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Event-by-Event Strategies for Modeling Amsterdam Masonry Structures

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ABSTRACT: In Amsterdam, the North-South metro-link is being constructed underneath the urban historical city centre. This implies that historical masonry facades and monuments face the risk of being damaged due to the settlements imposed by the subsurface building activities. The situation is critical because we have the combination of very soft soil, timber pile foundations and brittle un-reinforced masonry. A representative Amsterdam masonry façade from 18-19th century is analysed. A crucial aspect of large-scale masonry structural behaviour is that cracks may initiate, gradually propagate and then suddenly snap to a free surface, thus the final fracture is localized and often highly brittle. Existing Newton-Raphson based non-linear finite element techniques face difficulties in modelling this, whilst softening results in negative stiffness, sharp snap-backs and bifurcations. In this paper, the sequentially linear analysis is proposed as an alternative approach. The paper illustrates the potential of numerical models to gain insight in the risks and in the need for mitigating measures to preserve the architectural heritage.

1 INTRODUCTION

Ground movements may considerably affect the serviceability and safety of structures. In particular the behaviour of large-scale historical masonry buildings subjected to settlements induced by tunnelling activity has been analysed in this paper. The numerical difficulties encountered in modelling these full-scale buildings have been highlighted in previous works (Rots 2000, Boonpichetvong and Rots 2003) and are summarized in this paper where the case study of a representative Amsterdam masonry façades dated from 18-19th century is analysed. Analytical and empirical approaches are available for evaluating the cracking damage of surrounding structures subjected to settlement. Potts and Addenbrooke (1996) employed elastic beam elements to represent an overlying structure on a soil continuum. However, due to many provided assumptions, the direct application of these simplified methods in very sensitive cases such as fragile historical masonry buildings raise the need for more detailed analysis. For quasi-brittle materials like concrete and masonry especially, the introduction of tension-softening models, fracture mechanics and nonlinear finite element (NLFE) methods have improved the possibilities for predicting cracks in structures. Several NLFE analyses were performed in the past with the DIANA finite element code in order to investigate how mesh refinement, element typology, constitutive law, convergence method and loading can affect the façade response (Boonpichetvong and Rots 2003). The main remark arising from the results of these studies is that the façade response strongly depends on the adopted constitutive models and convergence methods. These previous investigations, revealed a very brittle snap-back response accociated with full fracture of the façade, which cannot be caught by smeared crack models. When the crack pattern develops the structural response obtained with smeared crack model shows a snap-through, due to the significant unbalance between high elastic energy stored in the façade and low fracture energy of masonry. For this reason the façade was re-analysed with discrete crack models and arc-length control technique. In this case snap-back response can be predicted, but practical use of this numerical procedure requires a lot of user's experience. In this paper it will be shown that the strong and sharp snap-back response, due to settlement, can be automatically detected by a sequentially linear analysis method. Sequentially linear analysis is an alternative to non-linear finite element analysis of structures when bifurcation, snap-back or divergence problems arise. The incremental-iterative procedure is replaced by a sequence of linear finite element analyses performed by decreasing Young's modulus and tensile strength at the integration point of elements, when their damage increases.

2 OVERALL EVENT-BY-EVENT PROCEDURE

The locally brittle snap-type response of many structures inspired the idea to capture these brittle events directly rather than trying to iterate around them in a Newton-Raphson scheme. A critical event is traced and subsequently a secant restart is made from the origin for tracing the next critical event. Hence, the procedure is sequential rather than incremental. The sequence of critical events governs the load-displacement response. To this aim, the softening diagram is replaced by a saw-tooth curve and linear analyses are carried out sequentially (Rots 2001). The global procedure is as follows. The structure is discretized using standard elastic continuum elements. Young's modulus, Poisson's ratio and initial strength are assigned to the elements. Subsequently, the following steps are sequentially carried out:

- Add the external load as a unit load.
- Perform a linear elastic analysis.
- Extract the 'critical element' from the results. The 'critical element' is the element for which the stress level divided by its current strength is the highest in the whole structure.
- Calculate the ratio between the strength and the stress level in the critical element: this ratio provides the 'global load factor'. The present solution step is obtained rescaling the 'unit load elastic solution' times the 'global load factor'.
- Increase the damage in the critical element by reducing its stiffness and strength, i.e. Young's modulus E_i and tensile strength f_{ti}, according to a saw-tooth constitutive law as described in the next section.
- Repeat the previous steps for the new configuration, i.e. re-run a linear analysis for the structure in which E_i and f_{ti} of the previous critical element have been reduced. Trace the next critical saw-tooth and element, repeat this process till the damage has spread into the structure to the desired level.

