

Monitoring and Modelling Strategies for the World's Largest Elliptical Dome at Vicoforte

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ABSTRACT: With its internal axes of 37.15 by 24.80 m, the Vicoforte dome is the fifth biggest dome in the world and by far the largest elliptical one. The dome-drum system has suffered since the beginning from significant structural problems, related in part to soil settlements, and, to a large extent, to the daring structural configuration. The paper deals with the monitoring and modelling strategies adopted in recent years following the strengthening intervention operated in 1985 with the application of active tie-bars around the perimeter at the top of the drum.

1 INTRODUCTION

The Sanctuary of Vicoforte, near Mondovì (Cuneo, Italy), is a building of great historical, architectural and structural significance and owes its fame primarily to its great elliptical masonry dome (Figs. 1a-b), which is the fifth biggest in the world (after the Pantheon, Saint Peter and S. Maria del Fiore in Italy, and Gol Gumbaz Mausoleum in India) in terms of overall dimensions. With its 37.15 m and 24.80 m long axes, it is also by far the largest elliptical dome the world over.

Originally conceived by the Duke Charles Emanuel I of Savoy as the mausoleum of the dynasty, the Sanctuary began to be constructed in 1596 by architect Ascanio Vittozzi. Since the earliest stages of construction, the building was adversely affected by important differential settlements of the foundations due to an unfortunate selection of the site, and remained almost abandoned for more than one century. The shallow baroque ribbed dome was erected only in 1732 – after the monument had lost its original role as a mausoleum of the royal family – on top of a slender drum completed in the preceding years.

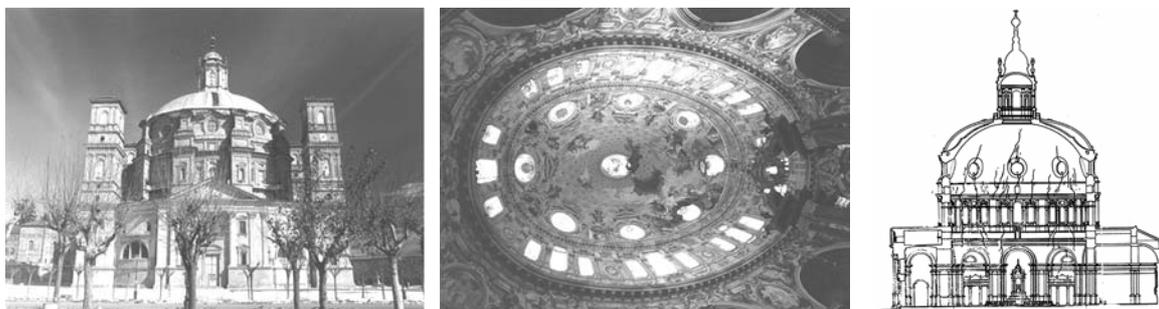


Figure 1 : External (a) and internal (b) view of the Sanctuary. Cracks in the dome (c): north-south section; view of the west side (Garro 1962).

The dome-drum system has suffered since the beginning from significant structural problems, related in part to the additional settlements induced by the new built masses, and, to a large extent, to the daring structural configuration of the system itself. In fact, the extended network of cracks that affects a large portion of the building becomes particularly severe in the dome-drum region (Figs. 1c and 2a), where the cracks are oriented along the typical meridian directions with maximum openings at window locations and an increased density next to the buttresses. Cracks extend over a large portion of the meridians and spread up to the summit of the dome, the largest meridional crack, at the oval window on the west side, almost reaching the base ring of the top skylight lantern. Its maximum width is 4.8 cm, and the sum of crack widths along the perimeter exceeds 40 cm. Important oblique cracks near the openings of the buttresses result from shear stresses induced by the radial thrusting action of the dome (Fig. 2b).

