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Seismic Vulnerability Assessment of Qutb Minar, India

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ABSTRACT: This study addresses the Seismic vulnerability of Qutb Minar, the tallest Minaret in the world. The seismic vulnerability assessment of the structure is explained in terms of the hazard assessment at the site, numerical modelling, site response analysis, and dynamic response spectrum analysis of the structure. Finally the results are discussed in terms of thrust lines and principle stress contours.

1 INTRODUCTION

The damage suffered by tall and flexible towers in the Kutch area of Gujarat in the 2001 earthquake had drawn attention to the seismic behaviour of Qutb Minar, one of the tallest stone masonry towers in the world. This 12th century architectural marvel is an important landmark of Delhi, the capital city of India. Delhi region falls in seismic zone IV in the seismic zone map of India. The zone has fairly high seismicity with a general occurrence of earthquake of 5-6, a few of magnitude 6-7 and occasional incidence of 7.5-8 magnitude shocks. Among these incidents the 1803 Gharwal-Kumaon earthquake had brought down the crowning cupola of the Minar and left the Minar seriously damaged. The seismic history of the city reveals the seismic threat on this world heritage monument and calls for the vulnerability assessment of the structure. In this background the paper reports the seismic vulnerability assessment of the structure including the hazard assessment at the site, the numerical modelling of the structure and the dynamic response spectrum analysis of the structure.

2 QUTB MINAR

The construction of the Minar started in 1198 AD and attained its present status in 1368AD through three phases. In plan the minar is circular, the base being 14.07 m in diameter and it tapers to a diameter of 3m at the summit along a height of 72.45m. Fig. 1 shows the sectional elevation and plan of the minar at each storey level. It consists of an externally fluted shell and an inner shaft, which supports a spiral stairway of Delhi quartzite stones. In its artistic aspect the most elegant features of the Minar are the balconies supported by a system of stalactite bracketing.

The Minar shell masonry consists of a thick rubble stone masonry infill faced externally with ashlars of red sandstone and internally with Delhi quartzite stone. The central shaft is of rubble stone masonry with quartzite stone facing. The foundation is 10.2m deep and was strengthened by cement grouting during 1971-1972 on observing a tilt in the verticality of the Minar.

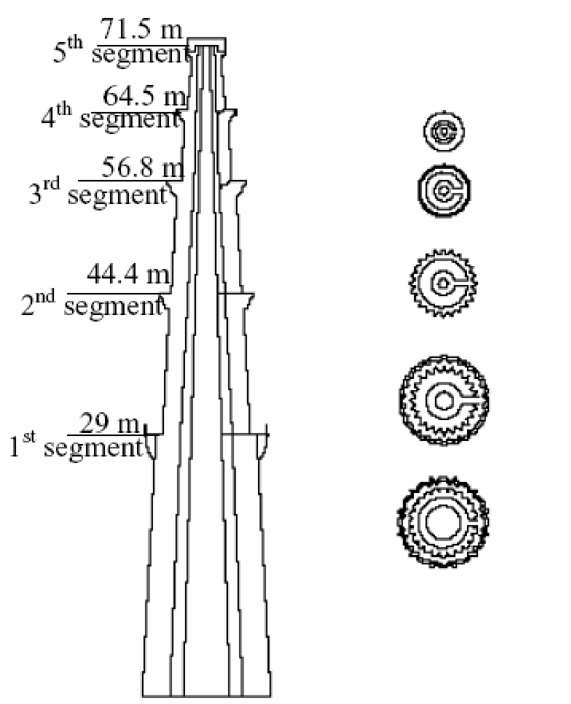


Figure 1 : The sectional elevation and plan of the Minar at each storey level

3 HAZARD ASSESSMENT

The seismic hazard assessment is the procedure of specification of the ground motion parameter at a particular site due to the nearby seismic sources. The probable seismic hazard varies spatially depending on the nature and location of the seismogenic faults. At a particular site this hazard estimation is further depends on the soil layers above and the depth of bed rock. Depending on the soil strata of propagation the surface level motion can get amplified to many folds of bed rock motion and vice versa. So the calculation of the site specific response spectrum is very important as far as a seismic vulnerability analysis is concerned.