2.1 Saw-tooth diagram for masonry in tension

The way in which the stiffness and strength of the critical elements are progressively reduced constitutes the essence of the model. In other words, it is necessary to provide a saw-tooth approximation of the constitutive stress-strain relation. A very rough method would be to reduce E to zero immediately after the first, initial strength is reached. This elastic perfectly brittle approach, however, is likely to be mesh dependent as it will not yield the correct energy consumption upon mesh refinement (Ba□ant and Cedolin 1979). In the sequentially linear procedure the consecutive strength and stiffness reduction is based upon the concept of tensile strain softening, which is fairly accepted in the field of fracture mechanics. Different approaches have been investigated in previous works (Rots and Invernizzi 2004), in the present paper a new generalized tooth size approach, already adopted for concrete (Rots et al. 2006), is presented and applied to masonry. The tensile softening stress-strain curve is defined by Young's modulus E, the tensile strength f_t, the shape of the diagram, and the area under the diagram, Fig. 1a. The area under the diagram represents the fracture energy G_f divided by the crack band width h, which is a discretisation parameter associated with the size, orientation and integration scheme of the finite element. Please note that the softening diagram is adopted only as a 'mother' or envelope curve that determines the consecutive strength reduction in sequentially linear analysis. For a linear softening diagram, the ultimate strain ε_u of the diagram reads:

$$\varepsilon_u = \frac{2G_f}{f_t h} \quad , \tag{1}$$

and the tangent to the tensile stress-strain softening curve is, see Fig. 1a:

$$D = \frac{f_t}{\varepsilon_u - \frac{f_t}{E}}, \tag{2}$$

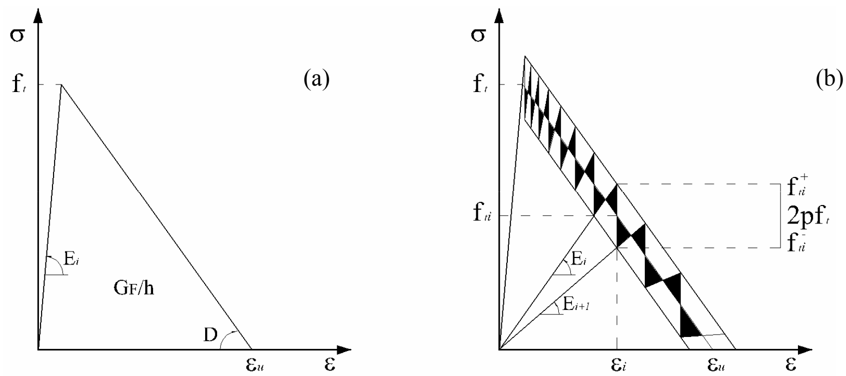


Figure 1: (a) The stress-total strain curve for linear softening (b) and the consistent saw-tooth diagram.

In a sequentially linear strategy, the softening diagram can be imitated by consecutively reducing Young's modulus as well as the strength. As shown in Fig. 1b, a strength range is set, as a percentage of the maximum tensile strength, *p*. As a result, a band or 'strip' is introduced into the softening diagram, delimited by two curves parallel and equidistant to the original diagram. The number of required teeth need to reach the complete damage of the element (*N*) and the values of Young's modulus (E_i) and tensile strength (f_{ti}) at the current stage *i* in the saw-tooth diagram are automatically obtained as values depending on this strength range, chosen by the user. We can interpret this as a pre-set 'ripple curve' on top of the base curve, so if *i* denotes the current stage in the saw-tooth diagram, the following relation can be imposed:

$$f_{ti}^- = f_{ti}^+ - 2pf_t \,. \tag{3}$$

The reduced strength f_{ti}^+ corresponding to the reduced Young's modulus E_i is taken in accordance with the envelope softening stress-strain curve:

$$f_{ti}^+ = \varepsilon_u^+ E_i \frac{D}{E_i + D} \quad , \tag{4}$$

where

$$\varepsilon_u^+ = \varepsilon_u + p\frac{f_t}{D}, \tag{5}$$

i+1 denotes the next stage in the saw-tooth diagram and a_{i+1} is a variable which increases the damage of the element by reducing Young's modulus:

$$E_{i+1} = \frac{f_{ti}^-}{\varepsilon_i} = \frac{E_i}{a_{i+1}} \,. \tag{6}$$

The value of a_{i+1} can be obtained from Eqs. 3 and 6, see Fig. 1b:

$$a_{i+1} = \frac{E_i}{f_{ti}^-}\varepsilon_i = \frac{f_{ti}^+}{f_{ti}^-} = \frac{f_{ti}^+}{f_{ti}^+ - 2pf_t} \quad . \tag{7}$$

Note that this is the softening curve in terms of stress versus *total* strain, i.e. the sum of elastic strain and crack strain of an imagined cracked continuum. The model always provides a solution: the secant saw-tooth stiffness is always positive, so that ill-conditioning or divergence does not appear in sequentially linear analysis. An advantage of this new formulation is that no special techniques are required to handle mesh-size objectivity.

3 SETTLEMENT DAMAGE PREDICTION OF HISTORICAL MASONRY FAÇADE

3.1 Historical masonry façade

Presently, in the Netherlands, bored tunnelling is planned to be driven in soft soil adjacent to historical masonry buildings founded on timber pile foundations. An example for an Amsterdam historical masonry building is as shown in Fig. 2a. The structure of these historical masonry buildings normally consists of un-reinforced façades and bearing walls. The layout of the selected masonry façade as in Fig. 2b is made of a block of three house units. A uniform thickness of one brick (220 mm) is adopted here for the whole façade. The opening pattern shows two large openings at ground floor and a regular pattern of three window openings at the three floors above. Above the window openings at ground floor, lintels in the form of steel beams are present to distribute the vertical load to either side of the opening. The load-bearing and house-separating walls perpendicular to the façade have not been included in the model. Also their connections to the façade and other 3D effects have been omitted. Although it is well understood that 3D effects can modify the damage pattern, the context of this study will be limited to 2D response.

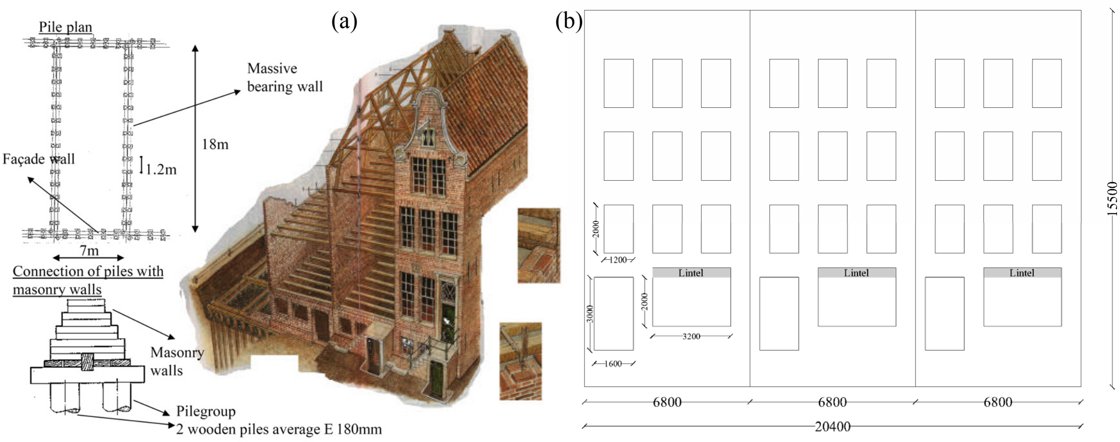


Figure 2 : (a) An example of historical masonry buildings in Amsterdam (after Netzel and Kaalberg, 2001); (b) the selected historical masonry façade (all dimensions in mm).