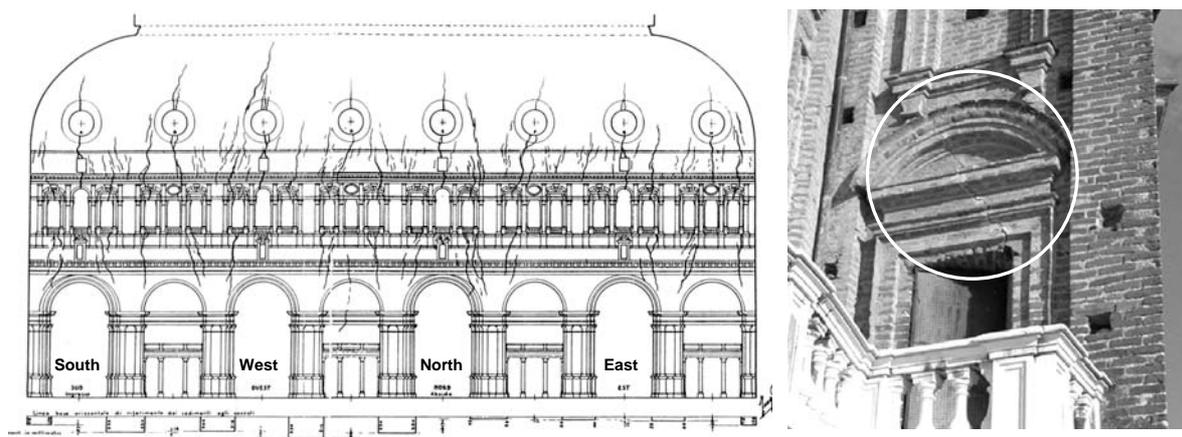


Figure 2 : (a) Crack distribution and total settlements along the perimeter; maximum differential settlements (W vs. NE) ca. 30 cm. (b) Diagonal shear cracks in the buttresses (Garro 1962).

In 1985, concerns over such severe cracking configuration, with particular regard to possible yielding or rupture of the original three levels of circumferential iron rings located in the transition region between dome and drum, prompted the decision to undertake monitoring and strengthening works. A system of 56 active tie-bars, slightly stressed by jacks, was applied (1987) within the masonry at the top of the drum along 14 tangents around the perimeter (Fig. 3), and a complex monitoring system was set up to measure movements of the structure and propagation of cracks, as well as stresses in the reinforcing tie-bars (re-tensioned in 1997).

In recent years a new project was started for a thorough renovation of the monitoring system and the updating of the structural models originally adopted to explore the static configuration and integrity of the construction and define the characteristics of the strengthening system.

The paper presents the general lines and the preliminary results of this new research program aimed to set on new advanced bases the general preservation and protection plans for the monument, in particular in what concerns the structural aspects.

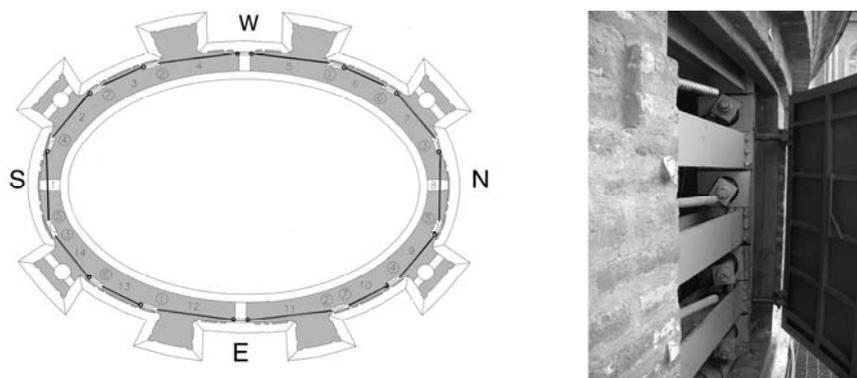


Figure 3 : Modern strengthening system.

2 THE NEW MONITORING SYSTEM

The new monitoring system is a technologically upgraded version of the instrumentation installed during the dome strengthening works of 1985-87. The goal of the monitoring process is to study the evolution of damage conditions and to identify the phenomena that caused them. The instruments used can be classified into two groups, i.e., tools for the measurement of strains, cracks and states of stress in the structure, and tools for measuring the boundary and ambient conditions. The first group includes 10 strain gauges applied to the main cracks at the impost of the dome and to the biggest west crack at the extrados; 20 horizontal pressure cells, for the determination of stress variations in the eight pillars and the buttresses; another cell, arranged vertically near the top of the dome above the main meridian crack, is used to determine the circumferential compression stress; 56 load cells are situated at the ends of the active cerclage bars to control their stress conditions; 2 pairs of stainless and invar wire gauges at the impost of the dome are used to assess the overall geometry of the structure, and 12 nails are provided for additional manual measurements of convergence. The second group includes: 1 hygrometer and 25 mechanical thermometers, 17 of them located inside the walls and 8 externally; 3 piezometric electric cells for monitoring the ground water level; 1 hydrometer for monitoring the water level in the drainage pipe collection tank. The entire monitoring system is managed by a PC working as a central acquisition system that enables the data obtained to be processed and transmitted in remote mode.

A dynamic monitoring system, presently under consideration, consisting of 10 tri-axial accelerometers to be placed along the main axes at significant points will enable to record the structural response to seismic events, even low-magnitude ones, and to ambient micro-tremors. It will provide a data-base for future analyses in the seismic domain and for the procedures of validation and updating of the numerical models.