Ghosh (2003) has calculated the bed rock level PGA at Qutb Minar with 10% probability of exceedance in 100 years as 0.167g. The PGA at the base of the minaret is obtained through a site response analysis. Thirteen natural earthquake records are scaled to the bed rock level PGA at the Minar. These scaled records have been propagated through the soil stratum at the Minar site. The response spectrum at the surface level is obtained for all the thirteen ground motions. An average spectrum of all the calculated spectra is taken as the site specific acceleration response spectrum for the seismic load input.

The site specific response spectrum for a return period of 1000 years obtained from the site response analysis is plotted in Fig. 2. The spectrum specified by IS 1893 for the same return period is super imposed on the obtained site specific spectrum. It is evident from the plots that the IS spectrum underestimates the structural response in the frequency range around 1-6 Hz and overestimates the structural response in the frequency range around 6-25 Hz.

4 NUMERICAL MODELLING

The finite element modelling is done using the commercial finite element package ANSYS. The modelling procedure includes two steps as explained:-

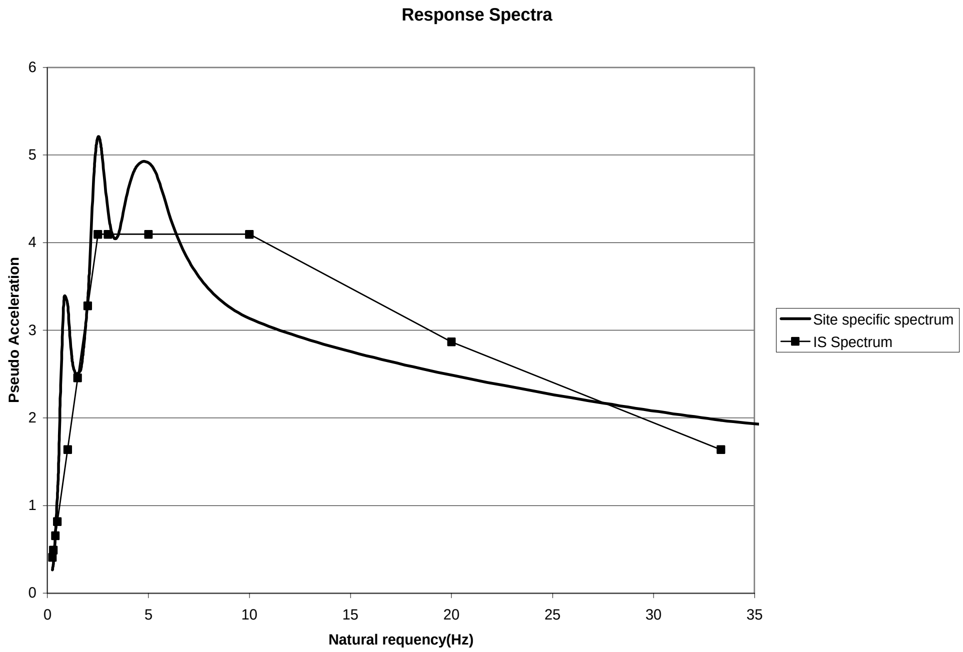


Figure 2 : Site-specific response spectrum

4.1 Geometrical modelling

Two different modelling strategies are adopted for the present study.

4.1.1 3-d beam modelling

The Minar is modelled using 3-D Timoshenko beam elements. These elements are suitable for analysing slender and moderately stubby/thick beam structures. A linear formulation is used with six degrees of freedom at each node. The tapering section of the Minar is modelled following the true geometry. The effect of central shaft and stairs are incorporated as 3-d lumped mass elements at each storey level. The tilt of the Minar is taken in to account from the base level.

4.1.2 Solid modelling

The 10 noded solid tetrahedron elements with three translational degrees of freedom per nodes are used for the solid modelling. The central shaft of the tower is modelled with the connection between the shaft and shell maintained only at the level of balconies through lintel slabs. The lintel slabs are modelled using SHELL 63 elements with membrane behaviour. It is found that this connection as adequate enough to ensure a proper coupled behaviour of the shell and shaft of the minar in the significant modes. So the stiffness of the shaft is also accounted in the 3-d solid model where as the same is ignored in the beam modelling. The main entrance and also the opening at the first balcony level are incorporated in the model. The externally fluted body of the Minar is idealised as circular.

