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Simplified Evaluation of the Horizontal Capacity of Masonry Arches

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ABSTRACT: Arches are among the most widely used and characterizing elements in masonry structures. At the same time, they are prone to severe damages or even collapse depending on the activation of cracking mechanisms. In this paper, the problem of the lateral strength evaluation of masonry arch with abutments piers, acted upon by its own weight and by seismic horizontal loads, has been studied. Within the framework of the limit analysis approach, as expressed for masonry structures by Heyman, the principle of virtual works for olonomic systems is applied to derive expressions for the collapse multiplier of a chosen load distribution. In this aim, three collapse mechanisms have been selected on account of mechanical and engineering considerations: local, semi-global and global collapse and the expression of the multiplier for each of them has been derived. An extensive parametric analysis on 60 structural schemes, varying the geometrical ratios that determine the arch and the pier configuration has been performed. As a result, ranges of the main geometrical ratios, in which each mechanism prevail, are derived. Finally, a simplified formula based only on the geometry of the structure is proposed and compared with a FEM analysis on a triumphal arch taken from a real church.

1 INTRODUCTION

Buttressed or triumphal arches in churches, colonnades at the base of buildings or in internal patios, are some examples of the use of arches in masonry structures. Unfortunately, they are also used to experience severe damage in case of exceptional load, such as seismic actions or foundation settlements, or in case of degradation of the material. In most of the cases, the proneness to collapse of these structures is dependent on the activation of cracking mechanisms, subsequent to the formation of a sufficient number of rotational hinges.

In (Abruzzese et al. 1990, 1995) the collapse mechanisms of buttressed arches were studied in detail according to different load conditions (vertical load alone or in combination with horizontal loads) and different arches shape (circular and pointed). Symmetric and non symmetric collapse mechanism, in the following referred as arch and mixed mechanisms, were analyzed through the application of the kinematic theorem. For determining the minimum collapse multiplier, a recursive mathematical procedure needs to be established, since each time that a hinge position is varied, several mechanical parameters and relevant displacement components have to be determined. The procedure is elementary from a conceptual point of view, but may turn out not very useful for practical purposes. In the light of both these considerations and of a previous study conducted by the authors on masonry portal frames (De Luca 2005), after introducing an additional failure mechanism, a simplified formula based on the geometrical dimensions of the buttressed arch is proposed.

2 PROBLEM FORMULATION

Because of the redundancy of the arch structure, a kinematic mechanism can be activated once four plastic hinges form and a bar chain develops. In theory, since the hinge formation may occur at any location, the possible hinge combinations are several. Mainly, the possible mechanisms can be divided in three groups, as shown in Fig. 1:

- arch mechanism, with the formation of 4 hinge within the development of the arch;
- mixed mechanism, having one hinge at the base of the pier on the opposite side of the horizontal force and 3 hinges on the springing of the arch;
- frame-like mechanism, which considers the rocking of the two piers about 2 hinges at the base and 2 hinges along the development of the arch.

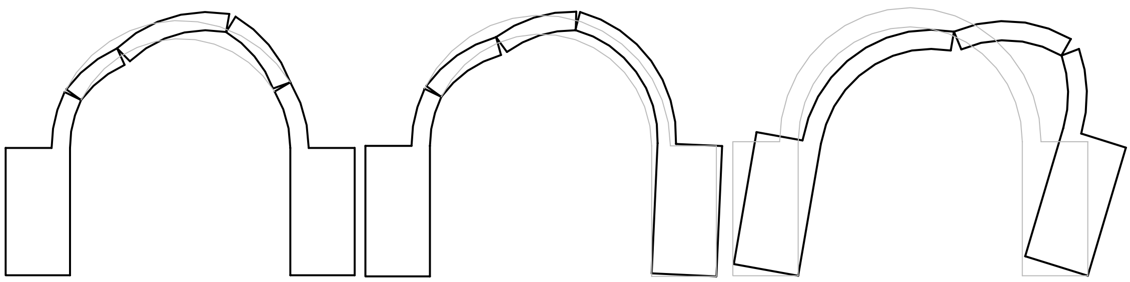


Figure 1 : Possible mechanisms: (a) arch mechanism; (b) mixed mechanism; (c) frame-like mechanism.