Some considerations can already be formulated from the data collected during the first 17 months. Convergence measurements and stress values in the active cerclage bars evidence that seasonal temperature variations constitute the primary cause of variability of the data. In particular, thermal expansion phenomena during the hottest periods are accompanied by a considerable reduction in the stress level in the active tie-bars. Some strain gauges detect a slight increment in the dimensions of some cracks, which, however, does not exceed one tenth of mm per year. Aside from these normal fluctuations, the measurements recorded by the various instruments can be presently rated as substantially stable. When the time horizon of the monitoring process will be long enough, it will be possible to explore the existence of a correlation between structural parameters and the variations in the boundary and ambient conditions.

3 DIAGNOSTIC INVESTIGATIONS

Diagnostic investigations were performed in the past years to characterise the foundation soil and the masonry structure. Geotechnical studies were conducted in two stages: the first campaign, carried out in 1976, consisted of 13 geotechnical surveys followed by laboratory tests on undisturbed soil samples. The surveys covered a vast area, inside and outside the Sanctuary, and were accompanied by the drilling of 29 bore holes at the base of the masonry structure through the foundations. As revealed by these investigations, a layer of marl declining in the south-west direction is situated from 3 to 9 m below the surface level. The foundation structures in the south-west zone of the monument, especially those of the pillars supporting the dome-drum system, rest on a silt-clay layer reaching a maximum thickness of 3 m (Fig. 4a).

The second campaign, carried out in 2004, coincided with the installation of the 3 electrical piezometric cells of the new monitoring system. It included 4 geotechnical surveys, each of which collected 5-6 undisturbed soil samples that were subjected to tests; 2 cross-hole seismic geophysical tests were performed in the bore holes for the determination of the velocity profiles of shear and compressive waves (S and P waves, respectively) in stratified media (Figs. 4a-c). On the basis of the available field and laboratory investigations a geotechnical characterization of the subsoil was carried out. The bedrock formation has on average a very high strength and very low deformability, and therefore, from the geotechnical point of view, it can be considered as a rigid support for the upper formation. The latter is slightly over-consolidated, mainly due to

seasonal oscillations of the groundwater, desiccation phenomena and, possibly, superficial loads applied and removed locally within historical times.

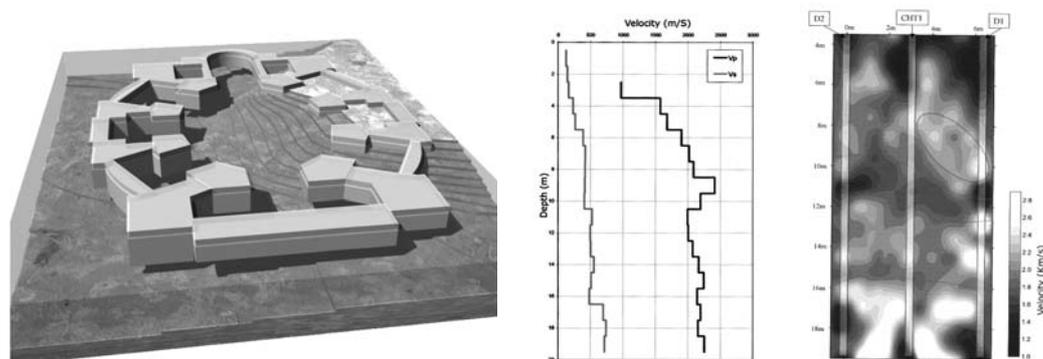


Figure 4 : (a) Axonometric view of foundations and foundation layers. Cross-hole test: (b) P-wave and S-wave velocity profile; (c) tomographic data elaboration.

As for the mechanical properties of the masonry, the following average data were obtained from compressive tests on masonry blocks taken from the building (Barosso 1979):

- Young's modulus: 1500 N/mm^2 ,
- compression strength: 3 N/mm^2 .

In 2004, non-destructive tests (NDT) were performed to evaluate the compressive strength and Young's modulus of the mortar and bricks by means of a scratch tester (scratch width) and a Windsor Pin System (penetration resistance). These tests were accompanied by compression tests on small brick and mortar cylinders ($\phi 33 \text{ mm} \times 50 \text{ mm}$) performed according to Japanese Standards JIS A 1108. Good correlations were found between the outcomes of two NDT tests and between these tests and the compression tests. In particular, as scratch width gets larger, penetration resistance and compressive strength decrease. On the other hand, as penetration resistance increases, compressive strength becomes greater. The relationship between compressive strength and Young's modulus is fairly linear (Aoki et al., 2004b).

4 MODELING STRATEGIES

Different analyses were performed with the finite elements method, using various material and structural models with different levels of sophistication, focusing on different aspects of the structural response. The results obtained from such models were also compared with the results of a limit state analysis for a rigid no-tension material.

