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A Multiscale Approach for the Analysis of Block Masonry Under Damage and Friction

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ABSTRACT: The topic of this paper is the numerical analysis of periodic masonry brickwork, seen at a microscopic level as a set of blocks interacting by non linear elasto--plastic softening interfaces. Two approaches are generally used in the literature in order to describe such a behavior: Discrete Models, aiming at the detailed solution of the microscopic problem but characterized by very high computational costs, and Continuum Models, providing a coarse description of the masonry material but perfectly fit for a coarse FEM discretization. It is our opinion that, once the analysis of a whole masonry building is defined as the primary objective, the use of Discrete Models is practically prohibitive. However, the use of Continuum Models implies the definition of a carefully tuned constitutive prescription, possibly based on microscopic analysis of the masonry material. A different approach, known as Multi-scale method, is proposed in this work. This method does not define any constitutive relation at the macroscopic level, but adopts a step-by-step homogenization technique that takes into account the presence of both micro and macro models throughout all the integration procedure.

1 INTRODUCTION

In the past, the traditional approach to the structural design of masonry structures was based on the building experience acquired along the centuries. Paradoxically, the great development of Continuum and Solid Mechanics, strongly encouraged by the birth and development of new materials has not actually changed the way in which masonry systems can be designed and calculated. Nowadays, in view of the rehabilitation of traditional historical buildings and of the updated reuse of masonry for the contemporary design, it is strongly felt the need for mechanical and numerical tools for the structural design and assessment of safety requirements, able to bridge the gap between the extraordinary techniques of the past and our sophisticated mechanical knowledge.

The starting point of this research work was a reflection about the ways in which, in the last decades, the analysis of masonry structures (and in particular of periodic brickwork) has been faced in the scientific circles. The different methods found in the literature can be ascribed to two main classes. A first class is that of *Continuum Models*, in which the masonry is described by means of an equivalent continuous medium and the generation of coarse FEM discretization becomes a natural and straightforward procedure. Of course, such an approach should be based on proper homogenization techniques, and the constitutive prescriptions have to be explicitly defined in advance. Unfortunately, this is just the crucial point, since it is quite difficult to obtain explicit constitutive relations on a micro mechanical basis if we wish to take into account for the coupling between plasticity and damage. A second class is that of *Discrete Models*, aimed at a very detailed solution of the mechanical problem (at the level of single blocks and joints): a very fine discretization is required and a large number of variables must be introduced in order to properly manage the problem. As a natural consequence, this approach is unpractical for the solution of problems at the an engineering scale. Each of the two mentioned approaches feeds on a preferential observation scale and each shows a number of advantages and shortcomings, as well. The research work that is here presented is just an effort to overcome the dichotomy between discrete and continuum models, exchanging some of the constitutive accuracy of the former for the computational feasibility of the latter.

The attention is focused on the mechanical modelling of periodic masonry brickwork (i.e. a regular, isodom arrangement of blocks and mortar joints). This field has been widely investigated in the last few years, and nevertheless presents a number of unresolved questions that will be shortly addressed. For example, the homogenization techniques for periodic media have been extensively and successfully applied in linearized elasticity, while in the non linear case we can't say that there are widely accepted and firmly stated homogenization procedures. Only in specific and well defined context the non linear homogenization of periodic media has produced relevant theoretical and practical results, for example in the field of non linear elasticity governed by convex potentials, rigid-plastic constitutive relations (Suquet 1987). Unfortunately, these procedures are not suitable to describe the constitutive behaviour of the mortar joints, governed by the coupling of damage (presence of micro fractures) and not associative elasto-plasticity (Coulomb friction). In order to overcome the intrinsic drawbacks of a homogenization approach, many attempts were tried in order to define an explicit macroscopic constitutive law. Of course, a drastic a priori simplification of the constitutive behaviour at macro-level is performed in this case, by describing either the plastic behaviour (Debuhan et al. 1997) or the damage (Gambarotta et al. 1994) of the joints.