4.2 Material modelling

The Minar masonry is idealised as linear elastic and the analysis is carried out for two different set of material parameters, one aligned to a stiffer end and the other one aligned to a more flexible end. In the 3-d beam analysis the material is considered as linear elastic and isotropic. The elastic modulus of the Minar shell is taken as the volumetric average of the modulus of each layer. At the higher stiff end the elastic modulus is calculated as 5.6GPa and at the lower flexible end it has been obtained as 1.4 GPa. A Poisson’s ratio of 0.2 is assigned for masonry. In the 3-d solid modelling the masonry is considered as linear elastic orthotropic. The homogenisation procedure followed is as explained in [5]. A Poisson's ratio of 0.2 is assumed for units and 0.35 is assumed for the mortar (Giordano *et al.* 2002). The ratio of elastic modulus of the mortar to that of the unit is assumed as 0.1. The rubble masonry is considered as linear elastic isotropic with an elastic modulus of 1GPa in the higher stiff end and 0.6GPa in the lower flexible end. The calculated mechanical parameters for masonry are listed in Table 1.

Table 1: Material parameters assigned for the ashlars masonry in Minar shell

		$E_{sandstone}$=39GPa	$E_{sandstone}$=7.8GPa
Ex (GPa)	Sand stone	20.6	4.1
	Quartzite	24.3	4.9
Ey (GPa)	Sand stone	14.4	2.9
	Quartzite	17.0	3.4
Ez (GPa)	Sand stone	27.0	5.4
	Quartzite	31.7	6.3
Gxy (GPa)	Sand stone	4.1	0.83
	Quartzite	4.9	0.98
Gxz (GPa)	Sand stone	7.5	1.5
	Quartzite	8.9	1.8
Gyz (GPa)	Sand stone	4.8	0.9
	Quartzite	5.6	1.1
Density (kg/m3)	Sand stone	2300	2000
	Quartzite	2600	2300

5 STRUCTURAL ANALYSIS

The structural analysis includes two parts: the static analysis under gravity loading and dynamic analysis under a feasible earthquake loading. The earthquake loading is represented by a site specific response spectrum and response spectrum analysis is carried out. The static analysis shows a very good margin of safety under self weight and tilt for both material parameters. The maximum principal compressive stress observed under dead weight is around 5MPa in the stiffer case where as it is about 2.5MPa in the flexible case. The seismic analytical procedure and the results obtained for the 3-d beam and 3-d solid models are discussed separately in the subsequent sections.

5.1 3-D beam analysis

5.1.1 Modal analysis

The modal analysis is carried out to obtain the natural frequency and vibration modes of the system. The dynamic properties of the tower corresponding to stiff and flexible material parameters are listed in Table 2 and 3 respectively. All the modes with in the first 33.33 Hz frequencies are extracted and these modal frequencies correspond to a total mass participation (along global X direction) of 87.5% in the case of stiff material parameters and 89.5% in the case of flexible material parameters.

5.1.2 Response spectrum analysis

The tilted Minar has been analysed for a combined loading under site specific response spectrum and dead weight. The spectrum shown in fig is used as the seismic load input. A maximum horizontal displacement of 21.7 cm is observed in the case of stiff material parameters and 30.3 cm is obtained in the case of flexible material parameters. In the considered level of seismic excitation the stress levels in the masonry seems to be rather very high. So the results are discussed in terms of the thrust lines.

Table 2 : The dynamic properties of the tower for the stiff material parameters

Mode	Natural frequency (Hz)	Individual modal participation mass (percentage)			Cumulative Modal Participation Mass (percentage)			Predominant Vibration Direction
		U_X	U_Y	U_Z	U_X	U_Y	U_Z	
1	1.11	0.00	0.00	38.82	0.00	0.00	38.82	Z
2	1.11	38.82	0.00	0.00	38.82	0.00	38.82	X
3	3.26	0.00	0.00	22.53	38.82	0.00	61.35	Z
4	3.26	22.53	0.00	0.00	61.35	0.00	61.35	X
5	6.5	0.00	0.00	9.24	61.35	0.00	70.58	Z
6	6.5	9.23	0.00	0.00	70.58	0.00	70.58	X
7	7.69	0.00	0.00	0.00	70.58	0.00	70.58	Torsion
8	8.12	0.00	64.31	0.00	70.58	64.31	70.58	Y
9	10.46	0.00	0.00	6.79	70.58	0.00	77.37	Z
10	10.46	6.79	0.00	0.00	77.37	0.00	77.37	X