The first step was the construction of a solid model (Fig. 5) reproducing as faithfully as possible the complex geometric characteristics of the structure of the whole monument, to be used as the basic term of reference for the construction of models having, in some cases, a simpler geometry or a more limited extension, and hence suitable for more complex investigations, in particular in terms of constitutive laws of the materials and of the domain of analysis. The reference model was created on the basis of a general structural topographic survey, completed in 2004, extended to the entire monument with the exception of the dome. The geometry of the dome was defined on the basis of the 1976 photogrammetric survey of its interior surface and of direct inspections for the determination of its thickness. .

4.1 Linear-elastic FEM analysis of the entire undamaged building

Based on the reference solid model, a FEM model of the entire monument was produced, consisting of ca 10000 20-node brick elements and 580 8-node shell elements, with a total of ca 193000 degrees of freedom. As a first step, a series of elastic analyses of the entire undamaged building were performed, with the aim of obtaining a preliminary rough description of the overall structural behaviour. Different analyses were undertaken in order to estimate the influence of different structural elements such as the old annular reinforcement and the recent strengthening bars, or the lantern at the top of the dome (Spadafora 2002, Aoki et al. 2004a).

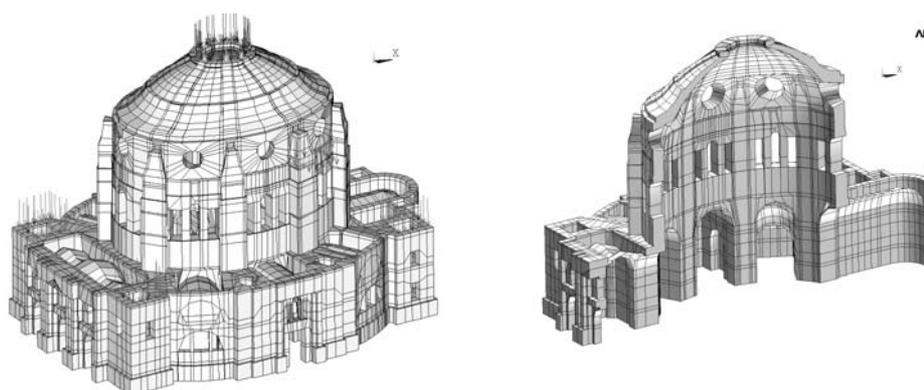


Figure 5 : Solid FEM reference model.

The results in terms of principal stresses show rather clearly that the skeleton of the building is essentially made up of the eight pillars, the buttresses above them and the ribbed segments of the dome. These structural components are responsible for transferring the dead weight of the structure to the ground; at the same time they serve as stiffening elements that counter the radial thrusts of the dome.

Interesting results are obtained in what concerns the states of strain and stress in the dome-drum system, the general response of the structure depicted by the model resulting in fairly good agreement with the experimental observations and the observed damages. In particular, significant tensile stresses are produced in the model at the dome impost level. These stresses are strong enough to generate sub-vertical cracks, especially in the proximity of the oval windows (higher values at E and W on the minor axis), i.e., at the weakest points in the dome. The upper part of the drum appears to be pushed outwards by the radial thrusts of the dome, which causes the widening of the dome oval at the impost level, especially in the section along the minor axis where a narrowing of the oval at the base of the drum is also detected. The original iron rings (locations in Fig. 9) reduce both hoop stresses and widening of the oval (Fig. 6a-b), resulting particularly effective, as confirmed by the limit analysis reported at section 4.4. Their integrity in the real structure is currently being investigated. The distribution of the principal stresses in the buttresses denotes the presence of a shear action consequent to the outward deformation of the drum, justifying the observed diagonal cracks in the structure.

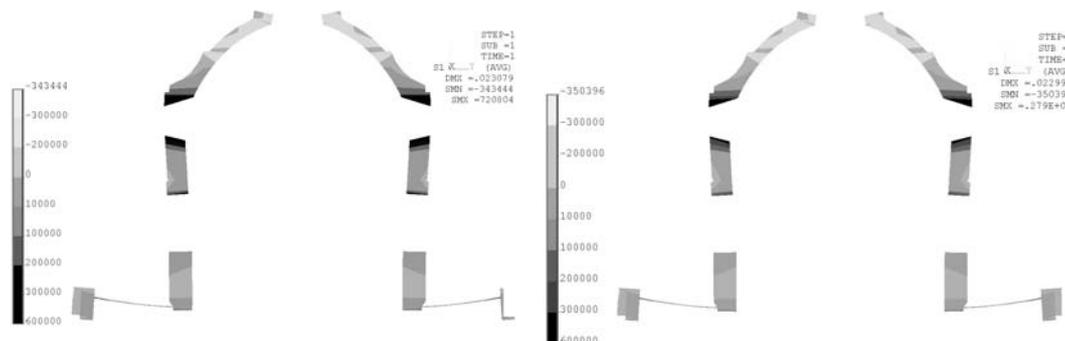


Figure 6 : Deformed shapes and maximum principal stresses of dome and drum at the cross section along minor axis, without (a) and with (b) original iron rings.

