

Non-Linear Dynamic Analyses for Seismic Assessment of Ancient Masonry Towers

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ABSTRACT: In 2002 the Minaret of *Jam* in Afghanistan was included in UNESCO's list of endangered world heritage monuments. The world's second tallest minaret is in a precarious state due to severe inclination and deterioration of masonry's mechanical strength. Probabilistic seismic hazard assessment (PSHA) of the site revealed a low hazard level with peak ground acceleration of 0.04g for a 475-year return period. Earlier investigations on the tower's seismic vulnerability under linear response showed that, despite low level of seismic hazard at Jam, current state of stresses at the minaret's base due to the inclination could be consistently increased by seismic loading probably initiating cracks that can threaten the tower's stability. This paper illustrates results of *non-linear* dynamic analyses of the tower which integrate findings from the previous study on linear analyses. Seismic input for the non-linear analysis was obtained from *deaggregation* of the outcome of the PSHA. This allowed identification of the controlling earthquake which was in turn used to select a suite of real accelerograms. The natural, strong-motion records were chosen by imposing spectral compatibility condition over the 475-year return period uniform hazard response spectrum from PSHA. Features of the non-linear model for performing time-history analyses are illustrated. Dynamic soil-structure interaction was taken into account through the sub-structuring method. Finally, the results from *non-linear* analyses are compared with those of the previous *linear* analyses.

1 INTRODUCTION

1.1 Seismic response of ancient masonry towers

Tall, ancient masonry towers have certain distinctive features. Firstly, their height and slenderness imply lack of redundancy for redistribution of stresses as well as lack or little energy dissipation along the structure. Their shape and material of construction are potential sources for concentration of stresses at their base. Their brittle behaviour is attributable to the prevailing vertical force coupled with the brittleness of deteriorated masonry (Macchi 1998). On the other hand, vis-à-vis their dynamic behaviour, a typical feature is the long natural period of vibration. The dynamic response is, therefore, confined to the descending branch of a response spectrum. However, whether this attribute would play a favourable role greatly relies on the seismic hazard at a site and on possible local site effects, and actual state of the structure and of its material. A combination of these contrasting features renders the assessment of seismic vulnerability of ancient masonry towers mandatory. This paper identifies various phases in this endeavour through the example of the Minaret of *Jam* highlighting recent non-linear structural investigations to corroborate earlier findings obtained within the realm of elastic analyses.

1.2 The Minaret of Jam in Afghanistan

The Minaret of *Jam* is located in a remote region in north-western Afghanistan, 260 km east of the city of Herat. The tower was erected between 1163 and 1203 AD during the reign of the Ghurid Dynasty. Situated in a river valley (see Fig. 1, left) at the junction of Hari-Rud and Jam-Rud rivers in the Hindukush range, the world's second tallest minaret (60.4 m) is of vital significance in the history of medieval Islamic civilization.

The minaret rises on an octagonal base that transforms into a tapering cylindrical shaft for the rest of its elevation. The original entrance of the tower is currently inaccessible lying below 4-6 m of alluvial deposits from the adjoining rivers. The inner chamber is accessible through a double helical stairway up to 40 m, supported on a solid, central shaft (see Fig. 1, right). Intricate motifs and inscriptions from the Koran in Kufic script are found on the façade.



Figure 1: (Left) The Minaret of *Jam* in the Hari-rud Valley (courtesy: A. Bruno). (Right) vertical section.

The monument is included in UNESCO's World Heritage List. Various threats have prompted the committee to incorporate it in the List of World Heritage in Danger since June 2002. The tower is inclined by 3.4° for a reason yet to be ascertained, but perhaps attributable to scouring by the rivers. The top of the tower is 3.35 m out-of-plumb.

Due to a combination of the lean and deteriorated masonry, the minaret is close to structural collapse purely under gravity loads. Therefore, the effect of even a moderate earthquake could be catastrophic for the tower, when the same could pass unnoticed for an ordinary structure. The 800 years of this minaret's survival may infuse a sense of optimism about its future. However, when the tower started leaning is not known. The temporal window over which the inclined minaret has been exposed to earthquakes is uncertain. Structural collapses of four, XIV century minarets in *Herat* since 1915 AD justifies a comprehensive assessment of the seismic vulnerability of the Minaret of *Jam* (Macchi 2004).

2 SEISMIC HAZARD ASSESSMENT AND DEFINITION OF SEISMIC INPUT

2.1 Seismotectonic setting and earthquake catalogues

Afghanistan lies on the southern fringe of the Eurasian plate and it is subjected to collision with the Arabian plate to the south and transpression with the Indian plate to the southeast (Ambraseys and Bilham 2003). Most parts of central and western Afghanistan lie in the interior of a wide deformation belt at the margins of the country and behave kinematically as rigid blocks. Such areas are characterized by relatively low seismicity. At the boundaries and within the Eurasian plate there are a number of morphologically predominant strike-slip lineaments.