Here, a Multiscale procedure is proposed, in which no explicit choice about a preferential observation scale is made. Indeed, three different observation scales (and three corresponding mechanical models) exchanging information one with the other are contemporaneously present throughout all the analysis: macro, meso and micro-level. Moreover, a peculiar feature of this method is that no constitutive relation is explicitly defined at the macroscopic level, but the structural response is obtained at each step of the integration procedure by resorting on the lower scales.

1.1 Observation scales and mathematical models

In order to describe the mechanical behaviour of a masonry panel, we distinguish three different scales, shown in the picture, respectively referred to as macro, meso and micro scales. However, we don't give to these terms exactly the same meaning as a solid state physicist would give them, by relating to each term a precise physical size, but we use them in order to define a hierarchy of scales and models. For us, the *macroscopic* scale is the "structural" scale, that is, that of the masonry panel, which defines the problem size, the space domain and the discretization variables in a FEM analysis. Then, at the *mesoscopic* scale we see a non homogenous systems, composed of bricks and the mortar joints, and we observe the role played by the brick interlocking, by the brick shape ratio and by the material characteristic length; finally, at the *microscopic* scale we are inside the mortar joint, where relevant micro mechanical phenomena, such as crack's growth and frictional sliding of the crack faces are observed.

To each observation scale corresponds a physical-mathematical model. The macro model is a two-dimensional Cosserat continuum model, used to generate rationally a FEM model. The choice of a microstructured continuum instead of a Cauchy one, is due to the need to keep trace at the macro level of an internal material length to avoid the pathological mesh sensitivity arising when a softening behaviour is foreseen. The meso model is a discrete one, made up by blocks interacting two by two through linear inelastic interfaces. In this case, within the scope of this paper, it is sufficient to restrict our attention to the case of rigid units. At last, the micro model is the two-dimensional mortar joint model. It is a three d.o.f. model (elongation, shear and curvature) whose constitutive behaviour is inelastic and defined by two variables β and γ, representing the fractured length of the joint and the residual frictional sliding

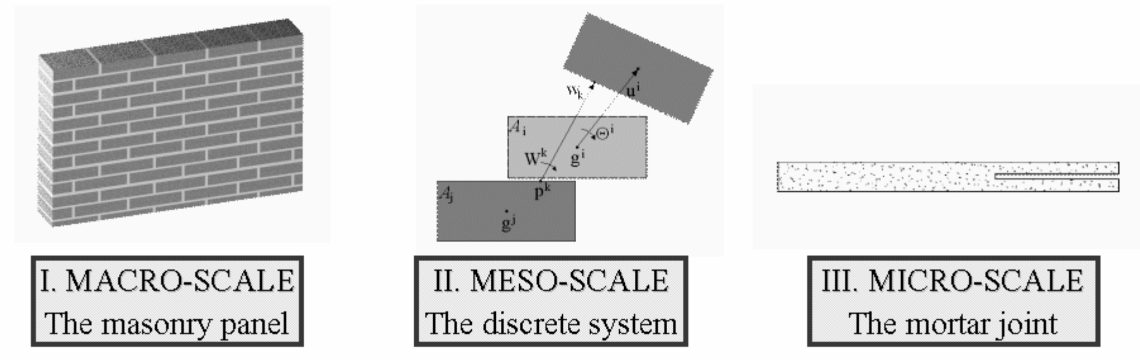


Figure 1 : The three scales.

2 STEP-BY-STEP EVOLUTIVE ANALYSIS

The multiscale approach can be combined within a classical arc-length step-by-step evolutive analysis, chosen as a prototype of a nonlinear analysis. In a step-by-step analysis, the equilibrium path is recovered by interpolation of a succession of equilibrium points.

In a multiscale approach, the most part of this integration procedure is performed at the macro scale, that is, at the continuum scale, after a suitable FEM discretization. By referring to the flow chart shown in the figure, the predictor phase is performed at the macro scale, while in the correction phase, every time we need to calculate the structural response, we have to resort to the lower scales. Moreover, since the constant iteration matrix is the stiffness matrix assembled at a suitable equilibrium point, also in that case we have to grasp information from lower scales. It should be clear that, since the problem is governed by a the non linear constitutive behaviour of the mortar joints, every time we need to update our constitutive information at the macro scale, we have to scale down to the micro level.