3.2 Overview of modelling approach

The currently available design practice for evaluating settlement effects on the surrounding structures due to soft-ground tunnelling can be classified into two types, namely uncoupled analysis and coupled analysis. In the first class, the greenfield settlement determined from empirical equations is directly imposed under building models, while the second class allows for the full interaction between above-ground structure and underlying soil. To account for the situation of historical masonry buildings founded on fragile piles, Rots (2000) introduced the semi-coupled scheme in which so-called bedding interface elements were employed to represent a simplified "smeared" model for fragile pile foundation. This strategy is adopted in the present study and the hogging situation i.e. the building sits at the point of inflection of ground settlement profile, is investigated here as being a common case in practice. The estimated settlement trough is taken in the form of Guassian distribution curve (Mair et al. 1996) for any percentage of expected ground loss (V):

$$S_v = S_{max} \exp\left[\frac{-x^2}{2i^2}\right] \tag{8}$$

where S_{max} = 0.31VD^2/i, i is the horizontal distance from the tunnel centerline to the point of inflection on the settlement trough (i=KZ$_o$). K is the trough width parameter for tunnels and is taken as 0.5 here. Z_o is the depth to the tunnel axis, which is 20 m herein. Tunnel diameter (D) is 6.5 m.

4 FINITE ELEMENT MODELLING AND RESULTS

4.1 Smeared crack analysis and results

The façade is modelled by eight-node quadratic plane stress elements. A three by three integration scheme is adopted. The non-linear behaviour of masonry is simulated by using a multiple fixed crack version of crack band model. Young's modulus (*E*) and Poisson's ratio for the façade material are 6000 N/mm^2 and 0.2 respectively. A linear tension softening has been adopted with tensile strength (f_t) equal to 0.3 N/mm^2 and fracture energy (G_f) equal to 0.05 N/mm. Constant shear retention value of 0.01 is adopted. A mesh with element size of 500 by 400 mm^2 is used. The crack bandwidth is chosen as 225 mm as a half of an average of the mesh dimension because the strain field in a quadratic element is likely to be lumped on one side. The loading scheme starts with the activation of self-weight and live load of 5 kN/m at each floor. Mass density of masonry is 2400 kg/m^3. The normal stiffness of the no-tension bedding interface elements is taken as 0.15 N/mm^3 by smearing out the stiffness of foundation system in an average manner. Finally, the settlement trough is applied incrementally. Fig. 3a illustrates the relation between the angular distortion (Boscardin and Cording 1989) and the maximum crack width in the façade. The maximum crack width is the maximum value that occurs somewhere in the façade. The position of the maximum crack width may differ with increasing the angular distortion. Smeared crack response reveals that initially the crack width remains very low, indicating that the façade behaves approximately elastically with only minor cracking. The maximum crack width gradually increases when the cracks at the corner of windows start to propagate. In correspondence of a critical value of angular distrorsion equal to 1/800 the top band of the façade is suddently fully cracked and the maximum crack width suddently increases very rapidly, while the angular distorsion remains almost constant. The moment of full through-depth cracking occurs when the top band of the façade is no longer able to carry the tensile stress in that region. Then, a separation occurs into two unloaded parts that rotate, Fig. 3b. The sudden breakage of the façade is accompanied by a sharp snap-back in the structural response.