2.1 Arch mechanism

On the base of surveyed damage patterns on real masonry arches, some assumptions on the position of the plastic hinges have been made. Namely, as shown in Fig. 2., it has been considered that the first hinge is located inside the springing of the arch at a distance of 5% of the internal span; the second one develops at an unknown distance from the first hinge, namely x_2, which is the variable in the problem; the distance between the second and the third hinge is assumed equal to half the span of the arch and the last one is placed on the extrados of the arch along the vertical of the pier.

In Fig. 2.b the kinematic chain is also shown.

Through the application of the principle of virtual works, it is possible to determine the relevant load multiplier:

$$\frac{F}{W_{tot}} = \frac{F_1 \cdot v_1 + F_2 \cdot v_2 + F_3 \cdot v_3}{F_1 \cdot u_1 + F_2 \cdot u_2 + F_3 \cdot u_3} \tag{1}$$

where:

$F/W_{tot}=\lambda$ = load multiplier;
F_i = self weight of the single parts of the structure involved in the mechanism;
v_i and u_i = vertical and horizontal displacements of the centroids of the rigid blocks forming the mechanism.

By varying the position of the second hinge, a set of load multiplier is obtained. The minimum of such values is assumed to be the searched collapse multiplier. This application clearly lays within the general concept of the limit analysis theorems. Quite obviously, the adopted approach does not ensure the selected load multiplier to be the minimum, since it does not involve each possible collapse mechanism. Nonetheless, the determined value is regarded as sufficiently reliable as far as this mechanism shape is concerned, since experience shows that the assumptions made on hinge positions are realistic.

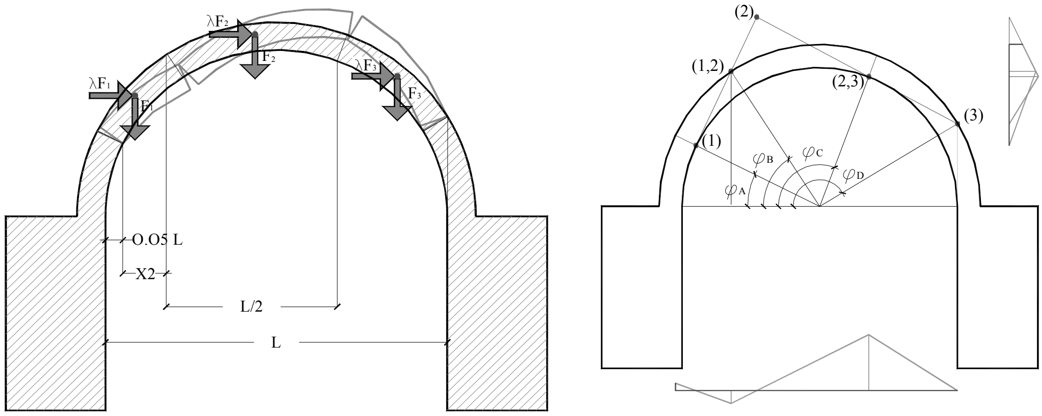


Figure 2 : Arch mechanism: geometry, loads and kinematic chain.

2.2 *Mixed mechanism*

In this case the rocking of one pier is involved in the mechanism. As in the previous mechanism, similar assumptions have been made on hinges position. Again, the first one takes place at 0.05 length of the span, the second one is assumed to be again at an unknown x_2 distance (or angle φ_b); the third and the forth ones respectively at a x_3 distance (or angle φ_c) form the second one and at the toe of the pier. Therefore, the problem is two variables dependent. In Fig. 3 the geometry, the load condition and the kinematic chain are depicted.