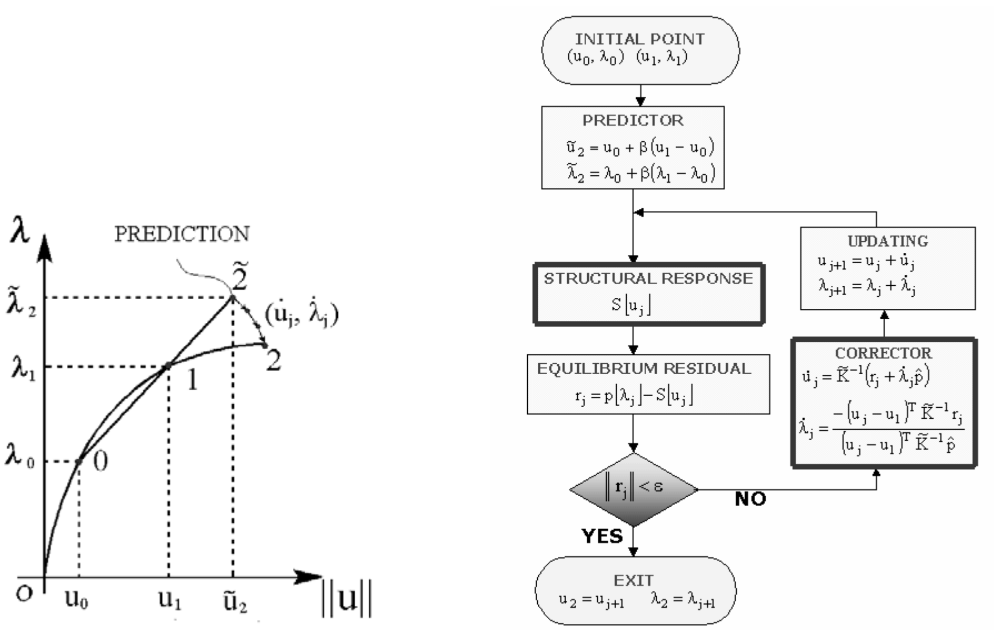
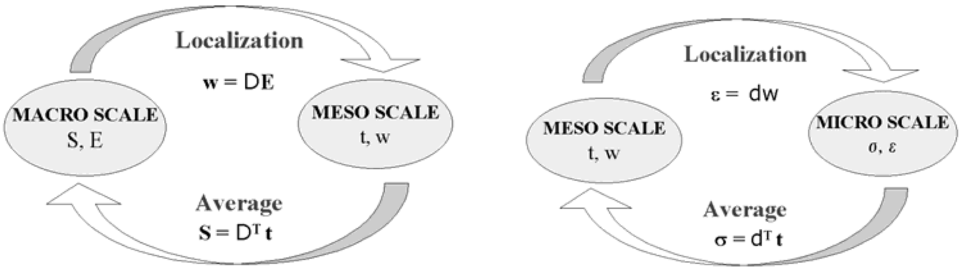


Figure 2 : Equilibrium path.

3 SWITCHING BETWEEN MACRO-SCALE AND MESO-SCALE

In order to resort to the lower scales is necessary every time we need to update the constitutive behaviour. The communication among the scales is rigorously hierarchic, that is, each scale communicates only with the adjoining scales and by means of linear operators of up and down scaling, suitably defined.



3.1 The microscopic model

When entering the microscale, we go inside the mortar joint, at the very scale of all those mechanical phenomena having a relevant role in the behaviour of masonry materials: nucleation and development of cracks, frictional sliding, energy dissipation. It is here that the constitutive law should be described with a certain detail and accuracy introducing a set of internal state variables. The experimental evidence shows that ruptures in masonry structures are characterized by decohesion and frictional sliding and are mostly localized at the contact surface between mortar and blocks rows. Considering that the mortar joint has been identified as an interface and that we are dealing with plane system, we will describe the microcracks as segments, and accordingly adopt a scalar measure of damage β, in the form of an internal variable controlling the cracks' length in the joint. Furthermore, it will be assumed that the nucleation and propagation of cracks always remains within a plane and is only governed by two modes: *opening* (I mode) and *sliding* (II mode), that are uncoupled (that is to say, dilatancy is not modelled).