Distribution of seismic activity in Afghanistan is spatially and temporally inhomogeneous. A catalogue of historical and instrumental seismicity of Afghanistan and neighbouring areas from 25 to 1975 AD (Quittmeyer and Jacob 1979) was available for the study in addition to a re-

cently compiled, updated and reevaluated catalogue of 1312 earthquakes from 734 to 2002 AD summarizing written history (Ambraseys and Bilham, 2003). The latter was the primary source of information for the seismic hazard study (Menon et al. 2004).

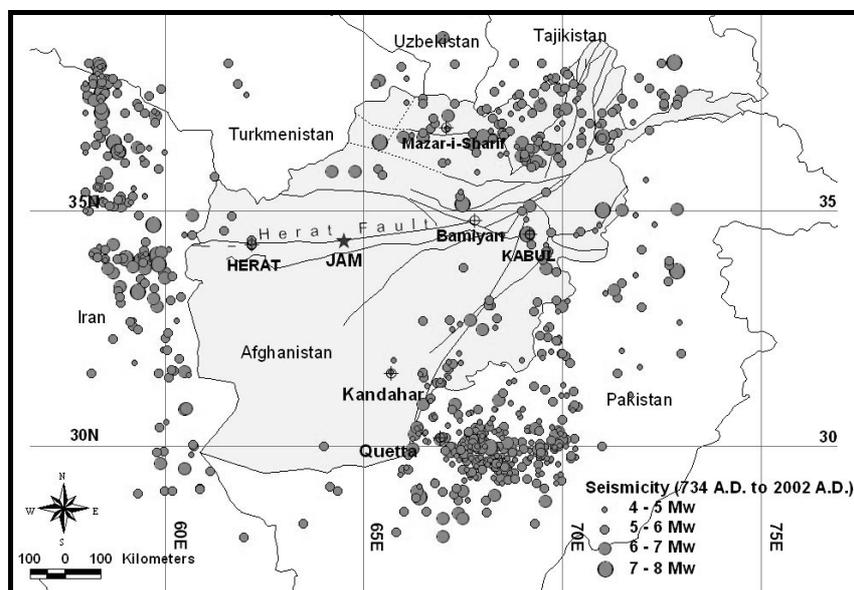


Figure 2 : Distribution of earthquakes in Afghanistan from 734 – 2002 AD (Menon et al. 2004).

Jam lies in a zone of relative seismic quiescence. Though located in close proximity to the *Herat* fault, a prominent strike-slip tectonic feature in northern Afghanistan, currently available data suggest that this lineament is characterized by a very low level of activity, if any. This inference is complemented by historical seismicity dating back to 734 A.D (Menon et al. 2004).

2.2 Methodology adopted for the definition of seismic input

A comprehensive Probabilistic Seismic Hazard Analysis (PSHA) at the archaeological site of *Jam* was performed by Menon et al. (2004). The scope of the seismic hazard analysis was definition of seismic input for linear, dynamic analyses of the minaret. PSHA was performed following the classical Cornell-McGuire approach incorporating a systematic treatment of uncertainties within the *logic-tree* framework. Alternative models for the logic tree branches were based on: a) seismogenic zoning scenarios, b) attenuation relationships and c) maximum earthquake magnitude. The analyses were performed using the computer code CRISIS99 whereas CRISIS2003 (Ordaz et al. 2003), a recent version, was used in this study to carry out the *deaggregation*. Real accelerograms from standard strong motion databases were selected by imposing a spectral compatibility condition over the 475-year return period uniform hazard spectrum obtained from the earlier study (Menon et al. 2004). Natural records are preferred over artificial accelerograms since the former are more realistic in terms of time duration, number of cycles and energy content in relation to seismogenic parameters. Besides, horizontal and vertical component of real accelerograms are characterized by a proper phase correlation.

2.3 Results from Probabilistic Seismic Hazard Analysis (PSHA)

The key results of the PSHA are shown in Fig. 3. The horizontal and vertical Peak Ground Accelerations (PGA) at *Jam* corresponding to a 10% probability of exceedance in 50 years (475-year return period) was found to be 0.041g and 0.024g, respectively. The values indicate *low seismic hazard* at *Jam* which is consistent with both the seismotectonic setting and historical seismicity of the region. Interested readers are referred to Menon et al. (2004) for a thorough description of the PSHA.