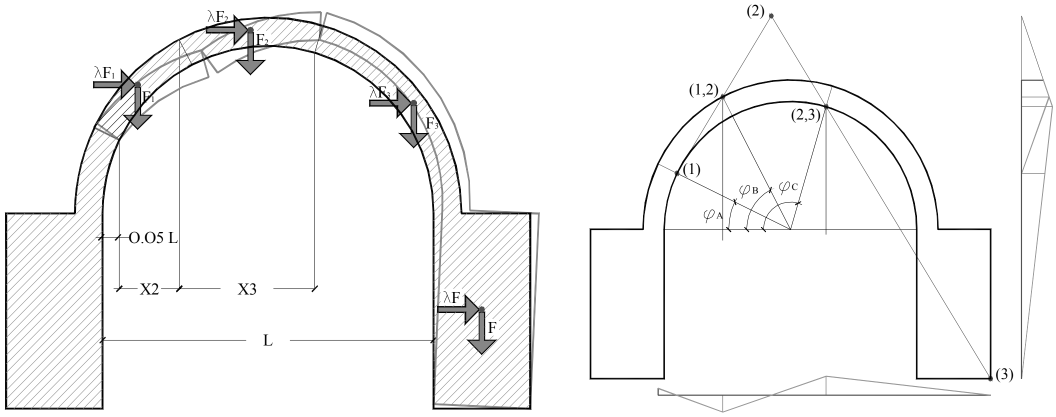


Figure 3 : Mixed mechanism: geometry, loads and kinematic chain.

The equation of the virtual works is formally the same as the local mechanism, except the fact that the rocking pier has to be taken into account.

2.3 *Frame-like mechanism*

This mechanism considers the rocking of both the two piers, so that a slightly different placing of the hinges is defined. The first and the fourth hinges develop at the toe of the piers, the second one is located at middle span and the third one at 90 % of the span. Under this hypothesis (Fig. 4), also confirmed by surveyed damage pattern, the position of one or two hinges does not require to be varied in order to seek out the minimum load multiplier. Trivial algebra provides its expression.

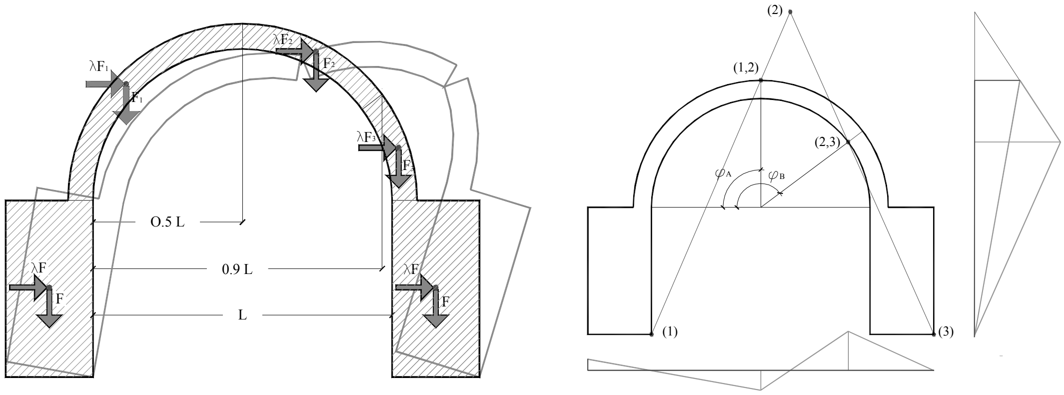


Figure 4 : Frame-like mechanism: geometry, loads and kinematic chain.

3 PARAMETRIC ANALYSES

The masonry arch superimposed on two piers, as shown in Fig. 5, is geometrically defined once the values of the radius *R* and thickness *s* of the arch, base *B* and height *h* of the pier are selected.