Table 3 : The dynamic properties of the tower for the flexible material parameters

Mode	Natural frequency (Hz)	Individual modal participation mass (percentage)			Cumulative Modal Participation Mass (percentage)			Predominant Vibration Direction
		U_X	U_Y	U_Z	U_X	U_Y	U_Z	
1	0.58	0.00	0.00	39.07	0.00	0.00	39.07	Z
2	0.58	39.07	0.00	0.00	39.07	0 00	39.07	X
3	1.71	0.00	0.00	22.6	39.07	0.00	61.66	Z
4	1.71	22.6	0.00	0.00	61.66	0.00	61.66	X
5	3.39	0.00	0.00	8.5	61.66	0.00	70.77	Z
6	3.4	8.5	0.00	0.00	70.77	0.00	70.77	X
7	4.04	0.00	0.00	0.00	70.77	0.00	70.77	Torsion
8	4.25	0.00	64.66	0.00	70.77	64.66	70.77	Y
9	5.47	0.00	0.00	6.73	70.77	64.66	77.51	Z
10	5.47	6.73	0.00	0.00	77.51	64.66	77.51	X

5.1.3 Discussions

Thrust line is a measure of the stability of the structure. Thrust lines are calculated at different sections along the height of the Minar and the stability of the structure against overturning is checked. Fig. 3 shows thrust lines calculated for a spectrum of 1000-year return period for the stiff and flexible material parameters, plotted along with the kern distance of the section. The kern distance represents the limiting eccentricity for the resultant thrust in order to prevent the development of tensile stresses at the section. This is a sectional property. For annular sections the maximum eccentricity limit of the resultant thrust so as to prevent the development of tension at the section can be calculated using the following formula.

$$e_{\lim} = \frac{r_o}{2}\left[1+\left(\frac{r_i}{r_o}\right)^2\right] \tag{1}$$

The eccentricity of the thrust line from the centre of the section is calculated as

$$e = \frac{M}{W} \tag{2}$$

where M is the Bending moment due to the seismic loading and W is the self weight of the structure.

It is clear from the Fig. 3 that once the material properties coincide with the higher stiff end the entire section of the Minar will be under tension. The thrust line falls beyond the Minar cross section from the top of the base segment. This is indicative of an overturning failure of the

Minar about any section above the top of the base segment. Where as, in the case of flexible material parameters the thrust line falls beyond the section only at the base of the third segment. So an overturning failure can occur about the base of the third segment. In the first segment the thrust line falls well inside the kern distance.

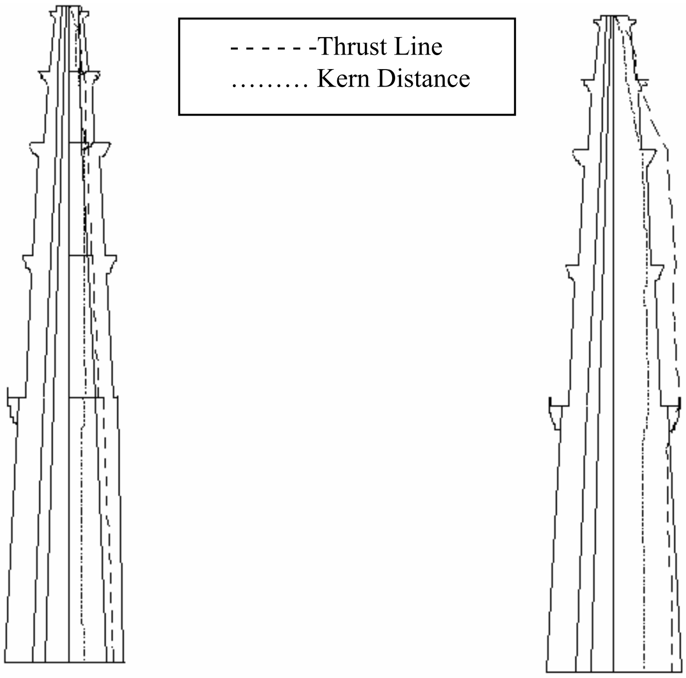


Figure3 : Thrust line for the Minar under calculated seismic and gravity loading

5.2 3-d solid analysis

The Minar is modelled in three dimensions and analysed for combined gravity and spectrum loading. The three dimensional model has incorporated the openings at the base of the first and second segments of the Minar. All the three masonry layers of the outer shell are integrated separately.