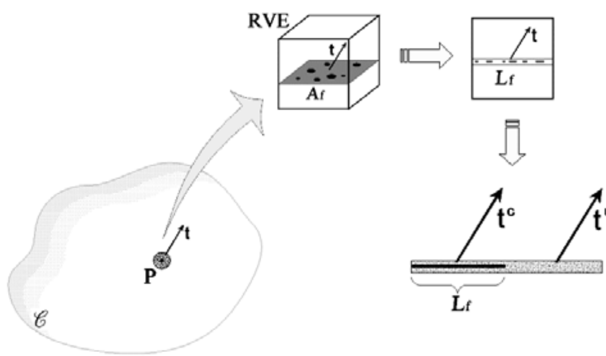
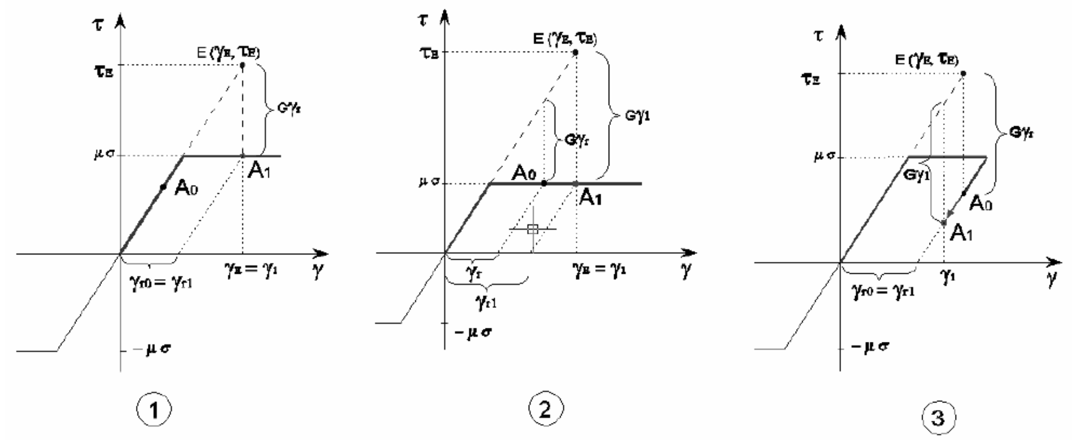


Figure 3 : Schematization adopted for the cracks developing within the mortar.

The scalar representative measure of the state of the material is chosen in order to represent -- in a proper metric - the cracked portion of the generic material point: $\beta = A_f/A = L_f/L$. The material is ideally divided into two parts connected in parallel (Uva 1998). One is perfectly intact (*unbroken fraction*) whereas the other has lost the material continuity (*cracked fraction*). Using this model, it is possible to describe the "averaged" microstress and microcouple ($\overline{\sigma}, \overline{m}$) acting on the point as the weighted sum of an elastic "`unbroken" (σ^u, m^u) and a "cracked" one (σ^c, m^c): $\overline{\sigma} = (1-\beta)\,\sigma^u + \beta\,\sigma^c$; $\overline{m} = (1-\beta)\,m^u + \beta\,m^c$

The behaviour of the unbroken fraction is linearly elastic in every step of the loading process. For the cracked portion, the stress-strain relation depends on the sign of the axial deformation ε, since it cannot support tensions when is open ($\varepsilon \geq 0$). As long as the faces of the crack have a contact ($\varepsilon \leq 0$) and allow the transmission of the stresses, the material fraction behaves as it is intact and elastic). If appropriate, it will develop inelastic deformations in the shear direction, involving frictional phenomena. Since the frictional sliding law is non holonomic, it is necessary to know the value of the residual sliding γ_r that has been reached in each instant of the time history (in Fig. 4 the algorithmic management of the frictional law is briefly described).



