Structural Analysis of Historical Constructions, New Delhi 2006
P.B. Lourenço, P. Roca, C. Modena, S. Agrawal (Eds.)

Strengthening of Neapolitan Domes Between the XVII and XVIII Century: Historical and Structural Analysis

M. Lippiello
Second University of Naples, Dipartimento di Ingegneria Civile, Aversa, (Ce),Italy

L. Dodaro and M.R. Gargiulo
University of Naples, Dipartimento di Costruzioni e Metodi Matematici in Architettura, Naples, Italy

ABSTRACT: The goal is to analyze reinforcement actions of some Neapolitan domes. At first some technical reports were critically interpreted. The engineers were called to express their opinion on the origin of the damages and to propose actions to be taken. Those conflicting opinions give rise to technical solutions completely different, whose validity will be analyzed in the light both of the development of the static theories on the vaulted buildings and of the instruments furnished by treatises of the time. Aim of the analysis is the comprehension of the domes structural behaviour, so as imagined by architects of the XVIII century.

1 INTRODUCTION

Between the end of the XVII and the beginning XVIII century, the Neapolitan architecture history was characterized by many reconstructions, due to various intensity earthquakes that unsettled the city. The first in 1688 was very serious; then, there were two tremors of smaller entity in 1694 and in 1702; finally, another strong quake in 1732. These sudden and unexpected events, happened in a short space of time, put to hard test most of the city buildings; not by chance, from the end of the XVII century, there were an intensification in the building sector, with smaller factories that were readapted as well as possible, while the great religious and civil constructions became a sort of "open building yard", continuously operating.

The seismic event of 1688 caused many damages to most of the buildings but it didn't determine substantial alterations of the city, because it didn't impact on the urban plan. On the contrary, the transformations suffered by the urban landscape were well visible: it lost some peculiar elements and also some important ancient monuments. Besides the famous Castore and Polluce's pronaos, in this occasion and during the other events that followed we assist to the collapse, of important elements that characterized its profile; for example the high and slender lanterns, the battlements and a lot of other architectural decorations that crowned the most representative buildings.

The Gesu Nuovo dome's collapse (fig.1b), built on Valeriano's design, between the 1629 and 1634, was the second great disaster in Naples after that of Dioscuri's temple. The great and famous structure, with a double cap, culminating in a very high lantern, was the symbol of Jesuits' power who stared immediately the reconstruction of it, engaging the Architect Arcangelo Guglielmelli for the design. He planned a single cap dome; static reasons induced to discard the solution of the double cap, certainly to lighten the underlying structures from the weight of the dome. Nevertheless, new cracks began to appear and the still acting compressive stresses exerted by the dome weight provoked new damages in many parts but also the increase of those already existing. From that moment, a real debate about the stability of Gesu Nuovo's dome was opened, and in 1774 it ended with the dome demolition and its substitution with a "scodella" (depressed spherical vault).

Similar considerations were probably done for the choices followed for the repair of St. Gennaro's Treasure's Chapel (fig.1c), matter of our search, whose dome suffered some damages that aroused many fears. To improve the stability, the first immediate interventation, that didn't solve the problem, was the removal of the lantern and its changing with a wooden copy, overlaid in lead. The problem of the conservation of the prestigious monument aroused, as we'll see, a wide debate between those experts who substained the urgency to encircle the exterior dome with a further iron ring and those one who opposed for fear of damaging it.

For the other domes, which hadn't the prestige of the previous ones, the removal of the lantern, that was the remedy adopted with great frequency during restoration works, compromised a definitive impairment so that, even today, it remains a proof of the damages caused by the earthquakes. The most known example is that of St. Maria Maggiore' church, called of the "Pietrasanta" (fig.1a), whose dome was seriously damaged by the earthquakes, so that it wasreputed essential to demolish the lantern in stone in order to lighten the loads upon the structure below. The Architect Giuseppe Lucchese was engaged for the demolition but also for the hoop of the dome and of the drum. After the earthquake of 1732, the dome was, once again, object of numerous restoration works. The greatest breakdowns interested the supporting big arches, that appeared seriously damaged; the Architect Lucchese believed that these, as the other structures of the church, of the convent and of the bell-tower, had to be reinforced affixing iron chains. In the case of St. Sebastiano, built to the beginning of the XVII century and notably damaged by the earthquakes of 1688 and of 1694, Ferdinando Sanfelice and other experts signed a report, in 1732, where they proposed a lightening of the dome weight, eliminating eight columns and eight parastades from the lantern, and rebuilding the little dome in Vesuvian stone because it resulted less heavy.

