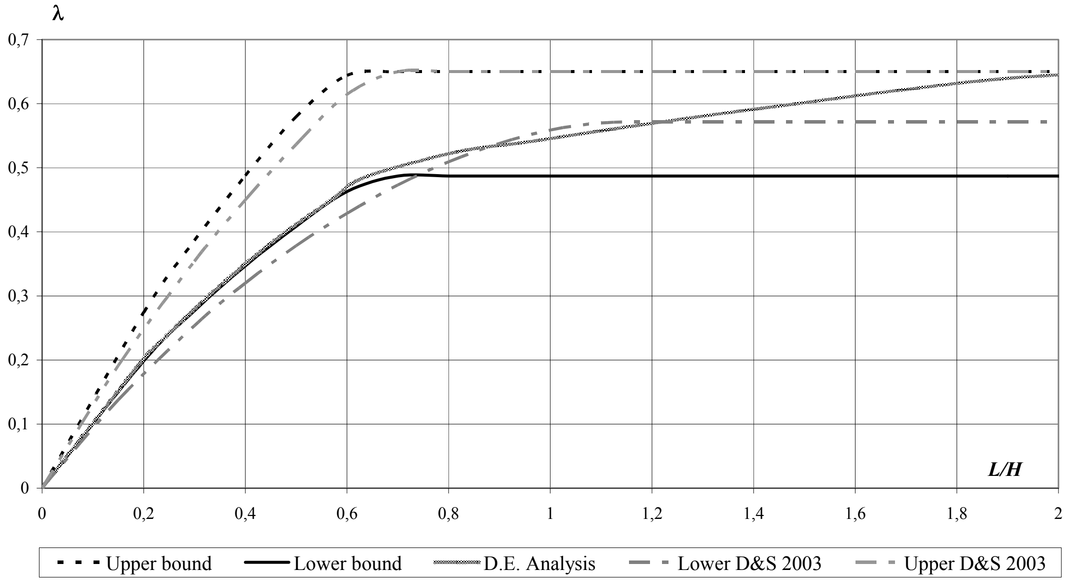


Figure 5 : Comparison for collapse load factor curves obtained with the discrete element analysis and two simplified approaches. Shape ratio s/h =1.143

A comparison with a discrete element analysis has been carried out to prove that the two values considered, lower and upper threshold, do indeed confine the real value. A wall constituted of bricks with an s/h ratio of 1.143 and a coefficient of friction $f = 0.65$, with no addition of weight by horizontal structure was chosen. The results in Fig. 5 are also compared with the upper and lower bounds obtained with the equations contained in D'Ayala and Speranza (2003).

4 IN-PLANE FAILURE OF WALLS WITH OPENINGS

The procedure described in the previous section, aimed at identifying collapse load factor and crack pattern for in-plane failure of walls, is very useful if it can be applied to real case structures, such as walls with openings.

A wall with openings can be subdivided in vertical piers and horizontal spandrels, depending on its number of storeys, while the lateral deformability of the wall, whether governed by flexure or shear, will depend on the relative stiffness of piers and spandrels. This in turn will define the redistribution of the external forces among piers, their crack pattern and their ultimate load factor. Hence the problem still reduces to the definition of a collapse mechanism (overturning or shear), a collapse load factor, and an angle of crack. The result will be dependent on the angle of friction chosen, the geometry of the masonry unit, and the geometry of the wall and its openings. In general in a real wall, opening will have different size and ratios, and will not be distributed regularly, so that there will be differences in geometry and hence capacity, among the piers. The spandrels can also have different height.

In order to extend the previous procedure, the assumptions made are that the wall as a whole is defined by one width and one height; the number of openings at each storey is variable; the size of the opening is constant at each storey and can be obtained as average of the real sizes.

Under these conditions the weakest pier alignment can be identified on the basis of the geometric parameters. However the procedure allows calculating more than one pier alignment in succession if so required.

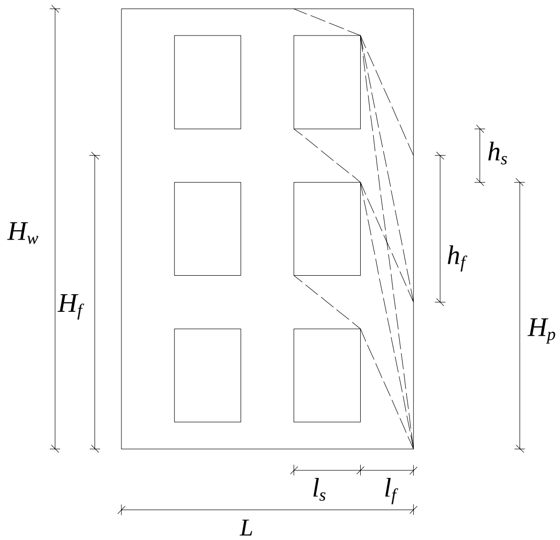


Figure 6 : Possible crack lines in a multistorey pier and spandrel. A uniformly distributed load due to the live loads and horizontal structures is assumed to act at each storey level.