.Figure 4 : Frictional model and residual sliding

A$_0$ = initial state; **A$_1$** = final state; **E** = elastic prediction

Calculation of the final state: $\begin{cases} \gamma_1 = \gamma_E \\ \tau_1 = \max(-\mu\sigma, \min(\tau_E - G\,\gamma_r, \mu\sigma)) \end{cases}$

Updating of the residual sliding: $\gamma_r = \gamma_E - (\tau_E - \tau_1)/G$

With regard to the damage evolution law for the interface, an energy-based propagation criterion is adopted, according to which the crack development will move forward when a sufficient part of elastic energy is available:

$$\mathcal{R}(\beta) = \begin{cases} R_{mcr}\dfrac{\beta}{1-\beta} & 0 \le \beta \le \beta_{cr} \\ R_{mcr}\left\{\dfrac{\beta}{1-\beta}\right\}^{-0.8} & \beta_{cr} \le \beta \le 1 \end{cases}$$

3.2 Scaling operators

The exchange of information between meso and micro-level is ruled by proper down-scaling and up-scaling operators. Actually, in the algorithm proposed, the switching between meso-level (the one-dimensional interface-joint) and micro-level (the micromechanical world contained within a single material point of the mortar joint) is roughly simplified.

The insight into the micromechanics of the mortar is performed only in a reduced set of points, which are the centres of the interfaces through which the interaction among the rigid blocks take place. Once the mesoscopic strains and stresses in the centres are known, a *specific* stress value is calculated by simply smearing the mesoscopic values over the entire length of the interface and it is used in order to calculate the actual value of β. This value is supposed to represent, in the average, the state of all the material points of the interface, that is to say, the micromechanical state of the centre turns out to be a smeared measure of the cracks developed.

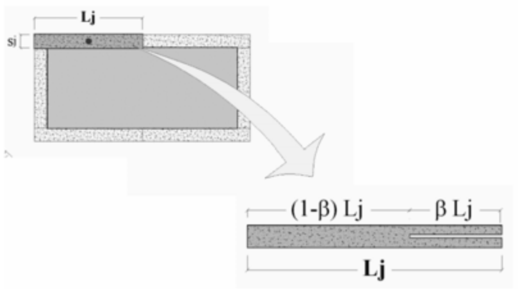


Figure 5 : Switching between mesoscale and microscale

4 THE MULTISCALE EVOLUTIVE ANALYSIS

At this point, all the operators to be used for switching from one scale to the other have been defined. It is now possible to precise where the multiscale algorithm is recalled within the evolutive analysis, which would otherwise be quite standard.

4.1 Operational definition of the iteration matrix

The first crucial moment is the definition of the iteration matrix $\tilde{\mathbf{K}}$ to be used in the correction procedure. For this task, it is chosen the tangent stiffness operator: $\tilde{\mathbf{K}} = \mathbf{K}_0$ assembled at the initial configuration (unstressed structure). In the commutative diagram of Fig. 6 the iteration matrix is given in terms of the macro/meso up and down-scaling operators ($\mathbf{D}$, $\mathbf{D}^T$) and in terms of the linearized meso-constitutive relation. Since we are in the unstressed configuration, all the mortar joints are uncracked and so further scaling is not necessary.

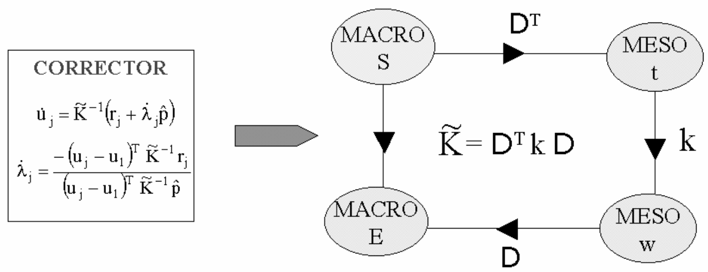


Figure 6 : Definition of the tangent stiffness operator to be used in the iterative process.