5.2.1 Modal analysis

Modal analysis is carried out in order to obtain the natural frequency and mode shapes of the structure for the stiff and flexible material parameters. A total mass participation of 90% is obtained within the 33.33Hz frequency range. A fundamental frequency of 1.064Hz is observed in the stiff end of material parameters and 0.53 Hz is observed in the flexible end of material parameters. The natural frequency values seem to be less than what observed from the beam model analysis. This is observed irrespective of the increased stiffness due to the incorporation of the central shaft in the model. But this reduction in the natural frequencies can be justified by the presence of the openings in the 3-d solid model, which had been neglected in the beam model. Fig. 4 shows the first two mode shapes of the Minar. Table 4 compares the various natural frequencies and mode shapes obtained for the 3-d beam and solid models. It should be noted that due to the unsymmetrical distribution of the openings the natural frequencies in the orthogonal directions are not exactly the same as in the case o the 3-D beam elements. In the solid model analysis the fifth mode is observed as torsional and the sixth mode as a translational mode. Similarly the sixth mode in the beam analysis is a torsional mode and the same in the solid analysis is a translational mode. These differences in the mode shapes are due to the inclusion of the opening in the solid model, which has been ignored in the beam model. So the solid modal results are more relevant even though it does not show much significant difference form the analysis results of the beam model.

Table 4 : Comparison of the modal analysis results of beam and solid models

Mode No	Natural frequency				Predominant direction	
	Solid model		Beam model		Solid model	Beam model
	For stiff material	For flexible material	For stiff material	For flexible material		
1	1.06	0.53	1.11	0.58	Z	Z
2	1.07	0.53	1.11	0.58	X	X
3	3.26	1.57	3.26	1.71	Z	Z
4	3.32	1.60	3.26	3.26	X	X
5	6.5	3.29	6.5	3.39	Torsional	Z
6	7.15	3.44	7.69	3.4	Z	Torsional