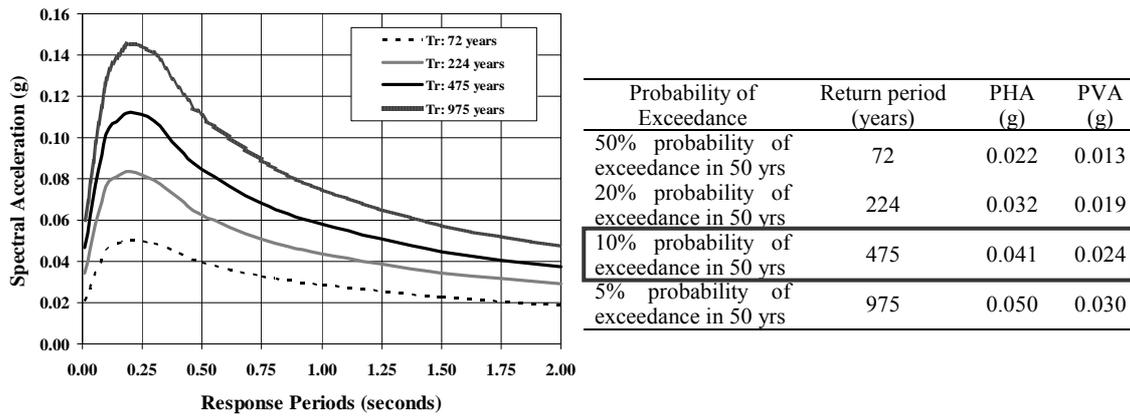


Figure 3 : Seismic input from PSHA at the archaeological site of *Jam*, Afghanistan (Menon *et al.*, 2004)

2.4 Deaggregation of seismic hazard

The mean annual rate of exceedance of a prescribed level of a ground motion parameter (e.g. PGA) computed in a PSHA is not explicitly associated with any deterministic scenario specified by a magnitude – source-to-site distance pair. Rather, it represents the contributions of different seismic sources to the hazard, each with its own probability distribution. The outcome of a PSHA can be profitably used to identify the seismic source (defined by a magnitude–distance pair) that contributes the most to the hazard at a site for a prescribed return period. This is done through a process called “deaggregation” and allows the definition of the “controlling earthquake” at the reference site (Reiter 1991). Deaggregation of hazard maybe performed for the PGA (intercept of a response spectrum at T=0) or for a spectral acceleration corresponding to any value of the structural period T.

Identification of magnitude–distance pairs controlling the hazard at the site enables selection of natural accelerograms from standard strong motion databases compatible with the regional seismogenic setting thus reflecting realistic earthquake scenarios for the site. These accelerograms can then be adopted for structural analyses.

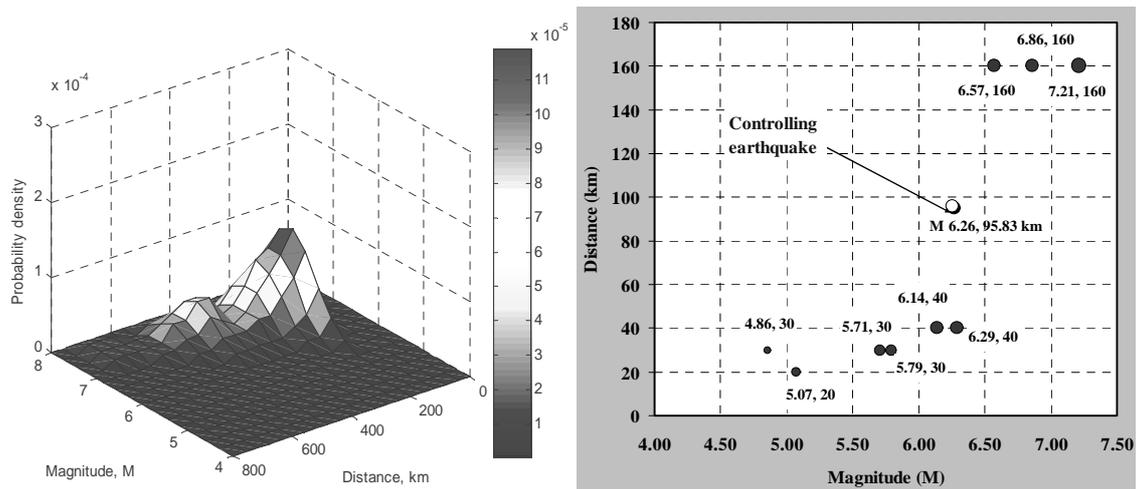


Figure 4 : Results of deaggregation for PGA (horizontal component) and return period of 475 years (logic tree branch from Menon *et al.*, 2004: seismogenic zoning based on seismicity + maximum historical magnitude + attenuation law by Abrahamson and Silva, 1997) and controlling earthquake at the site at *Jam*

Deaggregation of seismic hazard at *Jam* for the 475-year return period has been performed both for the PGA and spectral acceleration at T=1 sec, the latter being close to the fundamental period of vibration of the tower. The controlling magnitude-distance pairs resulted as M 6.26 at 96km and M 6.68 at 165km, respectively. Deaggregation was performed for alternative

branches of the logic tree developed by Menon et al. (2004). Resulting magnitude-epicentral distance pairs are plotted in Fig. 4.