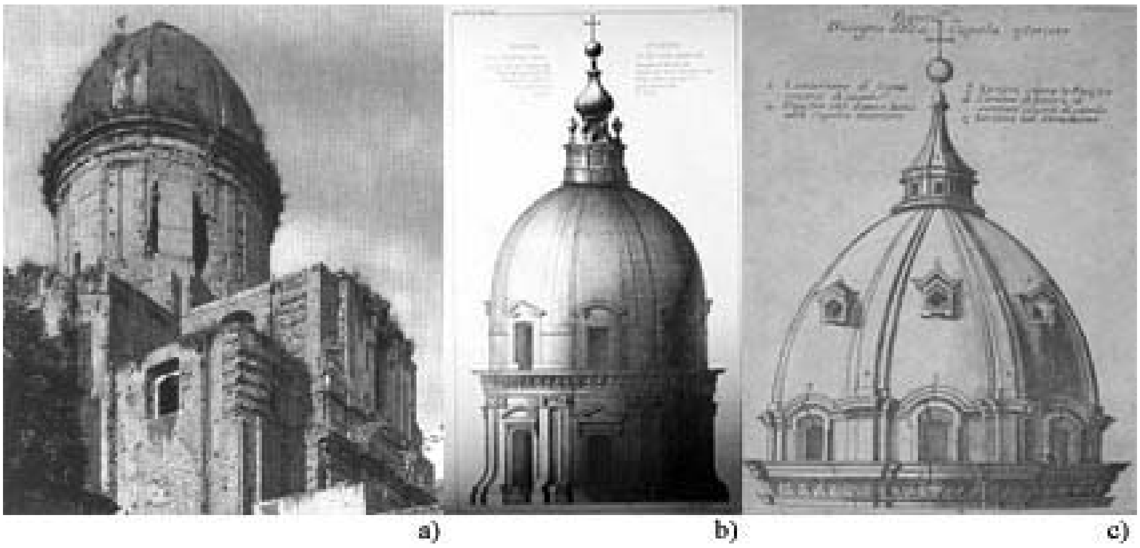


Figure 1 : The domes of: (a) S. Maria Maggiore; (b) Gesù Nuovo; (c) St. Gennaro.

The earthquakes of 1688 was stronger then the following ones but the operation of strengthening didn't give rise to the vast amount of works that were practiced after the tremor of 1732. The yards, opened between the 1733 and 1736, were so numerous that it is thought many city "congregations" took the opportunity to start works of reparation or of enlargement of their own buildings. The emblematic example for the way of operation was the restoration of the Trinità Delle Monache's monastery, done by the Architect Tagliacozzi Canale. The dome consolidation works consisted in lifting, and then putting again to place, all the "riggiole" (majolica- tiles) to close the cracks, restored with the insertion of "asche" (spells resulting from the working of construction blocks), forced in the masonry and jointed with the mortar. A curious detail concerns the rearrangement of the lantern, where the windows were wall up to obviate to cracks. The added masonry, in correspondence of the openings surely increased vertical efforts to which the dome was submitted, and the principle that had guided many interventions after the earthquake of 1688 had been really the opposite, to lighten the dome with the lantern demolition.

The characteristic that unites all these Restauration works consists in the absolute lack of measures of prevention of the damages; all the surveys show the conviction that the observance of the constructive rules in the only way to implore the repetition of other disasters.

2 CASE STUDY: THE ST. GENNARO TREASURE CHAPEL DOME

Of particular interest, for the wealth of documentation still available today, is the study of the surveys realized for the restauration of the dome of St. Gennaro's Treasure's Chapel.