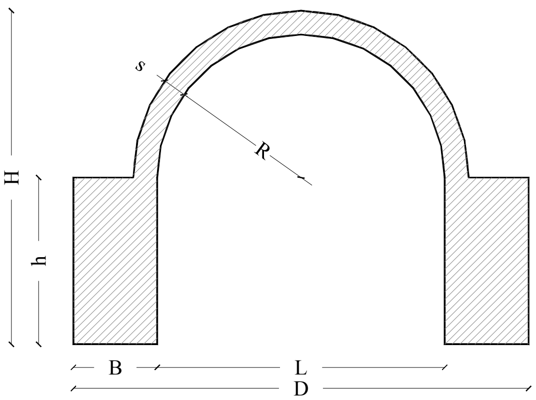


Figure 5 : Arch and piers geometry.

Consequently, the element geometry can be synthetically expressed adopting appropriate geometrical ratios (later on referred as fundamental ratios). Namely, we use the ratios B/R, h/R and s/R. Of course the dimensional definition needs the choice of a value for a parameter, for example the internal radius R, though it happens to be unnecessary for the determination of the collapse multiplier.

Actually, all the arches obtained fixing the fundamental ratios and varying one chosen parameter, namely the radius R, are characterised by the same kinematic multiplier. This consideration can be regarded as a sort of scale independency effect.

In order to study the behaviour of the most common arches in monumental buildings, a parametric analysis on a wide range of arches defined by the fundamental ratios has been carried out. In Fig. 6 the considered arches are depicted and the following parameters have been varied:
- h/R stepwise variability: 0,50 – 1,00 – 1,50 – 2,00 (arches with and height variable between half and twice the radius of the arch);
- B/R stepwise variability: 0,25 – 0,50 – 0,75 – 1,00 – 1,25 (base of the piers variable between 25 % and 125 % of the radius of the arch);
- s/R stepwise variability: 0,20 – 0,40 – 0,60 – 0,80 – 1,00 (thickness of the arches variable between 20 % and 100 % of the radius of the arch).

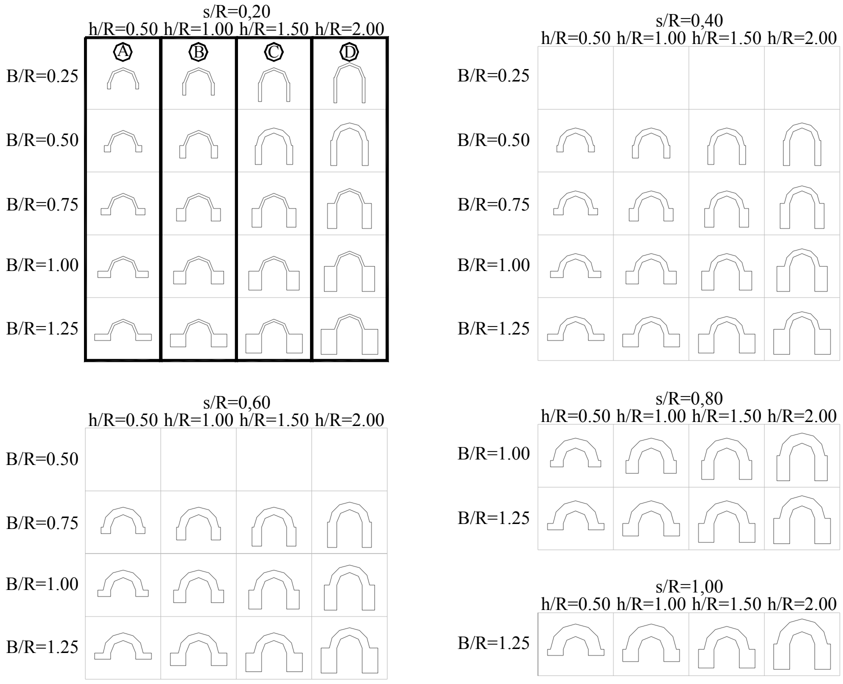


Figure 6 : Possible arch configurations fixing the ratio s/R.