4.2 Definition of the structural response

The second relevant point in the multiscale characterization of the step-by-step analysis is the definition of the structural response **S[u]**, that has to be constructed at each correction loop of every step as a function of the nodal displacements **u**. Starting from the macro displacements and strains, a scaling is first performed to the meso level (defining the mesoscopic deformation **w**), and then to microscopic scale (defining the micro-deformation ε). Once the microscopic strains are available: 1) the constitutive relations are applied; 2) the actual values of the damage parameters and of the residual frictional sliding for each interface in the REV are updated. Finally, up-scaling operators are invoked in order to come back to the meso level, first, and to the macro level, then, obtaining the macro-stresses and the desired structural response (Fig. 7).

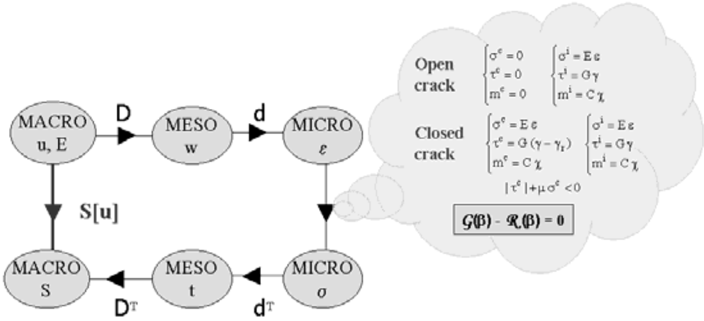


Figure 7 : Commutative diagram showing the definition of the structural response

5 NUMERICAL RESULTS

At this stage, the work was intended to provide a feasibility analysis for the use of the multiscale approach in the field of masonry mechanics. Some first basic tests have been performed, in order to investigate the effectiveness of the algorithm with respect to the elementary deformative modes. The first tests presented are simple tensile ones. Panels having different sizes have been subjected to an incremental loading perpendicular to the bed joints and to the head joints, respectively. The numerical results of the analysis are reported in terms of equilibrium paths, compared for the different cases.

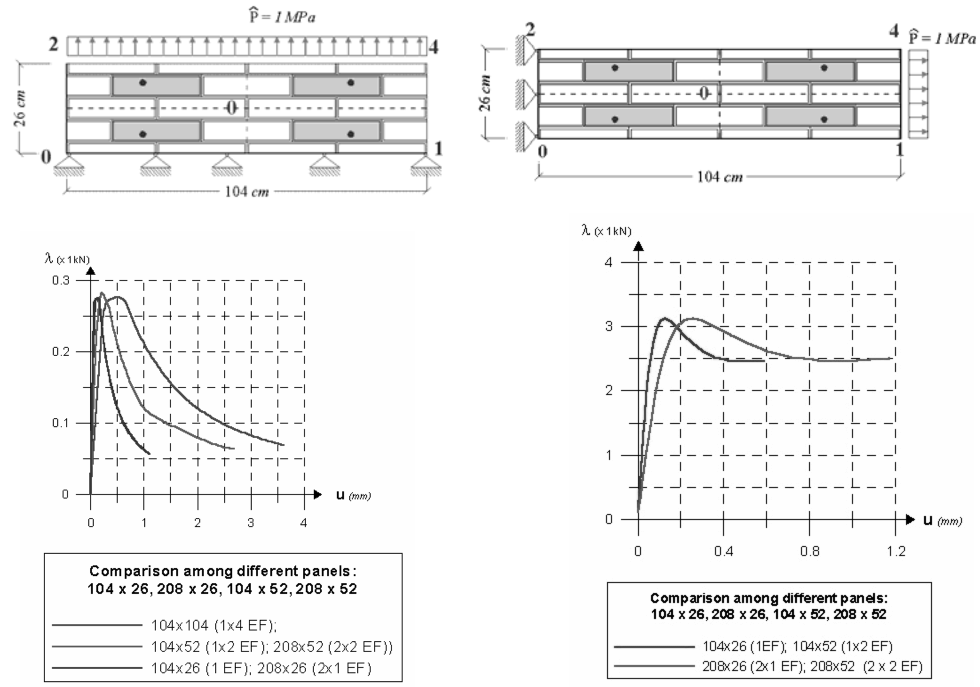


Figure 8 : Elementary damage modes. (Left) Tensile stress perpendicular to bed joints for panels of different height and width: the basic panel (1 finite element); equilibrium paths for the different panels. (Right) Tensile stress perpendicular to head joints for panels of different height and width: The basic panel (1 finite element); equilibrium paths for the different panels.