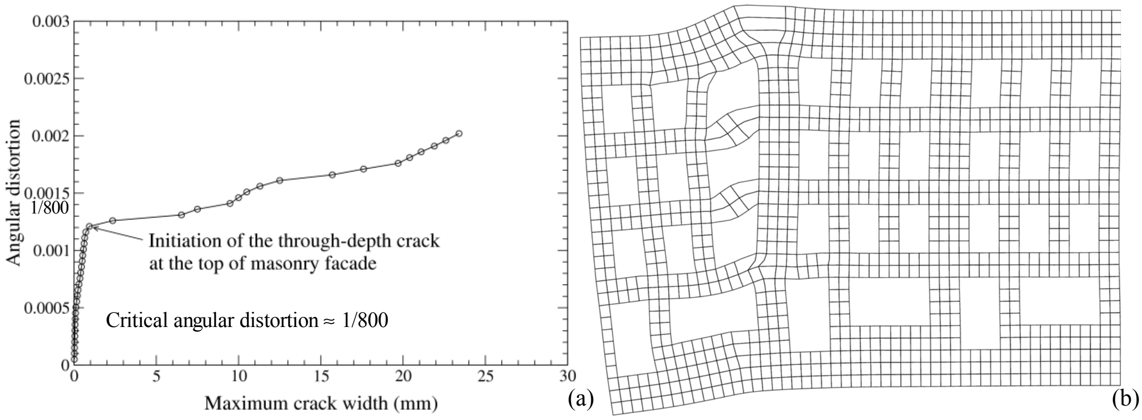
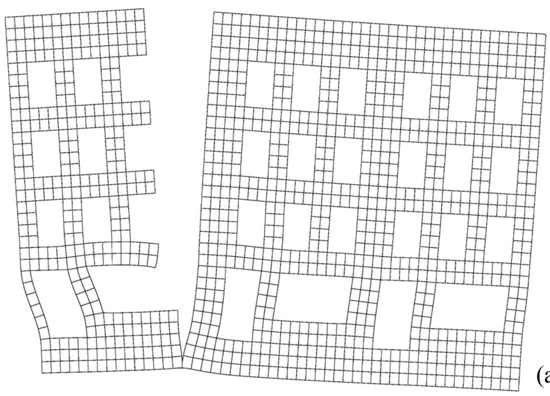


Figure 3 : (a) The façade response in smeared crack analysis, (b) localisation shown at the top of façade.

With displacement control, a snap-through response is captured in which, for a slight increase of the angular distortion, the dominant vertical crack shows an abrupt jump in the magnitude along the second branch of the façade response. Along this branch, the magnitude of maximum crack-width is rapidly increased and the quality of observed convergence performance in the nonlinear analysis becomes dramatically poor. The difficulty to achieve a good convergence is partly due to the significant residual stress in the fully open crack zone and mainly due to the significant unbalance between high elastic energy stored in the façade and low fracture energy of masonry.

4.2 Discrete crack analysis and results

The same façade is now re-analysed by placing six-noded quadratic interface elements at the vertical line of the final crack predicted in the former smeared crack analysis. The rest of masonry continuum is now assumed to be linearly elastic. Interface elements, having the same fracture properties of masonry in section 4.1, are placed at the vertical line of the final crack predicted by smeared crack analyses. Lobatto integration scheme is adopted to avoid the stress oscillation problem. Instead of steering the analysis by displacement control, the arc-length control technique is pursued here hopefully to reveal a stable crack propagation path of this façade. Switching to this arc-length control requires the modification of the previously prescribed displacement degrees of freedom at the bottom of the façade into the free displacement degrees of freedom. Note that as minimum constraints are still required to avoid the rigid body movement of the facade, few nodes at the right bottom of the façade are inevitably made as fixed supports. The effect of the live load and self-weight of masonry will be excluded here as it is found that this initial stresses result in severe false-cracking when a tying condition of the free displacement degrees of freedom along the bottom nodes of the façade is imposed. In addition, the effect of self-weight and live load is supposed to be negligible with respect ground settlements when the potential vertical crack line develops. A stable response can be predicted with the discrete crack corrector analysis and selecting into the constraint equation of the arc-length control procedure only the incremental displacements at the top of the façade. This approach, controlling the parameter which dominate the crack opening displacement, gives good convergences during all the course of loading. In fact, it is found that the façade snaps back during the stable cracking process along the corner of window openings at each floor, see Fig. 4a. With the classical incremental-iterative displacement control, the revisit of smeared crack analysis by the other types of smeared crack models, e.g., by total-strain rotating crack model (Feenstra et al. 1998) as enclosed in Fig. 4b cannot reveal this phenomenon. The strong and sharp snap-back response is indeed the brittle-failure characteristics for large-scale masonry structures. However, even with the discrete crack analysis, much effort is required and the approach is still somewhat not attractive. Application of the sequentially linear continuum concept is appealing and will be investigated in the following section.