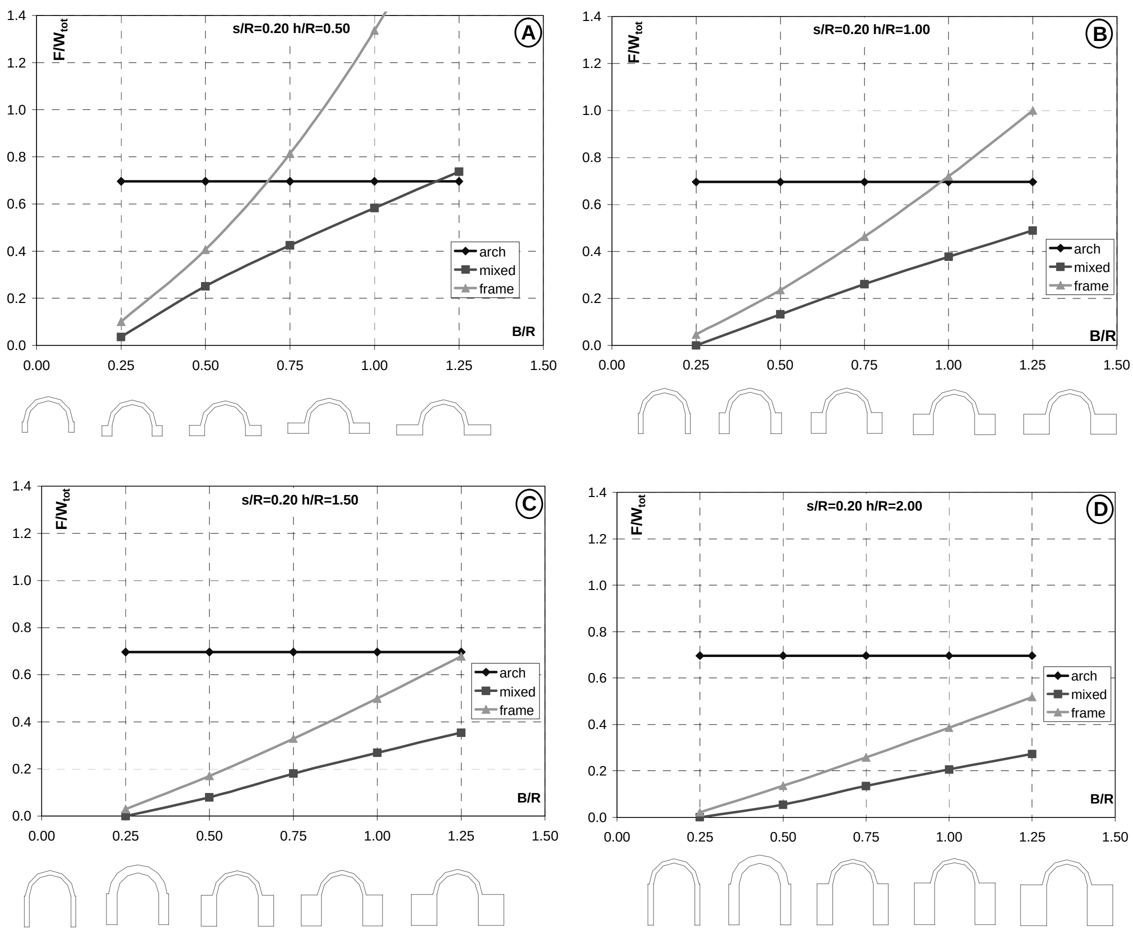


Figure 7 : Collapse multiplier for s/R = 0.2.