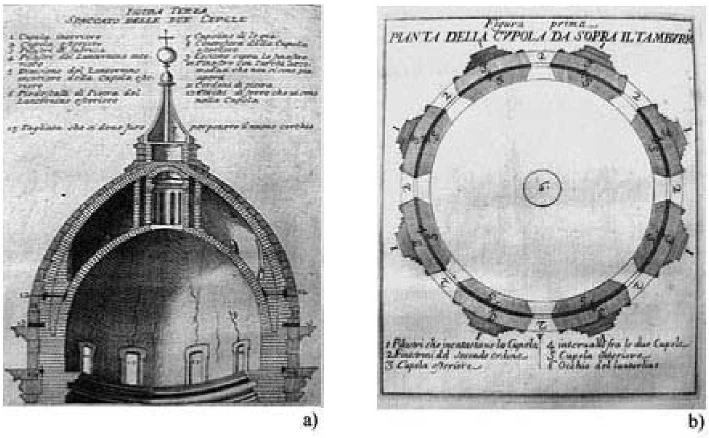


Figure 2 : Dome's section and plane drown by Sanfelice.

The dome, abutted on a high drum of a circular base, circled on the summit (fig.2b), has a double cap with non parallel generatrix curves (fig.2a). In reality, the dome is not abutted on the drum summit, but it rises with a straight wall of about 16 palms (1 palm = 26,4 cm approx), the so-called "false drum", a typical element of the Grimaldian architecture, characterized by the presence of small openings of depressed arches. The outer cap, with elliptical arches, visible to all city, which begins to curve at the height of 10 and ¾ palms from the summit of the false drum, has an external diameter of 73 palms and a maximum height from the lantern of 43 palms. The inner cap, on the other hand, set up on the summit of the straight wall, has an external diameter of about 60 palms and is approximately 37,50 palms high. The thickness of the two caps varies from 3 palms at the top of the false drum up to about 1,5 palms to the altitude of the small double drum that originally connected the two caps. The latter, a building of bricks of sixteen small pillars and sixteen small arches, is formed by two orders of tympanums: an external one, of 20 palms diameter from outside, and an internal one of 13 palms external diameter and an altitude of 14 palms. At the crowing of the dome, set up on the exterior cap, there is, at the moment of debate, a small blind lantern in oak wood, finishing with two ampullas, of about 18 palms external diameter, 19 palms high and 1 palm thick (fig.3b). This substituted the original stone lantern which was strongly damaged by an earthquake in 1688.

Technical reports displayed that the dome presented a fissure outline quite widespread (fig.3a), in both the caps, highlighting the presence of various types of cracks, different for gait and dimension. A first order of cracks, which are the widest present, can be found in the intrados of the two caps. From the survey of that time, the cracks in the drum resulted to be of little importance. They also pointed out a horizontal crack in the two orders of tympanums connecting the two caps as can be seen in Fig. 3a.

In formulating the restoration hypotheses, the scientific community is divided into two different currents. The former, inspired by the Architect Giuseppe Lucchese, sustained that the dome stability was seriously compromised; the cause of the cracks was attributed to the weakness of the wall structure of the straight wall (false drum) on which the two caps were planned and, therefore, it suggested to add to the three existing hoopping a further iron bandage in correspondence of the false drum summit. The latter, whose most important exponent was the Architect Ferdinando Sanfelice, disagreed with the cracks cause, and believed that the damages couldn't be such as to compromise the structure stability; for this reason they didn't accept the dome hoopping considering this operation "not only useless but also pernicious, being certain the damage and most uncertain the advantage" . They believed that the damages were caused by the excessive superstructures weight: the lantern in stone and the double small drum. Considering that the substitution of the old lantern in stone with the lighter small lantern in oak wood had been sufficient to stabilize the cracks of the outer cap, it would have been enough to eliminate the small drum external circle to lighten the inner dome and to guarantee its stability.

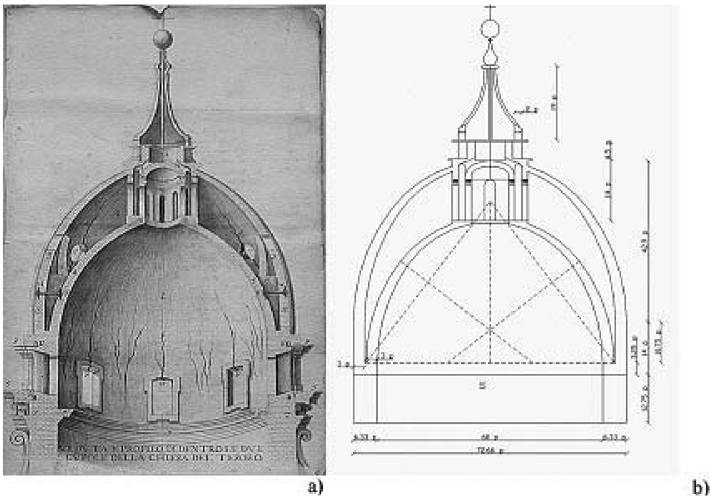


Figure 3 : (a) Lucchese's section; (b) section redrawn by the authors.