The FEM model also evidences the effect of the weight of the skylight lantern on the state of stress of the dome. This load enhances the circumferential tensile stresses at the impost of the dome and the tri-axial compressive stresses around to top crown circle of the dome.

A shell model limited to the dome-drum system was also produced. This model, characterised by a reduced computation burden, is essentially conceived for a use with more sophisticated material constitutive laws of anisotropic non-linear type, as indicated at section 4.3. On the other hand, the model incorporates a more refined description of some structural elements with respect to the general model, taking into account e. g. the cylindrical cavities present in the four buttresses adjacent to the major axis, and subdividing the dome-drum system in its main struc-

tural components, in order to be able to distinguish the individual behaviour of each. In a preliminary stage also this second model was associated to a pure elastic analysis. The results substantially confirm the outputs of the elastic analysis based on the general solid model.

4.2 Linear-elastic FEM analysis of the damaged dome-drum system

The principal feature of this model consists in the simulation of the main cracks observed in the structure through the creation of geometric discontinuities at the eight most important lesions. An effort was also done to make the numerical model as “light” as possible during the computation stage, and as “legible” as possible at the stage of interpretation, adopting a convenient blend of shell and beam elements, having regard to the specific geometric and structural characteristics of the portions under consideration (shell elements for the dome and bearing walls with curved geometry, beam elements for arches and pillars). Doing so, a special attention was devoted to some specific zones of architectural-structural relevance, as e. g. the transition region from the drum to the dome characterized by a reverse arch connecting the haunches of the dome to the drum. On the other hand the structures below the drum were represented rather schematically. The original iron rings were modeled as one-dimensional elastic elements; the aim of the investigation being the exploration of the static configuration of the monument before the recent strengthening intervention, the influence of modern tie-bars was disregarded. The analysis was performed in the linear elastic domain (D’Addato 2005).

The analysis of the results shows that the principal compressive stresses of the vault develop along the meridians and become tangential to the lantern impost oval, where the compressive stresses are tri-axial. The principal tensile stresses develop along the parallels and circumvent the cracks by assuming a 45° slant, and then concentrate at the bottom of the lesion, generating the “notch effect”, i.e., the tendency of the lesion to propagate through the undisturbed masonry (Fig 7a). The tensile stresses that have generated the cracks reappear in the segments between one main crack and another; the state of stress supplied by the model is in good agreement with the actual cracking configuration.

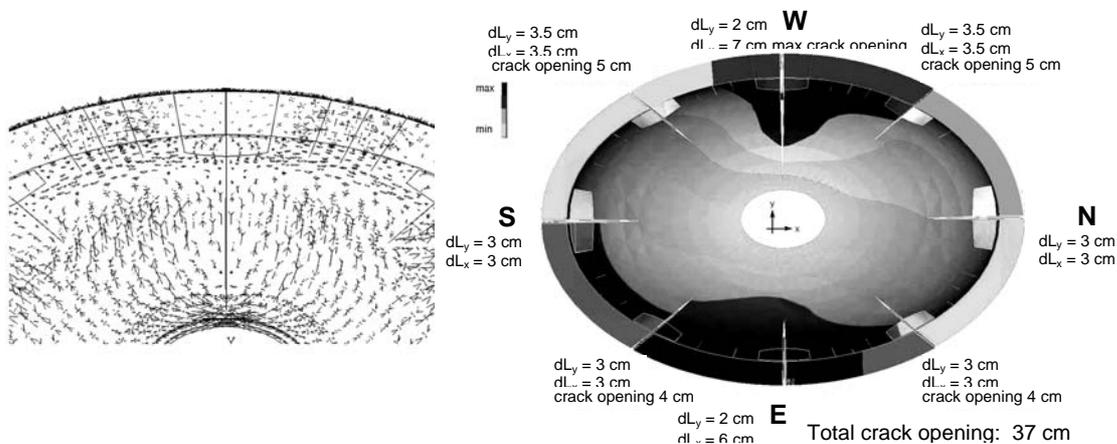


Figure 7 : Principal stresses (a) and deformations (b) of the cracked dome.