2.5 Selection of natural accelerograms

Natural earthquake records on rock sites satisfying the criteria of the magnitude-distance pairs from deaggregation were selected from strong motion databases such as PEER, COSMOS and the ESD Strong Motion Databases¹. 30 accelerograms were chosen for the controlling pair of M 6.26 at 96 km (deaggregation of PGA) whereas a satisfactory set of records could not be found for the M 6.68 at 165 km pair (deaggregation of spectral acceleration at T=1 sec). Compatibility of the mean response spectrum of 7 records randomly chosen from combinations of the above 30 records with the probabilistic 475-year return period hazard spectrum determined the selection of the optimum set of accelerograms, i.e. with minimum misfit between the two spectra.

Table 1: Seismological features of the 7 spectrum-compatible real accelerograms chosen for 475-year RP

Record	Station ID/Source	Earthquake	M	R (km)	Site Geology	PGA (g)
1	ILA051 (PEER)	Chi Chi'99	M 7.6	R _{jb} 88.5	USGS Soil A	0.033 (N)
2	KAU077 (PEER)	Chi Chi'99	M 7.6	R _{jb} 95.6	USGS A	0.023 (N)
3	TAP067 (PEER)	Chi Chi'99	M 7.6	R _{jb} 104.1	USGS A	0.042 (N)
4	Hemet Fire Station (COSMOS)	Whittier Narrows, CA'87	M 6.1	R _{jb} 101.9	Deep alluvium	0.034 (360)
5	Atene-A. Puketapu (COSMOS)	Weber 1, NZ'90	M 6.2	R _{hyp} 120	Rock/very stiff	0.039 (N40E)
6	(COSMOS)	Weber 2, NZ'90	M 6.4	R _{hyp} 117	Rock/very stiff	0.073 (N40E)
7	Kiparisia-bank (ES)	Kalamata'97	M 6.4	R _{epi} 103	Rock	0.013

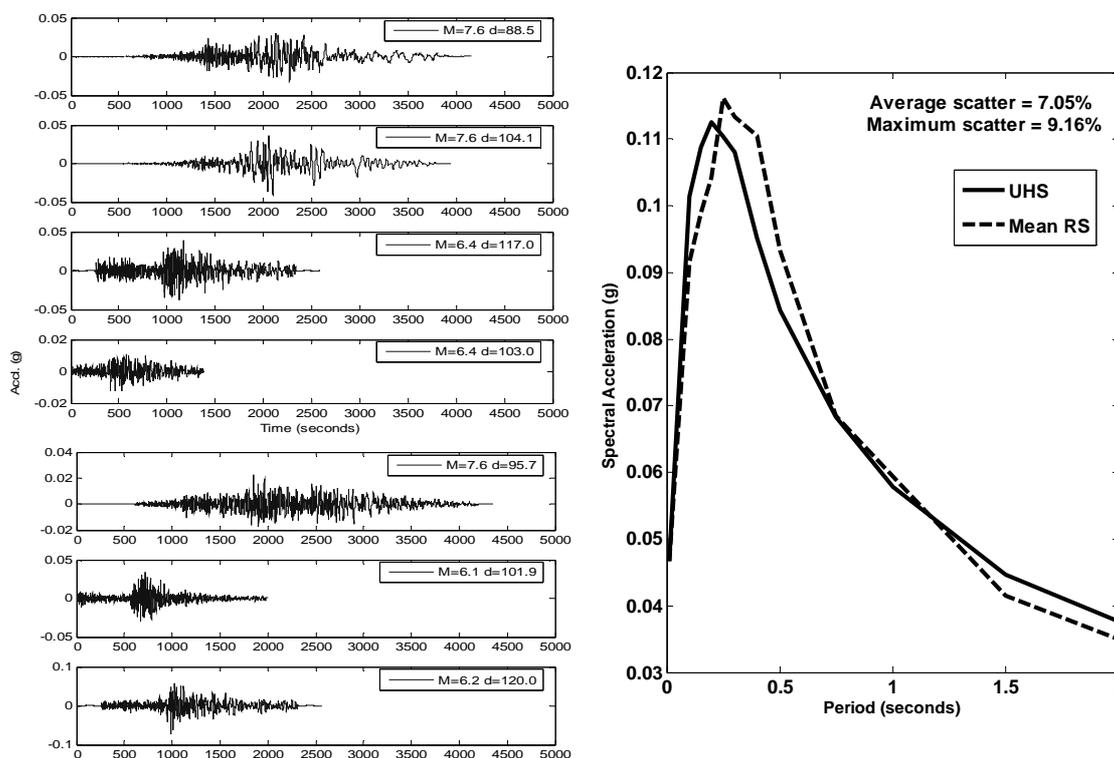


Figure 5: (Left) Suite of 7 natural accelerograms (horizontal component) selected with the criterion of compatibility with the 475-years probabilistic response spectrum. (Right) Comparison of the 475-years uniform hazard spectrum with the mean response spectrum of the 7 scaled set of selected accelerograms.