The opinions of Lucchese and Sanfelice repropose, within the Neapolitan ambit, a debate which was widespread in the nation and well documented in Poleni's work " Memorie Istoriche della Gran Cupola del Tempio Vaticano" printed in Padova in 1748. Their contrasting motivations find confirmation in similar disputes risen for the consolidation of the domes of S. Maria del Fiore in Florence, of the Cathedral of Montefiascone, of St. Marco in Venice ecc., and reveal how the new, but yet uncertain, scientific theories on materials' behaviour and of vaulted buildings begin to interfere with the tested "*art of building*", to offer justifications beforehand to a "*modus operandi*" based on a solid experience of building yards and on a careful observations of the details.

A point which characterizes the disagreement is the Lucchese and Sanfelice different cultural forming: Lucchese and his supporters had a yard experience but they knew the rising scientific rules. They showed a larger opening to the new consolidation techniques and a lesser elasticity in the interpretation of structural complex. Sanfelice, on the contrary, confirmed, also in this circumstance, his extraordinary structural insight. He was sure, despite of the evident cracks, that the structure was stable and he didn't deem necessary to avail himself of theoretical reasoning to justify his opinion. He refused any operation, unless it was essential, which could change the primary plan. The logic and coherence of both the positions justify the indecision of the Deputation to effect one of these consolidation projects.

The documents in the archives, following the publication of the opinions of Lucchese and Sanfelice, denounce how the Commission retained necessary to carry out other inspections before authorizing any intervention of restoration. Only in 1724, following a project of D. Teodoro Gallarano, one intervened on the dome eliminating the two small drums and building, in the cavity wall between the two caps, a wooden scaffolding braked by tie-beams, to support the exterior dome. Unfortunately, few indications have remained of this original intervention so it is difficult to reconstruct the logic of the author's project.

A point which characterizes the difference of opinions is the presumed weakness of the wall structure, and both the Authors use the comparison with some existing building to guarantee their respective positions. In a former work (Cantabene, Dodaro, Lippiello, 2006) to verify if the walls thickness were conform to the eighteenth century constructive practice, was made use of

Vincenzo Lamberti’s text “La Statica degli Edifici”, printed in Naples in 1781, in which the Neapolitan engineer, on the example of Belidor’s essay, creates simple rules for the measure of structural elements, with an explicit reference to the construction typologies, for the building work and for the characteristics of the materials used, typical of the architecture in Campania.

The results obtained, applying the above mentioned “ practices” to the inner cap, isolated by the rest of the construction, underline a weakness of the building thickness; if we consider the steadying of the outer cap, on the ground of our examinations, the false drum size should be similar to XVIII century Neapolitan constructive customs.

The Lamberti’ “pratiche”, based on levers theory, aren’t rich enough for domes analysis. In this paper the structure’ thickness will be verified using the well known model for the axial-symmetric domes, based on the safe theorem of the limit analysis, as formulated by Heyman.

3 HEYMAN ANALYSIS APPLIED TO MASONRY DOMES

According to the Heyman’ theory the material has been supposed to have the following properties:

1. masonry has no tension strength
2. the compressive strength of the stones is effectively infinite
3. sliding of one stone upon another cannot occur.

These classical assumptions reproduce in modern form the hypothesis on which the Couplet’ and Coulomb’ theories are based. The first property is safe, the second has been shown to be correct in practice and the third one is slightly unsafe.

“Accepting these postulates of the material behaviour, the uniqueness theorem may be stated for masonry as follows: if a line of thrust can be found which represents an equilibrium state for the structure under the action of the given external loads, which lies wholly within the masonry, and which allows the formation of sufficient hinges to transform the structure into a mechanism, then the structure is on the point of collapse; […]. “The safe theorem may be stated as follows: if a line of thrust can be found which is in equilibrium with the external loads and which lies wholly within the masonry, then the structure is safe (Heyman 1965).”