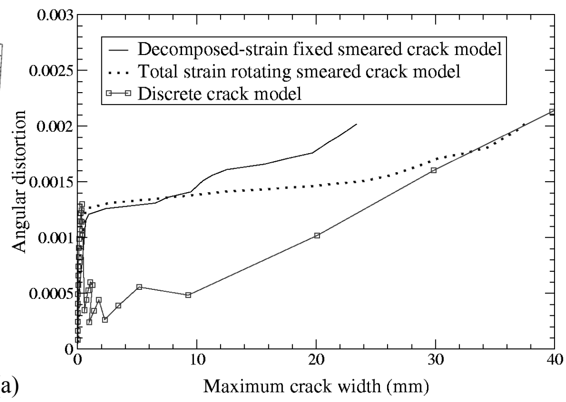


Figure 4: .(a) Incremental deformation during snap back response in discrete crack analysis, (b) Computed angular distortion versus maximum crack graph.

4.3 Sequentialyt linear analysis and results

The masonry façade is described with four-node membrane elements having size of 200x250 mm. From mesh refinement test presented in Boonpichetvong and Rots (2003), it came out that the structural response remains the same if 400x500 eight node elements or 200x250 mm four-node elements are used. So the results obtained with smeared or discrete crack models can be in any case compared with the ones obtained with sequentially linear analysis. The final crack predicted by smeared crack analyses, indicated the position of interface elements for discontinuum crack analysis carried out with the arc-length control technique. For the sequentially linear analysis, this same final crack pattern is also achieved by pre-defining the bands of elements to crack. A linear tension softening characterised by the same mechanical properties (Young's modulus, tensile strength and energy fracture) chosen for smeared crack model have been adopted for these predefined elements, while the remaining part of the façade is made linear elastic. According to section 2.1 a strength percentage *p* equal to 10% has been fixed for the saw-tooth diagram. Consequently, the total number of teeth needed in order to reach the complete damage is automatically evaluated (N=19). An isotropic reduction of stiffness was assumed. It means that Young's modulus was (saw-tooth wise) reduced in all directions. No-tension interface elements, previously inserted with the aim to ignore tensile resistance of soil boundary underneath buildings, are still included but now assumed to behave elastically. From the smeared crack analyses, these interface elements are shown to be in compressive mode, so this simplification can be made here without loosing the accuracy. Settlement is imposed as a displacement of nodes at the bottom of the façade. As in the case of NLFE analysis carried out with discrete crack models and arc-length control technique, selft-weight and live load are not considered.

Fig. 5 shows the curves in terms of angular distorsion versus maximum crack width obtained with smeared crack model, discrete crack model and sequentially linear analysis. In the case of sequentially linear analysis the crack width is determined as the maximum principal strain times the crack band width *h*, assumed equal to 200mm.

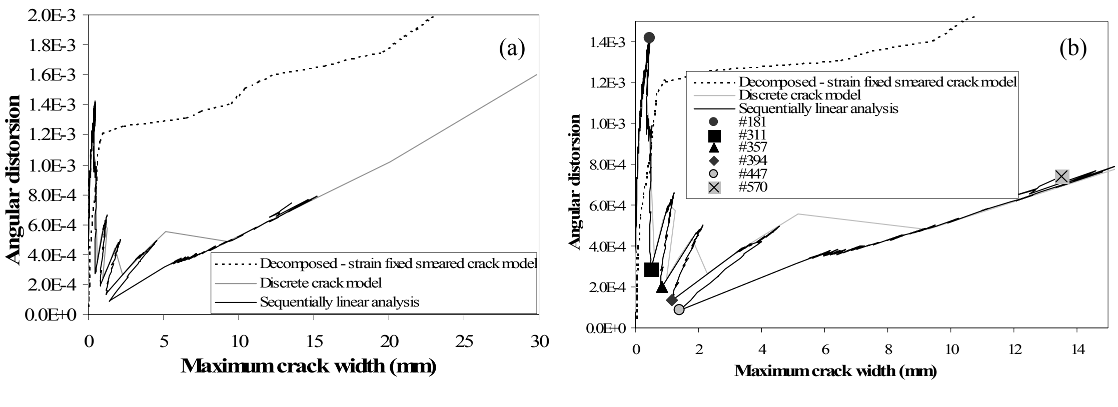


Figure 5: Angular distortion versus maximum crack width curves: (a) complete curves (b) zoom-in.