¹ <http://peer.berkeley.edu/smcat/>
<http://db.cosmos-eq.org/>
<http://www.isesd.cv.ic.ac.uk/>

The selected accelerograms are illustrated in Fig. 5 and the main seismological features are reported in Table 1. All the 7 records were scaled to the same PGA (0.041g) obtained from PSHA for the 475-year return period. According to Eurocode 8 (EN 1998-1 2004) the maximum allowable negative scatter between the two spectra is 10%. The mean response spectrum of the 7 selected accelerograms has an average and a maximum scatter compared to the probabilistic hazard spectrum of 7.05% and 9.16%, respectively. As shown in Fig. 5, the agreement between the two spectra is very good particularly around the fundamental period of the minaret ($T = 1$ sec). This suite of 7 accelerograms was therefore used to perform the non-linear dynamic, time-history analyses of the minaret.

3 STRUCTURAL ANALYSES AND SEISMIC VULNERABILITY ASSESSMENT

3.1 Results from previous studies

Originally, linear-elastic, lumped mass and 3D FE (shell) models of the minaret were developed, and modal and response spectrum analyses using the uniform hazard spectrum from the PSHA were performed (Lai et al. 2004). Seismic input pertaining to a 10% probability of exceedance in 50 years was chosen purely with the objective of demonstrating the effect of a low level ground motion on the state of stresses in the inclined tower. The stress distribution at the base of the minaret due to gravity and earthquake loads were estimated.

Solely under gravity load, the state of stress in the masonry at one end of the cross section at the base of the minaret (-6m and 0m levels) is close to zero (Fig. 5). The compressive stress at the other end is of the order of 1 MPa. Tensile stresses develop at the base of the minaret at the edge away from the inclination for a combination of earthquake and gravity effects. The authors concluded that the state of stress, due to the inclination, might be consistently amplified by seismic effects. That is, the tensile stresses may reach a value for which flexural cracks may be expected and hence a further increase of the inclination. Subsequently, a scheme for non-linear analyses was planned as a complementary phase of the seismic assessment with the objective of corroborating inferences from linear analyses.

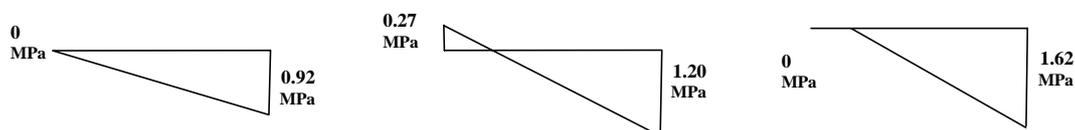


Figure 5 : Stress distribution at the base of the Minaret (0m) due to a) gravity load b) seismic load including gravity load c) due to seismic and permanent load after neglecting masonry tensile strength.

3.2 Non-linearity

Non-linear response of the minaret has been accounted for by means of a concentrated plastic hinge at the base of the minaret. A simple moment-rotation relationship has been calibrated for this plastic hinge. The moment-rotation relationship (Fig. 6) has been developed for the lower-most 6m of the tower by gradually increasing the eccentricity of a constant axial force on the base section. At every step, the moment and the corresponding rotation are calculated. As the eccentricity increases, progressively, one edge of the section experiences tensile stress. The cross sectional area under tensile stress is neglected and the moment of inertia of the remaining section about its new centre of gravity is calculated. The new moment and corresponding rotation are calculated. Initially the section, being circular, has negligible reduction in cross section, but soon after there is a significant reduction of area with increasing eccentricity. The procedure is continued until the compressed end fibre of the section reaches an ultimate strain of ϵ_u 0.003. Failure of the section in compression can be expected at this point.