Consequently the structure safety is assured if a static admissible state is found.

The thin shell theory is the most common approach to study the masonry domes: the membrane analysis assumes that stress resultants act in the middle surface of the shell and bending does not occur. As known this particular problem is statically determinate and the stress resultants can be found without reference to compatibility conditions or to material properties. Actually this theory is not completely suitable for masonry domes:

- tensile stress will not occur. The membrane theory has to be limited to the upper portion of the dome, until compressive hoop stresses are developed; the classical orange slices technique can be used for the lower part;
- dome thickness is not usually thin and therefore the thrust surface is not necessarily coinciding with the middle one.

3.1 The St. Gennaro Chapel dome modelling

In this section two approaches are applied and discussed with reference to the St. Gennaro dome.

The dome is made of two masonry caps, set on a big drum. The inner one, with a diameter of about 15.84 m, is the main support of the double drum of 5.28 m in diameter, from outside, and about 3.70m in height; the outer cap, 19.27 m in diameter, is much soaring and it is a support of the lantern of 4.75 m in diameter and 5 m in height.

Due to the particular shape of the dome [see fig.3b], with two different geometry caps, the equilibrium states of the two shells are investigated independently.

In this study, the two caps structural behaviour is evaluated through two tools: essentially, they are graphic methods of structural analysis that establishes the limits of the thrust surface for the equilibrium condition in the domes. The graphical method for the limit analysis, using the well known concept of a thrust line, is considered. This is a theoretical line, which represents the path of the compressive forces resultants though the stone structure. According to the “safe theorem”, for a pure compression structure to be in equilibrium with the applied loads, there must be a thrust surface that lies entirely within the section.

As a first stage of analysis, the dome structural behaviour is investigated on the basis of classical shell theory. In this approach, it is well known that, in an hemispherical shell under its own weight, the hoop stresses are compressive from the crown until an angle of 51.82° and tensile in the lower portion.

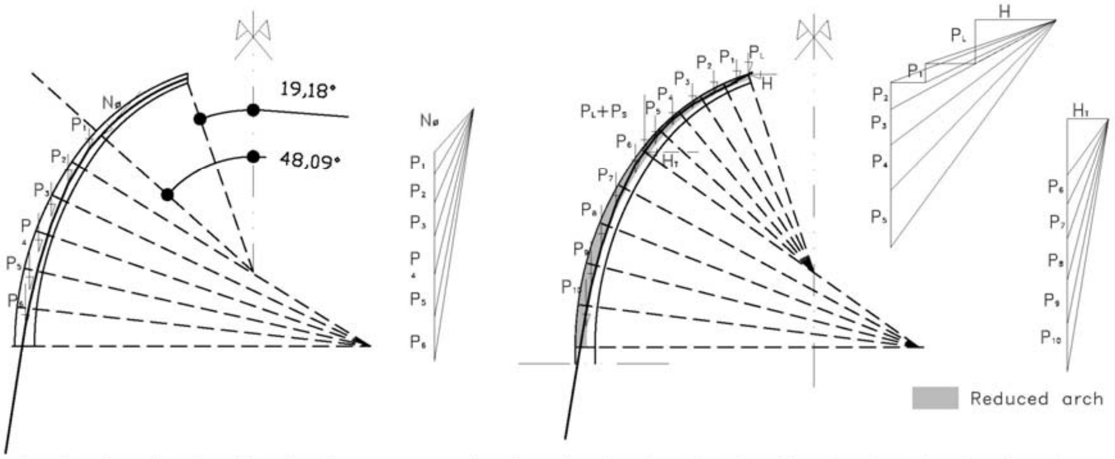
Being the shell upper part a spherical cap portion, the thrust surface is assumed coinciding with the shell middle surface up to the parallels undergo compressive stresses. For the dome in exam, the conventional membrane theory has to be modified to take into account the eye size and the lantern weight on the outer cap or the double drum weight on inner cap. As a consequence, the angle of change of sign in the circumferential stress is unknown and depends on these parameters.