Smeared crack models proved inadequate tools for predicting such structural response and the results detected a snap-through where the convergence was very poor. This problem was circumvented by using discrete crack models, which display a very sharp snap-back beyond peak and subsequently a rising branch again. On the other hands non-proportional loadings cannot be applied and several numerical difficulties related to the arc-length procedure must be overcame in order to obtain this curve. The response given by sequentially linear analysis fits very well to the one obtained with the discrete crack model, but the easiness of the input data implementation required by sequentially linear analysis represents one of the main advantage of this numerical procedure. In Fig. 5b, where a zoom-in of the curve is shown, are also indicated the reference numbers of sequentially linear analyses corresponding to the sudden jumps which characterise the snap-back of the structural response. In Fig. 6 the level of element damage in correspondence of these jumps is reported. The damage level is evaluated in terms of number of teeth reached in the saw-tooth diagram providing an indication of crack pattern and, if required, it can be transformed to crack width values.

5 CONCLUSIONS

The smeared crack models allow taking into account non-proportional loading (first the dead weight and live load, subsequently settlement are applied), but the very poor convergence performance makes the results untrustworthy. The advantage of smeared crack models is that cracks can occur anywhere in the mesh in any direction, while, with the discrete crack concepts, interface elements have to be predefined in the mesh. In the latter case snap-back response can be predicted, but practical use of this arc-length control technique requires a lot of user's experience. The response given by sequentially linear analysis fits very well to the one obtained with dicrete crack model and no special user's skill or experience are needed to predict with good accuracy the structural response. In this way, contact can be made between the fracture mechanics "science community" and the practical engineering design world. The implementation of non-proportional loading in sequentially linear procedure should be the next step of this work. This improvement is requested in order to make this proposed numerical procedure more general and able to consider not only the effects of settlement but also the ones of self-weight and live load.

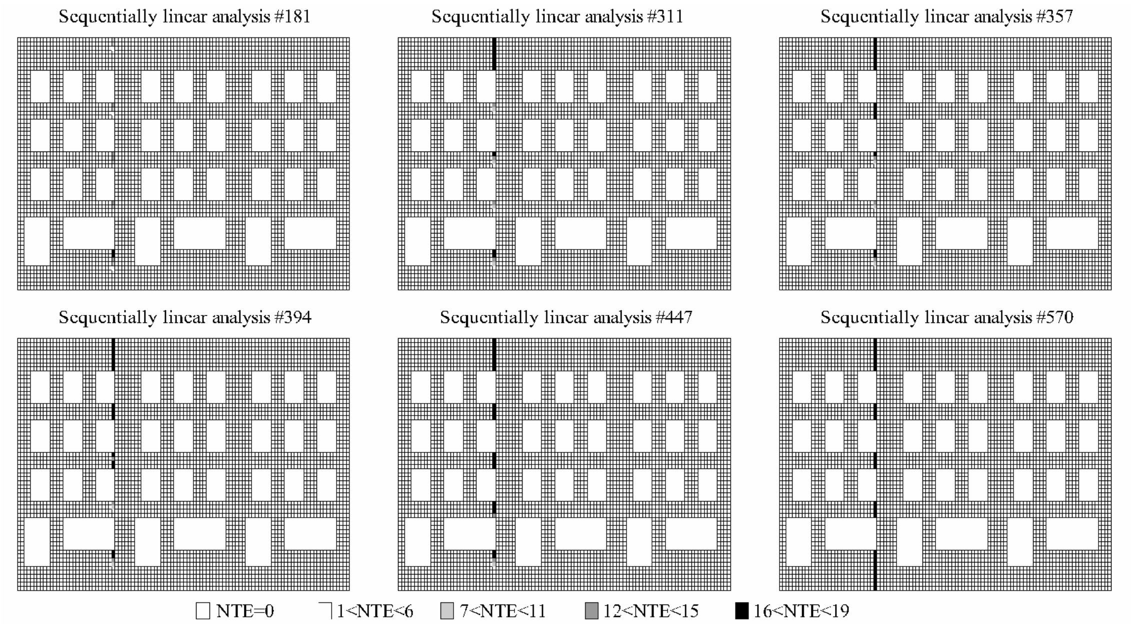


Figure 6: Level of damage in correspondence of jumps in the structural response.

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