Then, the collapse multiplier F/W_{tot} of the 3 kinematic mechanisms previously described has been plotted as a function of the fundamental geometrical ratios – h/R, B/R and s/R – fixing alternatively 2 parameters and analyzing the variation of λ as function of the third one. In Fig. 7, fixing the ratio s/R = 0.2 and varying H/R and B/R, the values of the collapse multipliers are reported. It can be noticed from each plot how the semi-global mechanism produces the minimum values so that it can be considered as the governing mechanism in these structures.

4 APPROXIMATE FORMULA

In the adopted procedure, the values of the collapse multiplier are numerically obtained. Conceptually, the procedure consists in computing the collapse multipliers for "frozen" hinge positions and subsequently selecting the minimum. As a matter of fact, for each mechanism, Eq. (1) could be treated via variational principles, using the well known functional minimization techniques. Although mathematically rigorous and somehow elegant, this way of proceeding certainly does not appear straightforward due to the difficulties in deriving the closed form expression of the terms in Eq. (1).

The use of a simplified formula, although with some unavoidable scatters with respect to the exact values, can be more appealing for practical uses. Using similar approach as in previous works on portal frames (De Luca et al. 2005), and in analogy with the "frame-like mechanism", we obtain what follows:

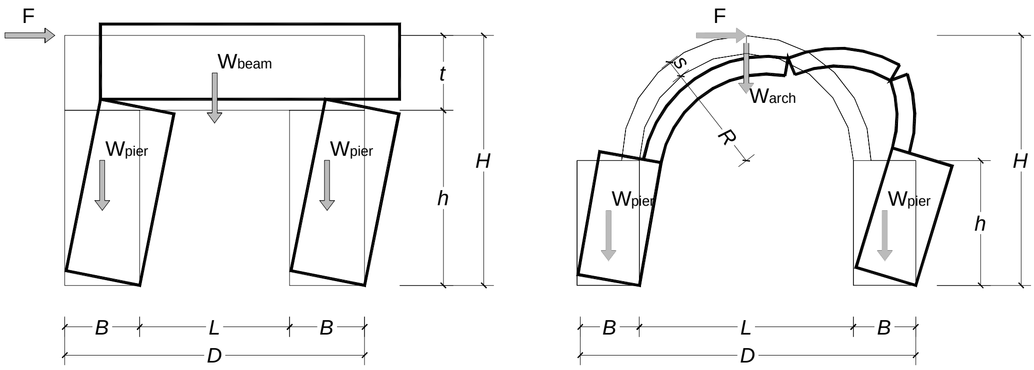


Figure 8 : Simplified mechanisms in portal frame and buttressed arches.

$$\frac{F}{W_{tot}} = \frac{B}{2 \cdot h} \cdot \left(1 + \frac{W_{beam}}{W_{tot}}\right) \cdot \left(0{,}50 + \frac{B}{D}\right) \quad \text{Simplified formula portal frame} \qquad (2)$$

$$\frac{F}{W_{tot}} = \frac{B}{2 \cdot (h + R)} \cdot \left(1 + \frac{W_{arch}}{W_{tot}}\right) \cdot \left(0{,}10 + \frac{B}{R}\right) \quad \text{Simplified formula buttressed arch} \qquad (3)$$

Where the variables are the ones of Fig. 8. The physical meaning of the three factors is similar for both the formulas except some necessary adjustments for the circular structure: the first one represents the column behaviour as a single panel element; the second one is the stabilizing effect of the upper part; the last one is the effect of the opening.

In Fig. 9 the comparison between the multiplier values obtained with exact expressions and the approximated formula is reported. Scatters increase for small values of the ratio h/R and high values the ratio s/R In the first case, the dimension of the pier is small compared to the arch springing so that a frame-like mechanism is very unlikely to occur; in the second case, the thickness of the arch is comparable to the width of the piers. Therefore, in both the cases more complex mechanisms are involved in the collapse and the simplified approach is not suitable.