Under this level the cap is sliced in sixty parts and each of them is assumed to behave as a cross section varying arch whose solution is obtained by a funicular polygon: the aim is to verify the forces polygon existence within the arch section.

Applying the above procedure to the outer dome and considering a constant thickness of the dome upper part, the following data have been assumed:

- dead load $q = 0,675\ t/m^2$
- eye radius $r = 8,2317$ m
- lantern weight $P = 1,5788\ t/m$

and an admissible static solution has been found (fig.4a) in which the membrane behaviour is valid until an angle of 48,09°, value not far from the angle (≅ 46°) corresponding to the starting point of Lucchese’ crack pattern.



Nø	P1	P2	P3	P4	P5	P6
3,71	0.90	1.15	1.37	1.56	1.69	1.76

PL	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	H1	H
0.44	0.19	0.27	0.36	0.46	0.57	1.15	1.42	1.64	1.80	1.89	1.67	0.82

Figure 4 : Outer dome trust lines: (a) shell solution; (b) funicular polygon.

By this procedure a satisfactory result has not been obtained for the inner cap. The considered data are:

- $r = 2,64$ m
- $P = 1,9206\ t/m$ (considering the double drum);
- $P = 1,0749\ t/m$ (considering only the first enclosure of the drum).

Near the cap “eye”, the lantern weight gave rise to tensile forces.

Masonry domes are not thin shells therefore the thrust surface can differ from the middle surface of the dome both for position and shape. Instead of looking for different suitable surfaces, on the basis of the previous theory, a different approach to the dome stability verification has been attempted.

Then, in this step of analysis the problem reduces to the one of a cross section varying arch, subject to self-weight and external loads. The dome can be imagined as composed by a series of arches obtained slicing the cap by meridian planes: on each voussoir, the compression action resultant of the hoop forces is applied as an outside horizontal force (fig.5). If it is possible to draw a thrust line within this arch, then an equilibrium state in compression is found and the dome is safe, it will not collapse.

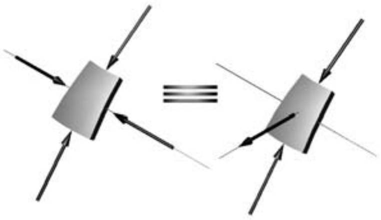
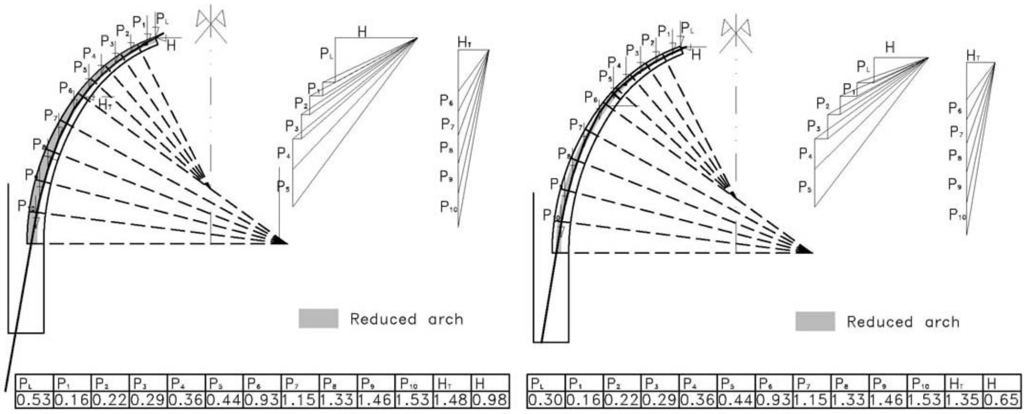


Figure 5 : Hoops stresses resultant.

Because of dome axial-symmetry, horizontal forces are independent from loads and meridian stresses, therefore their value can vary arbitrarily in order to adapt the thrust line to the thickness of the dome.

The procedure applied to the outer dome, dividing each arch into ten voussoirs, furnish a thrust line, wholly inside the arch thickness, very close to the middle line except in the last two voussoirs. The relative funicular polygon is sketched out in fig.4b. The pier control was not carried out since the reinforcement ring absorb the thrust horizontal component.