One interesting output of the analysis is represented by the estimation of the opening of the pre-established cracks at the base of the dome. The biggest width, of 7 cm, is associated with the crack having the longest vertical development located at the oval window on the west side, while the values of secondary cracks range from ca 4 to 5 cm (Fig. 7b). These values should not be compared with the maximum amplitudes of the cracks observed at the same level in the real structure (4.8 cm for the west crack). Rather, the comparison should be performed with the sum of the crack openings registered in the corresponding sectors of the real structure, which range from 2.9 cm to 8.2 cm, with an average of 5.2 cm for the 8 sectors corresponding to the 8 oval windows.

4.3 Damage FEM analysis of the dome-drum system

The model concerns only the dome-drum system. An anisotropic damage model for the masonry under plane stress (Calderini and Lagomarsino 2006) is employed. It considers different in-plane damage mechanisms, involving both mortar joints and blocks. The model is implemented in 4-nodes non-linear shell elements with transverse shear strain capability (ANSYS). Thus, except for the original tie-rods, the entire structure is modelled by means of shell elements. The description of the masonry geometrical pattern is obtained through the orientation of the local coordinate system of the elements. In particular, the surface of the dome-drum system is modelled considering its internal system of relieving arches.

Two different analyses were performed: in the first one, only gravity loads were considered; in the second, the additional effects of the foundation differential settlements were studied. After a reasonable estimate of the amount of settlements occurred subsequent to the completion of the dome, on the basis of historical notes and of recent surveys, the corresponding values were applied as vertical displacements at the base of the drum. The non linear damage model, while confirming in general the structural interpretations derived from the other models, evidences some specific features associated with the non-linearity and anisotropy of the masonry.

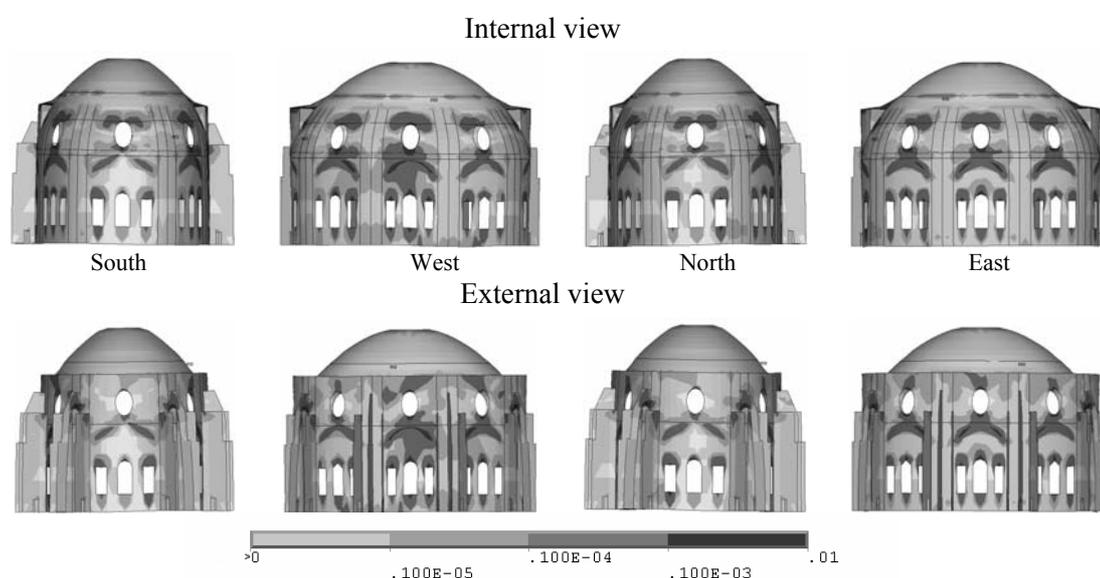


Figure 8 : Non linear damage analysis: principal inelastic strains considering foundations settlements.

In particular, it is worth noting that, if only gravity loads are considered, inelastic strains and failures denoting the formation of cracks are concentrated at the base of the dome, in correspondence of the oval windows and of the relieving arches, and in the drum above and below the triple windows (more intensely, in all cases, along the major sides of the monument perpendicular to its minor axis). In fact, in these regions, besides the concentration of the horizontal tensile hoop stresses acting orthogonally to the masonry head joints, due to the presence of the openings as already noted in the linear elastic FEM analysis of the undamaged building, the meridional compressive stresses on the bed joints of the masonry are very low, and thus the friction contribution to the strength with respect to horizontal stresses is limited. One other cause favouring damage mechanisms is represented by the presence of the arch rings of the oval openings and of the relieving arches above the triple windows of the drum, implying the presence of mortar bed joints nearly orthogonal, in the crown regions, or significantly inclined with respect to the circumferential tensile stresses acting on the structure.