The motivation behind lumping plastic deformation at a single level arises from the following rational considerations. The base section of the minaret, due to the inclination, is close to a state of zero stress at one end. Bearing in mind, brittleness of ancient masonry, any increase of tensile stress under this condition would imply possible initiation of flexural cracks. Once cracking initiates at a specific section, any increase in unfavourable loading will lead to widening of the

cracks until a state when the crack jeopardises the tower’s stability. Therefore, cracking is basically confined to this section and not spread as in reinforced concrete. Maximum rotation is concentrated at the height of the cracked section. Sudden collapse can be expected when the compressed end reaches the compression limit of masonry (~ 4MPa for the masonry of this minaret). This concept is substantiated by observations of damage and advancement of damage in Minaret 5 at Herat in Afghanistan, which is 2.70m out-of-plumb and has a large horizontal crack at a height of 3m from the base. The cross section is cracked for about one-third of its diameter and the edge of the masonry is subjected to high compression (1.20 MPa). The current inclination could be due to several successive earthquakes but currently available information suggests that the tower was out-of-plumb only by 0.90m in 1977 (Macchi 2004).

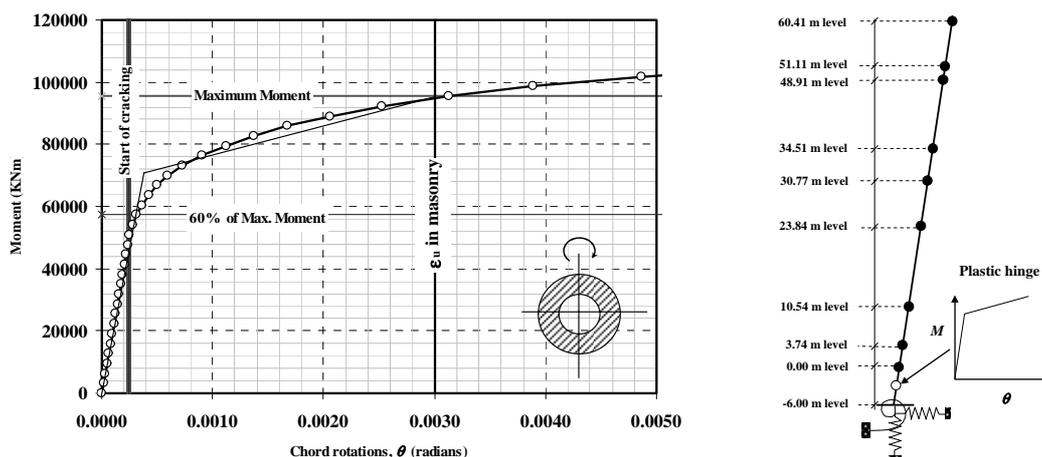


Figure 6 : (Left) Moment-rotation behaviour of the base shaft of the minaret with bilinear idealization and (Right) the lumped mass model of the minaret

In the lumped mass model of the tower, an idealised bilinear moment-rotation curve is defined at the plastic hinge at mid height of the lowermost element. All the other elements of the model are intended to remain elastic. Structural analysis was performed using the numerical code SeismoStruct. Global sources of geometric non-linearity (P-Δ, large displacement/rotation effects) are accounted for albeit the use of the elastic non-linear frame element. A co-rotational formulation, where the local element displacements and resulting internal forces are defined with respect to a moving local chord system, is employed. Exact transformation of element internal forces and stiffness matrix obtained in the local chord system into the global coordinates permits the large displacements and rotations to be accounted for (Seismosoft 2003).

3.3 Dynamic soil-structure interaction

Table 2 : Stiffness and damping coefficients calculated with DYNA4 for the two foundation soil scenarios

DOF	$V_s = 1000 \text{ m/s}$		$V_s = 200 \text{ m/s}$	
	STIFFNESS	DAMPING	STIFFNESS	DAMPING
X	8.515E+07 kN/m	1.508E+06 kN/m/s	3.404E+06 kN/m	1.945E+05 kN/m/s
Y	8.515E+07 kN/m	1.508E+06 kN/m/s	3.404E+06 kN/m	1.945E+05 kN/m/s
Z	7.475E+07 kN/m	1.477E+06 kN/m/s	3.097E+06 kN/m	1.996E+05 kN/m/s
RX	2.244E+09 kN.m/rad	2.176E+07 kN.m/rad/s	8.884E+07 kN.m/rad	1.626E+06 kN.m/rad/s
RY	2.244E+09 kN.m/rad	2.176E+07 kN.m/rad/s	8.884E+07 kN.m/rad	1.626E+06 kN.m/rad/s
RZ	4.501E+09 kN.m/rad	3.706E+07 kN.m/rad/s	1.784E+08 kN.m/rad	1.941E+06 kN.m/rad/s

$\rho = 1.9 \text{ Mg/m}^3$ (Mass density), $\nu = 0.35$ (Poisson's ratio), $\beta = 0.02$ (Damping ratio)

Given that the reason for the inclination of the minaret still remains unexplained and given the lack of geotechnical data from the site, the role of foundation soil in the dynamic response of the minaret was tackled by assuming two distinct scenarios: soft soil (V_s 200m/s) and stiff soil (V_s 1000m/s). Dynamic soil structure interaction (DSSI) was accounted for by the sub-

structuring method with soil response modelled as frequency dependent foundation stiffness and dashpot damping coefficients. DSSI was performed using the numerical code DYNA4 (Novak *et al.*, 1993) for an embedded foundation on homogeneous half space. The spring stiffness and dashpot damping constants for the 3 translational and 3 rotational degrees of freedom are defined at the base node of the lumped mass model. These values are reported in Table 2.