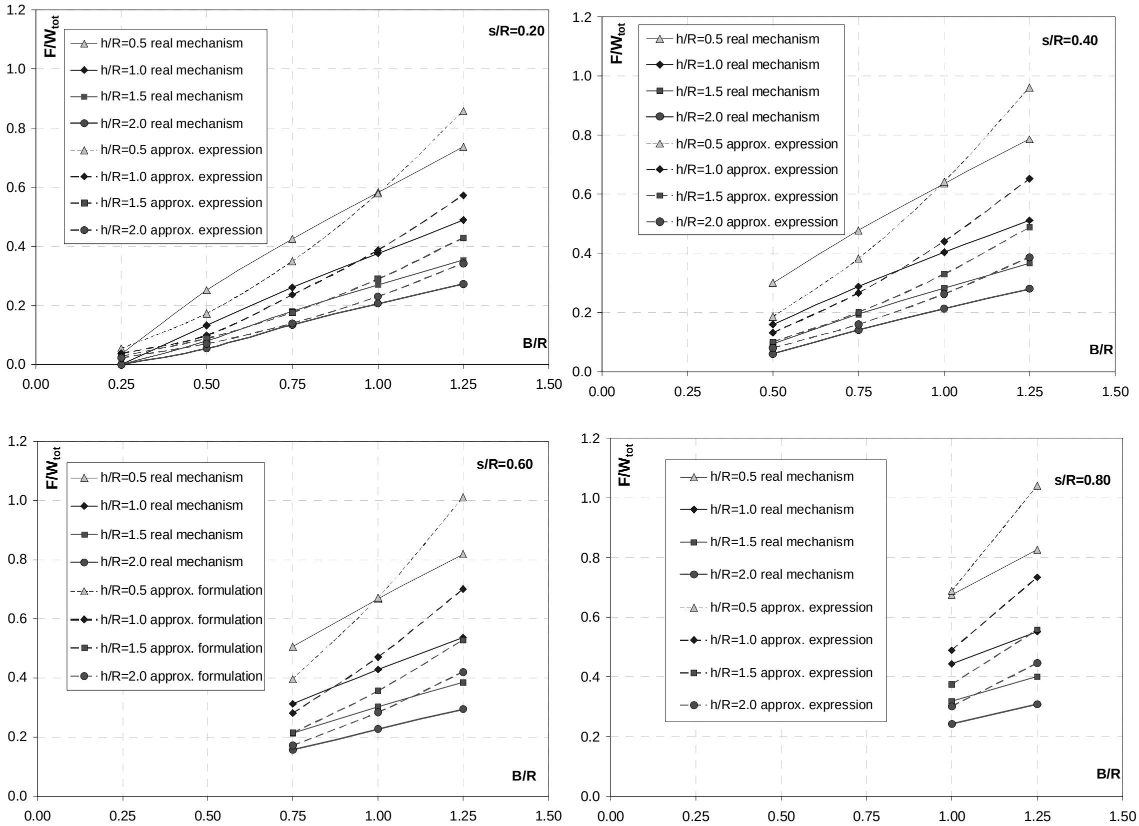


Figure 9 : Comparison of actual mechanism and approximated formula.

This statement is also confirmed by the graph plotted in Fig. 10 where the exact collapse multiplier and the approximated ones are reported for all the parametric study cases. The central line on the diagonal represents the value with zero scatter; the other lines represent the scatter of ± 10 and 20 %. Generally, the values are inside the sheaf of the straight lines, except for the values representing arches with the different collapse behaviour before cited.