P_L	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	H_T	H
0.53	0.16	0.22	0.29	0.36	0.44	0.93	1.15	1.33	1.46	1.53	1.48	0.98

P_L	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	H_T	H
0.30	0.16	0.22	0.29	0.36	0.44	0.93	1.15	1.33	1.46	1.53	1.35	0.65

Figure 6 : Inner dome trust lines: (a) with double drum; (b) with one drum.

The inner dome arch, divided into ten voussoirs too, was tested both with the double drum (P = 0,531 t) and without the first enclosure (P = 0,2972 t), as proposed by Sanfelice (fig.6). Since in both cases the last force of funicular polygon is within the false drum bottom, its stability is certainly verified.

The fundamental theorems of the limit analysis permit also to calculate the safety of masonry arches. Heyman has proposed a geometrical factor of safety obtained comparing the geometry of the actual arch with that of the "limit arch" which will just support the loads. The basic assumption of this approach is that each thrust line represents one of the infinite solutions of arch possible equilibrium configuration.

The thrust lines obtained for the arches of the St. Gennaro Treasure Chapel dome do not generate any collapse mechanism and then it has been drawn a reduced thickness arch which do not correspond to the "limit arch" defined by Heyman. On the basis of that, a factor of safety, for the dome in object matter, has been calculated: maintaining the dome ratio (variable cross section and polycentric arch generant) "reduced" thickness arches able to fully lodge the thrust lines, have been sketched out: both for outer and inner cap, a collapse mechanism has not been found; consequently, the obtained safe factors (actual thickness-reduced arch thickness ratio) represent an "upper bound" for the effective coefficients. Since the geometrical safe factor results to be about two, a further refinements of the problem it seemed to be unnecessary.

4 CONCLUSIONS

In this paper the safety of the dome of the Chapel of St. Gennaro is analysed. Two different approaches are followed in order to evaluate the safety of the structure and the related results are compared in order to evaluate the consolidation suggestions, proposed by Lucchese and Sanfelice.

The aim of this work, particularly, has been the rereading of two historical and technical reports, using the limit design criteria applied to masonry constructions. Utilizing this modern structural language, in our opinion, the traditional geometrical theory of the old master builders, can be re-interpretate with a great immediateness.

The results, that have derived from these approaches, confirmed the Treasure dome stability, in spite of the wide fissure outline showed. Considering the normal operating conditions, the dome does not show stability problems, and its geometrical factures assure a good safety coefficient. In the light of the modern knowledges, the Lucchese' perplexities were then unjustified. Equally, the intervention, proposed by Sanfelice, to eliminate the first enclosure of the drum, resulted little meaningful. In fact, as already showed in a previous paper (Cantabene 2006), the obtained advantage, eliminating this structure, is paltry. The used technique, for the purposes of this research, has furnished satisfactory results.

The research is still work in progress; particularly, it will be extended to different domes taking into account not axial symmetric loading conditions and seismic actions too. Besides the application of a numerical algorithm will be investigated at the light of the actual knowledge and technical laws.

REFERENCES

Cantabene, G. 2002. *Al tempo dei tremuoti. Restauri ed ammodernamenti a Napoli dopo i terremoti del 1688 e del 1732*, unplished PhD thesis. University of Palermo.

Cantabene, G., Dodaro, L. and Lippiello, M. 2006. *The repairs to the dome of the Chapel of St. Gennaro's Treasure: the Eighteenth-century dispute between F. Sanfelice and G. Lucchese*. In M. Dunkeld, J. Campbell, H. Louw, M. Tutton, B. Addis and R. Thorne (eds), *Proc. Of the Second International Congress on Construction History*, p. 561- 577. Cambridge.

Heyman, J. (Cambridge University Press) 1965. *The Stone Skeleton*. Cambridge.

Heyman, J. 1967. *On shell solution for masonry domes. International Journal of Solids and Structures* 3, p. 227-241.

Lucchese, G. 1707. *Parere circa il riparo da darsi alla Cupola della Cappella del Tesoro di S. Gennaro. Archivio Storico della Deputazione del Tesoro di S.* Gennaro, Napoli.

Sanfelice, F. 1708. Parere circa il riparo da darsi alla Cupola della Cappella del Tesoro di S. Gennaro. Archivio Storico della Deputazione del Tesoro di S. Gennaro, Napoli.