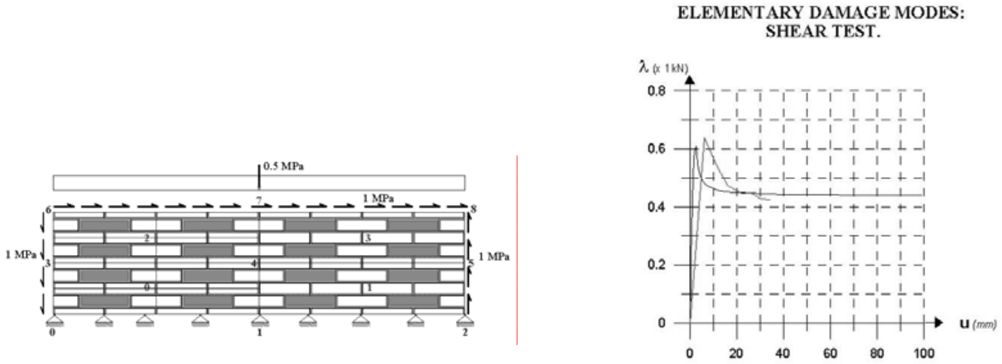


Figure 9 : Elementary damage modes: shear stress.

In Fig. 9, two curves are shown: the equilibrium path obtained by the multiscale approach (right) and one obtained by a fully discrete analysis (left). The multiscale approach slightly overestimates the limit load of the test panel, showing a stiffer behaviour. This can be motivated by the compatible character of the used homogenization procedure. The difference in terms of damage and displacement maps at the final configuration is not relevant. The number of d.o.f. of the multiscale analysis is about ten times lower than discrete systems.

6 CONCLUSIONS

The multiscale algorithm proposed has been designed with the idea to take advantage of a computationally effective transfer of information between a macroscopic level and a microscopic one. Simplifications have been introduced that would not restrict the generality of the problem, retaining the essential features necessary to explore the possibility of the method and build a numerical code to be gradually improved and enriched. Results indicate a good potentiality with regard to the requirements of computational convenience and accuracy. The low cost of the method relies on the fact that each scaling is very cheap (scaling operators are written in an explicit form), whereas the good accuracy is related to the circumstance that every single REV is considered in the analysis (by means of the Gauss point), in the spirit of a truly Discrete Approach. However, some crucial aspects emerged. The most critical point is related to the strong non-linearity induced in the reconstruction of the equilibrium path by damage development and friction. In some tests, severe difficulties of convergence have been encountered: the use of a Cosserat macro model does not seem to regularize the incremental problem. It could be supposed that the failure in the convergence depends on a fictitious locking phenomenon (Garcea 1998), to be further investigated by means of a spectral analysis. A possible remedy could be to resort to improve the path following analysis by explicating the variables governing damage, which are presently managed as classical internal variables.

REFERENCES

De Buhan P., de Felice, G. 1997. A homogenization approach to ultimate strength of brick masonry. *J. Mech. Phys. Solids*, n. 7, vol.45, pp. 1085—1104

Gambarotta, L., Lagomarsino, S. 1994. Damage in brick masonry shear walls, in Z.P. Bazant, Z. Bittnar, M. Jirasek, J. Mazars eds., *Fracture and Damage in Quasi brittle Structures: Experiments, Modelling and Computer Analysis*. E\&FN Spon, pp.463-472.

Garcea,G., Trunfio, G., Casciaro, R. 1995. Mixed formulation and locking in path-following nonlinear analysis. *Computer Methods in applied mechanics and engineering*. 165, pp.247-272.

Suquet, P. 1987. Elements of homogenization for inelastic solid mechanics. in: *Homogenization Techniques for Composite Media*, *Lecture Notes in Physics*, 272. Springer-Verlag, pp. 193-279.

Uva, G. 1998. A constitutive model with damage and friction for masonry structures: theoretical and numerical aspects. *Proceedings of the Workshop on Seismic Performance of Monuments "Monument-98"* Lisbon.