If the differential settlements of the foundations are considered as well, although the role of the various structural elements and the damage mechanisms remain basically the same, the damage state appears to be more severe and becomes asymmetric. The West side of the dome, characterised by the higher values of differential settlements, appears to be significantly more damaged than the East side, and the same applies, with less difference, to the North and South faces,

respectively (Fig. 8), with a close relation to the crack distribution observed on the real structure (Figs. 1c and 2a). A detailed description of the non linear damage analysis can be found in the parallel paper by Calderini et al. (2006). Further researches on soil-structure interaction in an historical perspective and with reference to the recent campaign of geotechnical investigations are presently under way.

4.4 Limit state analysis of the dome-drum system

A synthetic approximate estimation of the safety conditions of the dome-drum system may be obtained by limit state analysis introducing the simplifying assumption that considers masonry as a rigid material with no resistance to tensile stresses. The principles of this type of analyses based on extremum theorems of the theory of plasticity (static or kinematic approach) and the criteria for their application to masonry domes have been well established by Heyman (1977, 1988) and Oppenheim et al. (1989).

Although affected by some fundamental conceptual and/or computational errors that have undermined the validity of the results obtained at the time, these approaches were already adopted in the 18th century debate on the structural integrity of St. Peter's dome, namely by the *Tre Mattematici* in 1742 (kinematical approach applied to the dome-drum system with errors in the definition of the most efficient rigid body collapse mechanism and in the related computations) and in 1748 by Poleni (static approach erroneously limited to the sole dome).

The limit state analysis for the rigid no-tension material rests on the concept that masonry structures achieve their structural integrity through their geometry by providing, with adequate geometrical margins, a funicular load path to the ground without any contribution of tension stress fields (static approach), or, equivalently, by providing an adequate degree of stability for the most efficient rigid body collapse mechanism (kinematical approach). Rather seldom these conditions are reached in the case of domes, if the consideration is limited to the pure masonry. In fact, in most cases the obtainment of a safe funicular path, or equivalently of a stable rigid body configuration, is not guaranteed by the slender geometry of the dome-drum systems, affected by high symbolic values, normally found in the architectures of the Christian era. Rather, structural integrity generally requires the contribution of the tension stress field provided by the system of iron cerclage rings, and of the buttressing architectural elements.

The problem can be reduced to a two dimensional case by conservatively admitting, coherently with the assumption of absence of tensile strength of the masonry, the full development of meridional cracks evenly spaced along the perimeter, and consequently dividing the dome and the drum into *lunes* (vertical slices equivalent to half arches with varying cross section for the dome portion) of limited angular width. Doing so, it is apparent that the stabilizing contribution of buttresses is ignored, the evaluation of the structural stability being restricted to the isolated lune element. On the other way, the tensile contribution of the circumferential iron rings can be accounted for; if the kinematical approach is adopted, this contribution results from the plastic dissipation that takes place because of the circumferential expansion the rings are subjected to during the development of the mechanism.

This last approach was used in the case of Vicoforte dome-drum system extending its domain of application from the domes of revolution, to which it was originally applied, to the case of an elliptical dome (Reffo 2002). The two extreme cases of lunes ideally cut by meridional planes in correspondence of the minor and major axes of the base ellipse were consequently considered, taking into account the voids produced in the masonry mass by the large windows in the drum and the dome. The presence of the skylight lantern was considered only in terms of applied weight at the top of the lune with reference to its angular width, disregarding by way of approximation the spatial interaction between sectors of the dome of different deformability characterising the elliptical configuration. The approach consists of trying to identify the most probable collapse mechanism for each of the two extreme lunes and the relative safety levels through the principle of virtual works.

The typical mechanism through which the dome-drum half-arches reach collapse envisages the formation of three plastic hinges, arranged alternatively at the extrados and the intrados, and the presence of an uncracked zone at the top of the dome, which translates in the vertical direction only. In the case of Vicoforte dome the most efficient mechanism was found by a specifically programmed iterative procedure determining, by trial and error, the optimum position of

the plastic hinges. Lunes of the dome and drum of 1° of amplitude were considered and they were divided in vertical thin finite elements.