3.4 Time-history analysis

Seven dynamic non-linear time-history analyses (THA) have been performed for each foundation scenario with the 7 accelerograms illustrated in Fig. 5 scaled to 0.041g. For all the seven THA for the stiff soil case, plastic rotation at the base hinge is noticed, whereas for the soft soil case, all, except that with the Whittier Narrows record enters the non-linear range. Rotation at the plastic hinge corresponding to ultimate strain of masonry does not occur in any THA. Maxima of the response quantities have been averaged for 7 THA following the prescriptions of Eurocode 8 (EN 1998-1, 2004). Salient aspects of the results of THA are discussed below:

- Averaged maximum rotation demand at the height of the plastic hinge for the stiff soil case is 0.00137 radians (includes rotation due to the lean), whereas 0.00098 radians for the soft soil case. Cracking in the calibrated hinge initiates at 0.00025 radians.
- Averaged maximum displacement demand at the top of the minaret is 0.153m and 0.135m for the stiff soil and soft soil cases, respectively. The values are in addition to the inclination-induced deformation at the top (0.059m and 0.074m, respectively).
- Averaged maximum overturning moment induced by seismic load for the stiff soil case is 125% of the bending moment at the base due to the inclination (41,280 kNm) and 98% of the moment due to inclination (41,432 kNm) for the soft soil case.
- Significant oscillations at the top storey with frequency components corresponding to higher modes are noticed. Consequently, local damage can be expected at this level. Amplitudes of oscillations are appreciably reduced for the soft soil cases.

3.5 Discussions

From linear response spectrum analyses, the maximum displacements at the top of the tower due to the seismic load were 0.048m and 0.057m for the stiff and soft soil cases, respectively. Stress at the base of the tower, for the stiff soil case, at two extremities of the section was 0.27 MPa (tension) and 1.2 MPa (compression) for gravity plus seismic load (Menon *et al.* 2004).

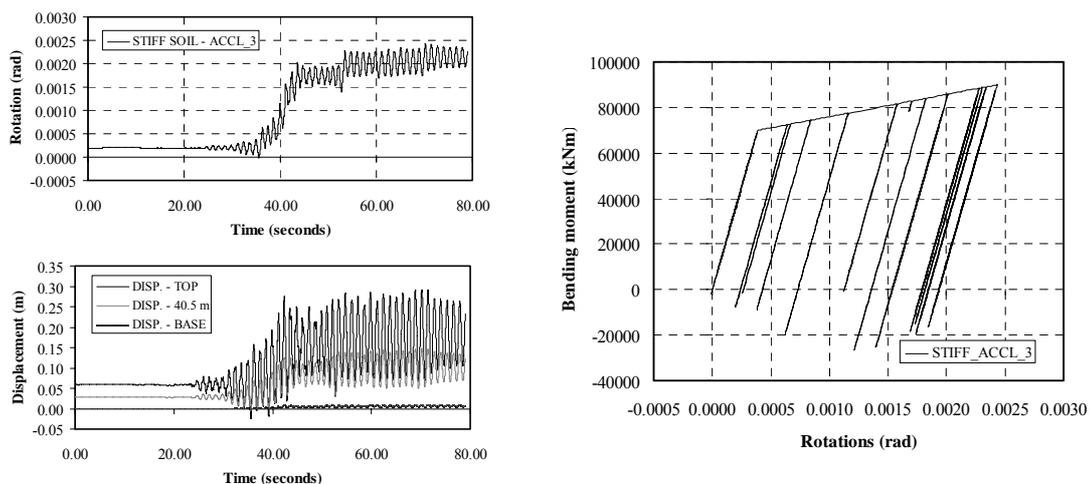


Figure 7 : Rotation history and hysteresis at plastic hinge, displacement history at 0m, 40.5m 60.4m levels

The accelerograms scaled to the 475-year return period PGA (0.041g) are capable of producing non-linear response in the minaret. Only for a single case an elastic response to the ground motion is seen. From the rotation time-histories in Figs. 7 and 8 it is seen that residual deforma-

tion can be expected once non-linear cycles begin. That is, crack opening and subsequent widening is imminent. Higher displacements at the tower's apex can be expected in case of crack formation at the base. For the singular case of elastic response in the THA, the maximum top displacement is 0.022m, consistent with the prediction of the response spectrum analysis.