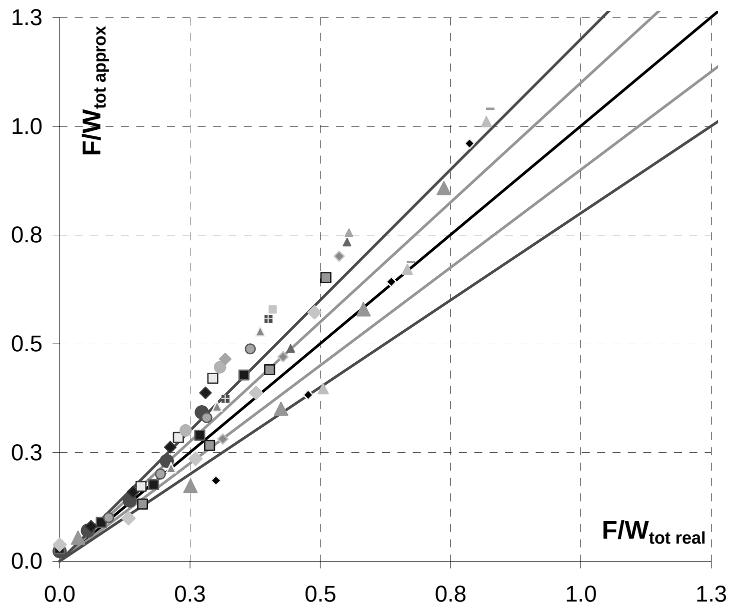
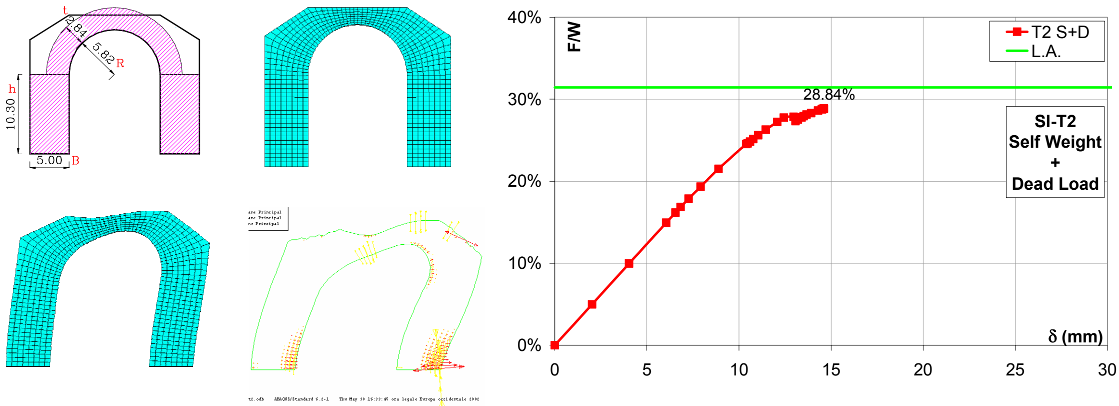


Figure 10 : Scatters of the simplified formula for different ach configurations.

Furthermore, in order to asses the validity of the simplified formula, a FEM non linear analysis on the triumphal arch of S. Ippolisto church was carried out. In Fig. 11.a the real geometry and the adopted schematization, the mesh, the deformed shape and the plastic strain tensor are reported. The FE analyses are carried out using a smeared cracking approach as in De Luca et al. (2004). The value of the collapse multiplier obtained from limit analysis matches well with the more refined curve from non linear analysis as can be noticed in Fig. 11.b.



(a) (b)

Figure 11 : Examples of: (a) Geometry, Mesh, Deformed shape, Plastic strain tensor; (b) Force displacement curves.

5 CONCLUSIONS

Applying the well known principles of the limit analysis, this paper gives a practical contribution to the issue of evaluating the horizontal collapse multiplier of masonry arches. Namely, analytical expressions and visual plots are provided. Three mechanisms for the buttressed arch have been considered: the local, the semi-global and the global one, involving the springing arch but also the rocking of the piers. The exact values are then compared with the ones obtained through a simplified approach. This last one gives good results providing that common values of the main geometrical ratios are used. The comparison on a real study case of non linear analysis vs simplified approach shows a good matching. In the opinion of the authors, the availability of a simple expression, based only on geometrical characteristics, is valuable for practical uses in which rapid seismic capacity assessment is required.

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