The safety of the pure masonry lunes was first explored, ignoring the presence of the iron cerclage rings. It was found that the stability is not guaranteed, the resisting virtual works being significantly inferior to the pushing works for the most efficient mechanisms, especially for the slender and flat lune along the major axis. This is due to the particular slenderness of the drum, and, above all, to the excessive voids produced in the resisting masonry masses by the large windows of the drum and the dome. The application of the static approach provides a check of these results showing the impossibility to locate any thrust line within the geometrical boundaries of the two lunes. It is therefore shown that if the attention is conservatively focused on the sections without buttresses (extending over more than 70% of the total development) the stability can be guaranteed only accounting for the contribution of a cerclage system. If the presence of the original iron rings (total cross section of 140 cm^2) in their well selected favorable positions is considered (Fig. 9; $a = b = 56 \text{ cm}^2$, $c = 28 \text{ cm}^2$), adopting once again the kinematical approach and assuming a yield strength for the iron of ca. 170 Mpa, together with an assumption of sufficient ductility, adequate safety margins can on the contrary be restored, the minimum ratio L_r/L_p between resisting and pushing virtual works amounting to 1.49 for the section along the major axis and to 2.63 along the minor axis (Fig. 9). Fig. 9 allows also appreciating the beneficial stabilizing effects of the masonry masses of the tiburio below the reverse arch marking the extrados of the dome, as they are located at the left hand side of the centers C_i of instantaneous rotation of the dome element. However, it should be noted that in the real structure they are affected by the presence of some voids, not considered in the calculations.

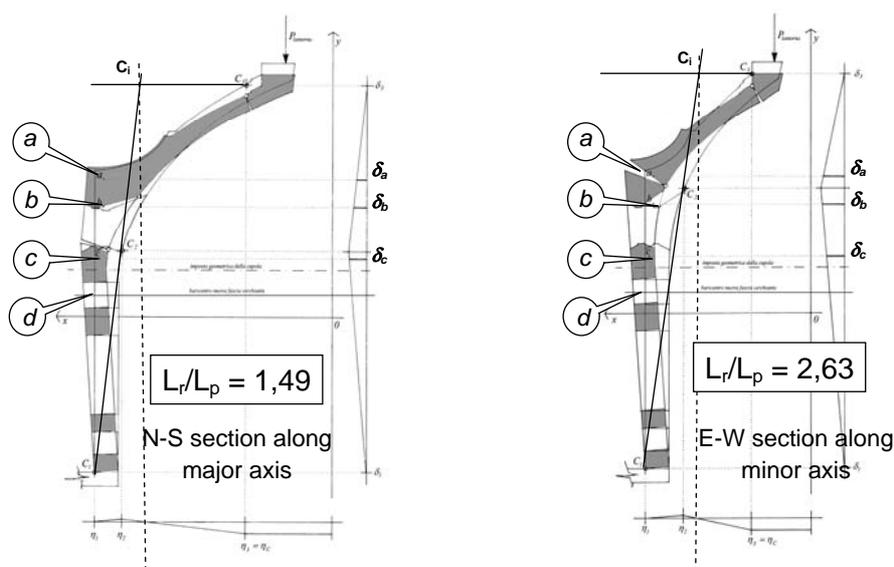


Figure 9 : Determination of the minimum ratio L_r/L_p between resisting and pushing virtual works.

Although these results were obtained through the schematic and partially conservative model of the rigid no-tension material for masonry, the emphasis they place on the role of the original cerclage brings further justification to the investigations currently under way on their structural integrity and prompts a positive evaluation of the strengthening intervention performed in 1985-87 through the application in d of the active tie-bars (Fig. 3) along the perimeter, and of the re-tensioning operated in 1997, which appear as appropriate safeguard measures.

5 CONCLUSIONS

From a comparison of the results of the different mathematical models the following conclusions can be reached:

- the primary causes of the structural damage that has affected the dome-drum system of the Sanctuary of Vicoforte are its daring structural configuration and the differential settlements of the foundations occurred after the completion of the dome;

- the first cause generated a network of meridional cracks, distributed over the entire perimeter, accompanied by diagonal shear cracks in the buttresses; the second cause resulted in more significant damages, enhancing the cracks due to the first cause in the zones most affected by differential settlements;
- the structural integrity of the dome and the drum relies consistently on the confining effect of a cerclage system.

The combination of these results with the data obtained from diagnostic investigations and monitoring allows to formulate hypotheses and outline possible scenarios and strategies regarding the structural preservation conditions of the monument. It permits to identify phenomena, structural components and indicators that have to be monitored to be able to evaluate the evolution of the structural integrity conditions. Due to the central role played by cerclage, monitoring of the new system of tie bars is mandatory, while significant information may derive from investigations on the integrity of the original iron rings. From a comparison between the current monitored data with those recorded before the 1985-1997 strengthening intervention, it seems clear that this intervention proved effective. On the other hand, geognostic surveys describe a foundation soil so well consolidated that, assuming the hydro-geological conditions remain stable, the occurrence of further settlements of the foundations, and hence further damage to the masonry, can be ruled out. Dynamic monitoring, presently under study, will offer valuable opportunities for the validation of current models and the seismic assessment of the monument.

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