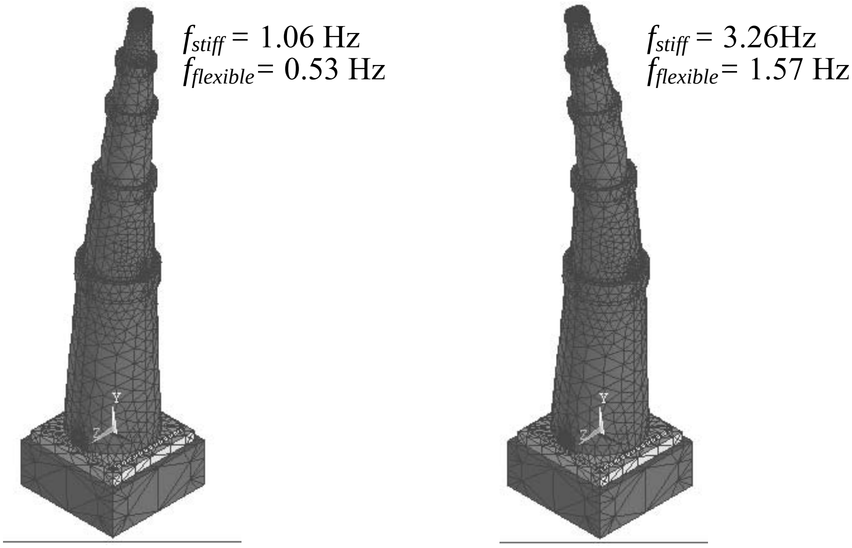


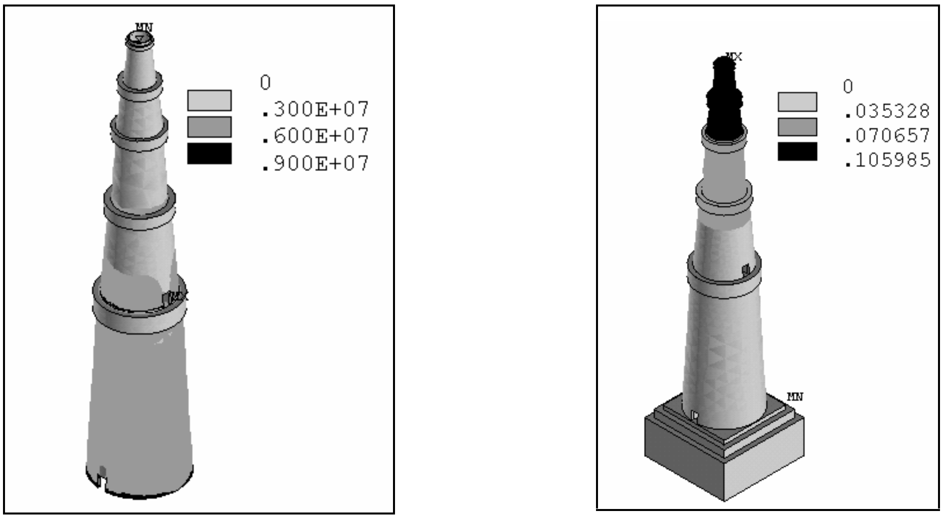
Figure 4 : First four mode shapes of the Minar obtained in the modal analysis of the solid model

5.2.2 Spectrum analysis

The Minar has been analysed for a loading of site-specific response spectrum and dead weight. The spectral load is applied along the global X direction. The deformation and principal tensile stress plots for the stiff material parameters are shown in Fig. 5.

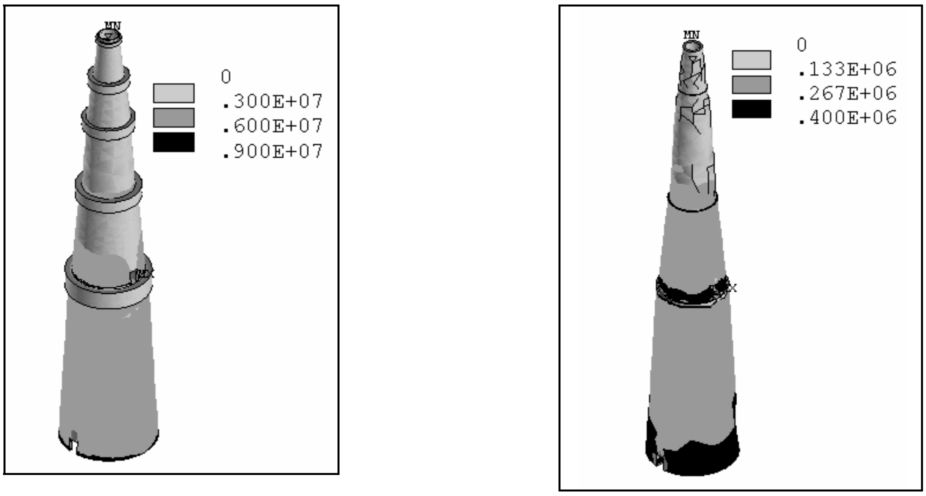
5.2.3 Discussions

As in the 3-D beam analysis higher tensile stress values are observed in the stiffer model. The flexible model showed comparatively smaller stress values. But the values are high considering the brittle nature of masonry. Very high stress values are observed in the facing masonry layers. The rubble masonry infill, which is of very less stiffness, exhibits the least stress levels. The maximum stress value is observed at the base of the second segment as in the case of the beam analysis. Stress concentrations are shown at the level of openings.



(a) Deformation plot (units in m)

(b) Principal tensile stress plot of the outer veneer (units in MPa)



(c) Principal tensile stress plot of the inner veneer (units in MPa)

(d) Principal tensile stress plot of the rubble infill (units in MPa)

Figure 5 : Response spectrum analysis results of 3-D solid model analysis

6 CONCLUSIONS

From the study it is clear that the Minar is potentially unsafe to an earthquake of considered intensity. Historic masonry, which is highly brittle in nature, cannot support tensile stresses. The material will fail once tensile stresses develop in the section. The non-realistic high tensile stress values obtained from the analysis are due to the linear elastic material mode assumption for masonry. Nevertheless the results indicate a possible potential collapse of the structure under an earthquake loading of considered intensity. The stiffer the Minar the more is the possibility of collapse. Under seismic events of reduced intensities a satisfactory behaviour of the Minar can be expected only if the natural frequency of the Minar falls in the flexible range (<0.5Hz) Further evaluations are required regarding the mechanical parameters of the Minar masonry in order to make comments on the design of seismic restoration.

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