A single pier alignment can be considered as a wall restrained to overturn or slide by the presence of the spandrels. For a wall with more than one storey is not known a priori how many storeys will be involved in the collapse mechanism and hence a variable height of wall needs to be considered in successive iterations to find the portion of active wall which will yield the smallest collapse load factor. This is accommodated by considering that the crack can be initiated at each inter storey height, as shown in Fig. 7. The set of equations developed in the previous section is still applicable. For instance Eq. (6) becomes:

$$\begin{aligned}\lambda_1 P_h H_p + \lambda_1 P_{1h}\frac{2}{3}H_p + \lambda_1 P_{2h}\frac{H_p}{2} = P_h\frac{H_p\tan\beta + s}{2} + P_{1h}\left(\frac{H_p\tan\beta}{3} + s\right) + \\ + P_{2h}\frac{s}{2} + F_j\frac{H_p}{3} + \sum_{i=1}^{n}\left(ih_f - \frac{h_s}{6}\right)F_s\end{aligned} \tag{9}$$

where in the second term appears the contribution due to the friction forces developed in the spandrels. In Eq. (9) h_f and h_s are the inter storey height and the height of the spandrel, respectively, $H_p = (H_f - h_s/2)$ is the variable height of the pier, with $H_f = nh_f$ the variable height of wall considered at each iteration depending on the number of storeys *n*, and hence $h_f \le H_f \le H_w$, with H_w total height of the wall. The vertical loads P_{ih} relate to the width and variable height of pier considered at any given iteration, once the number of storeys over which the crack propagates is chosen. Eq. (9) is only valid for cases with $\alpha_c \le \min(\alpha_b, \alpha_f)$ where $\alpha_f = \arctan(l_f / H_p)$ (see Fig.6 for symbols) and so the friction force F_j within the pier will assume the format of Eq. (1). The friction force F_s for each spandrel is calculated assuming that for the spandrel $\alpha_c \le \alpha_s$ (with $\alpha_s = \arctan(l_s / h_s)$), which is reasonable in most real cases:

$$F_s = \left[ph_s \tan\alpha_b + \frac{(h_s \tan\alpha_b + s)h_s}{2}\gamma \right] bf \tag{10}$$

As it can be seen this has the same format of forces F_1 or F_2 above.

Equation (9) can be solved for λ and minimised by differentiation with respect to $\tan\beta$, as seen before. However, for a wall with multiple storeys, the point of initiation of the crack and its length are not known a priori, like in the previous section, and hence if every possible condition is to be considered, this will result in a set of equations of type (9), each needing to be solved for $\tan\beta$ in order to find the collapse load factor for each possible condition and then chose the smallest and its corresponding mechanism as the one effectively taking place. Results in terms of collapse load factor are compared with the ones obtained with a discrete element procedure for the case of one or two storeys (Fig. 7).

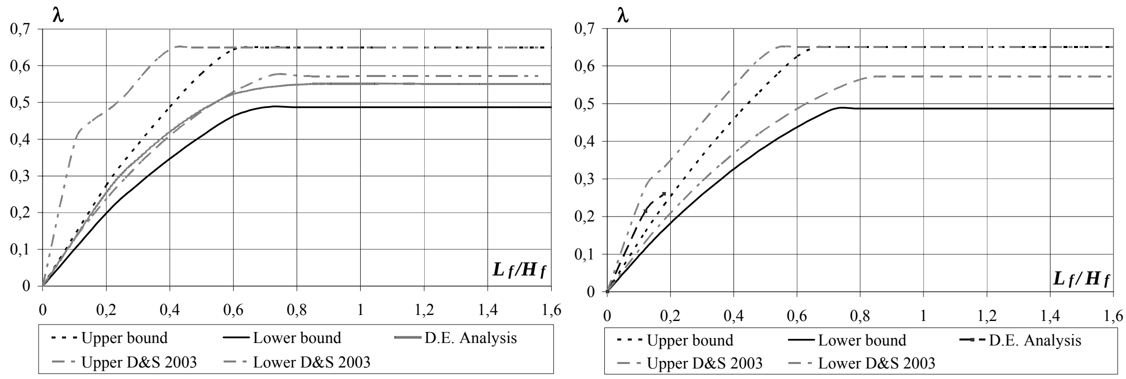


Figure 8 : Comparison for collapse load factor curves obtained with the discrete element analysis and two simplified approaches. Shape ratio s/h =1.143 and case a) one storey, b) two storeys.

5 CONCLUSIONS

The paper presents a simplified method for the calculation of the collapse load factor and the identification of the crack pattern in dry stone masonry walls with regular units and staggering. By using a limit state analysis approach and assuming that the interaction at surfaces is governed by friction, the collapse load factor and the corresponding crack inclination are computed, under the assumption that maximum friction will occur on all contact surfaces crossed by the crack. The three parameters governing the results are the shape ratio of the panel, shape ratio of the units and friction coefficient, which define the uppermost value of the collapse load value. It is shown that by removing the friction, once the crack inclination has been identified, a minimum value of the collapse load factor depending only on the relative ratio of unit to panel shape can also be defined. These two values for a given panel bound the correct solution and this is shown by comparing the output with results obtained with a discrete element analysis. The width of the range of existence is inversely proportional to the shape ratio of the units and increases with increased live loads.

The procedure, modified to take into account the restraining effects of the spandrels, has been applied to single piers in walls with opening. The results presented show that the procedure works well for one storey, where the D.E. analysis solution differs little from the lower bound solution. This is due to the fact that clean crack lines coinciding with the inclination of the unit's shape form in the wall and hence most surface rotate relative to each other without activating friction. In contrast with the previous, in the case of two storeys the simplified analysis is still able to bind the exact solution but the lower bound provides underestimated values of the collapse load factor. This is due to the fact that, in order for the crack to develop, some significant sliding and reverse rotation occur at the intermediate spandrel, which are not correctly simulated with the simplified approach. Further developments will be presented in another work.

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