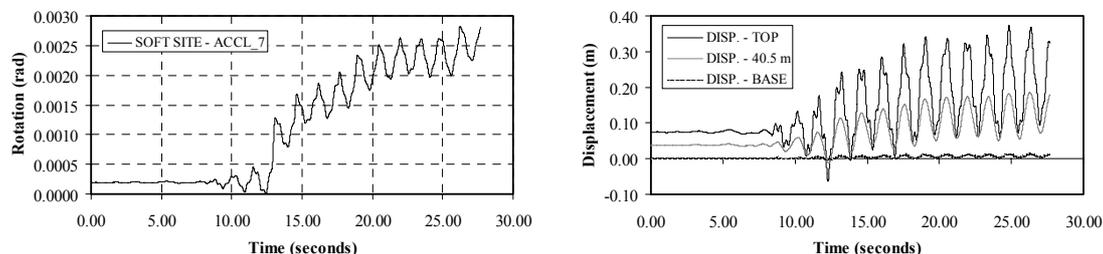


Figure 8 : Rotation history at plastic hinge and displacement history at 0m, 40.5m 60.4m levels

4 CONCLUSIONS

Subsequent to a two-phased study on the seismic vulnerability of the Minaret of *Jam* that dealt with seismic hazard analysis on the one hand and structural assessment within the elastic range on the other, the current paper deals with non-linear dynamic analyses. Deaggregation of the seismic hazard and identification of the controlling earthquake at *Jam* aid in selecting natural accelerograms to serve as input to time-history analyses. Spectral compatibility with the uniform hazard spectra from PSHA is imposed in this selection. The concept and details of the simplified non-linear model apart from the results of the time-history analyses were discussed.

The non-linear time-history analyses substantiate the conclusions drawn from the preceding phase based on linear-elastic analyses. Though seismic hazard at *Jam* is characterised by low level of exposure, the state of stress at the base of the minaret due to the inclination, is amplified by the seismic loading and tensile stresses reach a value for which formation of flexural cracks may be imminent with potential danger to the minaret's stability.

Despite relatively low seismic hazard at *Jam*, the occurrence of a "characteristic earthquake" with very long return period triggered by the *Herat* fault (though currently available data seems to support the antithesis), could devastate the tower due to its proximity to the fault. The authors recall an analogous event involving the Citadel of Bam in Iran in December 2003.

REFERENCES

- Abrahamson N.A. & Silva W.J. 1997. Empirical Response Spectral Attenuation Relations for Shallow Crustal Earthquakes. *Seismological Research Letters* 68 (n. 1), pp. 199-222.
- Ambraseys, N. N. & Bilham, R. 2003. Earthquakes in Afghanistan, *Seism. Res. Letters*, 74, pp. 107-123.
- EN 1998-1 2004. Eurocode 8: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings. European Committee for Standardization. Brussels, 229 pp.
- Lai, C.G., Menon, A. & Macchi, G. 2004. Seismic vulnerability of the Minaret of Jam in Afghanistan. XI National Congress of Seismic Engineering in Italy. ANIDIS 2004, Genoa, (electronic format).
- Macchi, G. 1998. Seismic Risk and Dynamic Identification in Towers. Keynote Lecture, Proc. Monument - 98, *Workshop on Seismic Performance of Monuments*, Lisbon, Portugal, pp. K7-K17.
- Macchi, G. 2004. Saving minarets at risk in Afghanistan. Proc. Structural Analysis of Historical Constructions - Modena, Lourenço & Roca (eds), pp. 1375-1382.
- Menon, A., Lai, C.G. & Macchi, G. 2004. Seismic hazard assessment of the historical site of Jam in Afghanistan and stability analysis of the minaret. *Journal of Eq. Engg.*, Vol.8, SP 1, pp. 251-294.
- Novak, M., Sheta M, El-Hifnawy, L, El-Marsafawi, H, Ramadan, O. 1993. DYN4: Computer Program for Calc. Foundation Response to Dynamic Loads. Geotech. Res. Centre, Univ. W. Ontario, Canada.
- Ordaz, M., Aguilar, A. & Arboleda, J. 2003. CRISIS2003 - Version 3.0.1: Program for Computing Seismic Hazard, Instituto de Ingenieria, UNAM, Mexico.
- Quittmeyer, R. C. & Jacob, K. H. 1979. Historical and modern seismicity of Pakistan, Afghanistan, NW India and SE Iran, *Bulletin of the Seismological Society of America*, 69 [3], pp. 773-823.
- Reiter, L. 1991. Earthquake Hazard Analysis: Issues and Insights. Columbia University Press, pp. 254.

SeismoSoft. 2003. "SeismoStruct - A Computer Program for Static and Dynamic Nonlinear Analysis of Framed Structures" [online]. Available from URL: <http://www.seismosoft.com>.